Filtered Latent Dirichlet Allocation: Variational Bayes Algorithm

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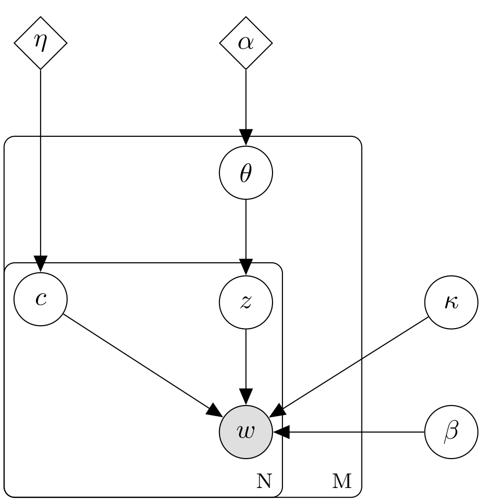
July 13, 2016

Filtered latent Dirichlet allocation is a natural extension of latent Dirichlet allocation that organically captures and filters out corpus-specific stop-words which are not otherwise caught by generic stop-word lists.

Probabilistic Graphical Model:

$$
\begin{aligned}
&\theta_d && \sim \text{Dirichlet}(\alpha) && \\
&c_{d_n} && \sim \text{Binomial}(\eta) && \\
&z_{d_n} && \sim \text{Categorical}(\theta_d) && \\
&w_{d_n} && \sim \text{Categorical}(\kappa) && \textbf{if } c_{d_n} = 0 \\
&w_{d_n} && \sim \text{Categorical}(\beta_{z_{d_n}}) && \textbf{if } c_{d_n} = 1
\end{aligned}
$$



The mean-field lower bound for the fLDA marginal log-likelihood, in expected value form, is:

$$\log p(w|\alpha,\beta,\kappa,\eta) \geq \sum_{d=1}^{M} \Big[\mathrm{E}_q[\log p(w_d, z_d, \theta_d|\alpha,\beta,\kappa,\eta)] - \mathrm{E}_q[\log q(z_d, c_d, \theta_d|\phi_d, \tau_d, \gamma_d)] \Big]$$

$$= \sum_{d=1}^{M} \Big[\mathrm{E}_q[\log p(w_d|z_d, c_d, \beta, \kappa)] + \mathrm{E}_q[\log p(z_d|\theta_d)] + \mathrm{E}_q[\log p(c_d|\eta)] + \mathrm{E}_q[\log p(\theta_d|\alpha)]$$
$$- \mathrm{E}_q[\log q(z_d|\phi_d)] - \mathrm{E}_q[\log q(c_d|\tau_d)] - \mathrm{E}_q[\log q(\theta_d|\gamma_d)] \Big]$$

In particular:

- $$\begin{aligned}\sum_{d=1}^{M} \mathrm{E}_q[\log p(w_d|z_d, c_d, \beta, \kappa)] &= \sum_{d=1}^{M}\sum_{n=1}^{N_d} \mathrm{E}_q[\log p(w_{d_n}|z_{d_n}, c_{d_n}, \beta, \kappa)] \\ &= \sum_{d=1}^{M}\sum_{n=1}^{N_d} \mathrm{E}_q[\log(\beta_{z_{d_n}, w_{d_n}}^{c_{d_n}} \cdot \kappa_{w_{d_n}}^{1-c_{d_n}})] \\ &= \sum_{d=1}^{M}\sum_{n=1}^{N_d} \big[\mathrm{E}_q[c_{d_n}] \cdot \mathrm{E}_q[\log \beta_{z_{d_n}, w_{d_n}}] + \mathrm{E}_q[1 - c_{d_n}] \log \kappa_{w_{d_n}}\big] \\ &= \sum_{d=1}^{M}\sum_{n=1}^{N_d} \big[\tau_{d_n}(-\log \kappa_{w_{d_n}} + \sum_{i=1}^{K} \phi_{d_{in}} \log \beta_{i w_{d_n}}) + \log \kappa_{w_{d_n}}\big]\end{aligned}$$
- $$\sum_{d=1}^{M} \mathrm{E}_q[\log p(c_d|\eta)] = \sum_{d=1}^{M} \big[N_d \log(1-\eta) + \log(\frac{\eta}{1-\eta}) \sum_{n=1}^{N_d} \tau_n\big]$$
- $$\sum_{d=1}^{M} \mathrm{E}_q[\log q(c_d|\tau_d)] = \sum_{d=1}^{M}\sum_{n=1}^{N_d} \big[\tau_{d_n} \log \tau_{d_n} + (1-\tau_{d_n})\log(1-\tau_{d_n})\big]$$

The remaining expected values are identical to those found in Blei's paper, *Latent Dirichlet Allocation* (2003).

The relevant update equations are as follows:

- $\eta = \frac{\sum_{d=1}^{M} \sum_{n=1}^{N_d} \tau_{d_n}}{\sum_{d=1}^{M} N_d}$

- $\kappa_j \propto \sum_{d=1}^{M} \sum_{n=1}^{N_d} (1 - \tau_{d_n}) w_{d_n}^{j}$

- $\tau_{d_n} = \frac{\eta}{\eta + (1 - \eta)\kappa_{w_{d_n}} \prod_{i=1}^{K} \beta_{iw_{d_n}}^{-\phi_{d_{in}}}}$

- $\phi_{d_{in}} \propto \beta_{iw_{d_n}}^{\tau_{d_n}} \exp\big(\psi(\gamma_i) - \psi(\sum_{l=1}^{K} \gamma_l)\big)$

- $\beta_{ij} \propto \sum_{d=1}^{M} \sum_{n=1}^{N_d} \tau_{d_n} \phi_{d_{in}} w_{d_n}^{j}$