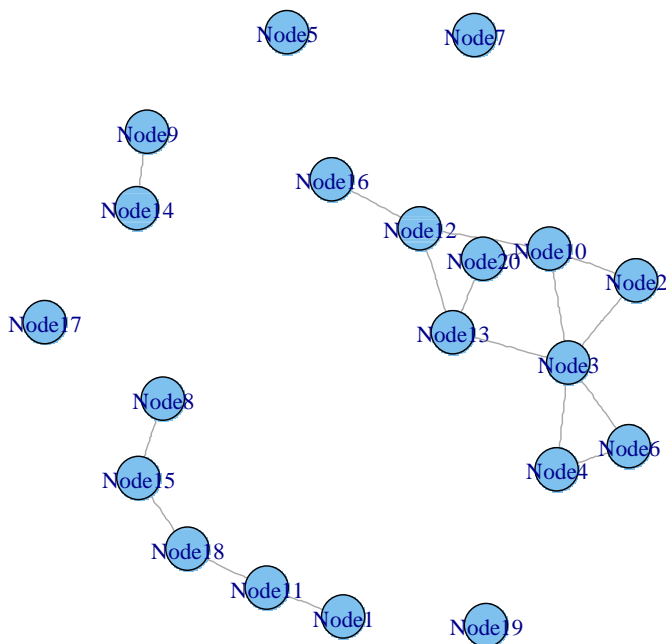# Problem 7: Friends and Smokers

Consider the following graph (available as `problem-7-friends.csv`):

Define a Markov Random Field as follows:

People have a prior probability of smoking of 0.2

All else being equal, it is 3 times as likely that two friends have the same smoking habit than different.

$$P(S) \propto \exp\left[\sum_i \phi(s_i) + \sum_{(i,j)\in Friends} \psi(s_i, s_j)\right]$$

$$\phi(s_i = 1) = \log 0.2 = -1.60943$$

$$\phi(s_i = 0) = \log 0.8 = -0.22314$$

$$\phi(s_i = 0, s_j = 0) = 0.549306$$

$$\phi(s_i = 1, s_j = 1) = 0.549306$$

$$\phi(s_i = 1, s_j = 0) = -0.549306$$

$$\phi(s_i = 0, s_j = 1) = -0.549306$$

Query: Compute the marginal probability of smoking for each of the unobserved nodes given the six observed nodes:

Node	Observation of Smokes
1	1
2	
3	
4	0
5	
6	0
7	
8	1
9	1
10	
11	
12	
13	
14	
15	1
16	
17	
18	
19	
20	

Additional graphs can be generated using the code in `problem-7-generator.R`. This file also contains a very slow implementation of Gibbs sampling as a baseline method.

Query 1: For each of the unobserved nodes, compute the posterior marginal probability that that person is a smoker.

Metric 1: The sum, over all of the queried nodes, of the absolute difference between the computed and the true posterior probability.