

1. $A=B^{-1}$, $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$, $x+y=?$

$A \cdot A^{-1} = I_n$ $\rightarrow \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} \cdot \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{cases} 3x-5=1 \text{ I,} \\ xy+10=0 \text{ II,} \end{cases}$

I $3x-5=1$ II $xy+10=0$ $xy=2+(-5)$
 $3x=1+5$ $2y=-10$ $=2-5=-3$
 $3x=6$ $y=-10 \div -5$
 $x=6 \div 3 = 2$ 2 $R = \text{alternativa C}$

2. $K=?$, $A = \begin{vmatrix} 1 & 0 & 1 \\ K & 1 & 3 \\ 1 & K & 3 \end{vmatrix}$ ($\det=0$)

$1+3K+0 = 3K+1$

$\det A = \begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ K & 1 & 3 & K & 1 \\ 1 & K & 3 & 1 & K \end{vmatrix}$
 $= K^2+3-(3K+1)$
 $= K^2+3-3K-1$
 $= K^2-3K+2=0$
 $3+0+K^2=K^2+3$

$K^{\text{II}} = \frac{3 \pm \sqrt{1}}{2}$
 $K^{\text{II}} = \frac{3-1}{2}$

$\Delta = 9 - 4 \cdot 1 \cdot 2$
 $\Delta = 9 - 8 = 1$

$K^{\text{II}} = \frac{3+\sqrt{1}}{2} = \frac{3+1}{2} = 4 \div 2 = 2$

$K^{\text{II}} = \frac{2}{2} = 1$

Resposta: alternativa C

$$3. B = A^{-1}, A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} : 2$$

$$\det A = 12 - 10 = 2$$

$$B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$$

Respuesta: alternativa c

$$4. \begin{bmatrix} x & 1 & 2 \\ 3 & 1 & 2 \\ 10 & 1 & x \end{bmatrix} \begin{bmatrix} x & 1 \\ 3 & 1 \\ 10 & 1 \end{bmatrix}$$

$$20 + 2x + 3x = 20 + 5x \quad x^2 + 20 + 6 = x^2 + 26$$

$$\det = x^2 + 26 - (20 + 5x)$$

$$x^2 + 26 - 20 - 5x = 0$$

$$x^2 - 5x + 6 = 0$$

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$$\Delta = 25 - 4 \cdot 1 \cdot 6$$

$$\Delta = 25 - 24 = 1$$

$$x^2 - 5x + 6 = 0$$

$$x^2 - 5x + 6 = 0$$

Respuesta: alternativa a

$$5. A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix}, A + A^{-1} = ? \quad \det A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$2 \cdot 2 + 2 = 6$$

$$1 + 2 + 4 = 7$$

$$A = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} (-1)(-2) & (-1)(-2) & (2)(-1) \\ (1)(-2) & (1)(-2) & (-1)(-1) \\ (2)(-2) & (2)(-4) & (-1)(-2) \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\bar{A} = (A^{-1})^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$^{-1} = \bar{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A + A^{-1} = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

Resposta: alternativa b

$$b. (x, A)^T = B$$

$$((x, A)^T)^T = B^T$$

$$x, A = B^T$$

$$x, A, A^{-1} = B^T, A^{-1} \rightarrow A x = B^T, A^{-1}$$

Resposta: alternativa b

$$7. B = \begin{bmatrix} x \\ y \end{bmatrix}, C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}, A^{-1} = ?, AB = C$$

$$A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix} \rightarrow A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \quad \det A = 24 - 25 = -1$$

$$A^{-1} = \begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} : \det A = -1 \rightarrow A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix} \quad \text{Resposta: alternativa d}$$

$$8. A = \begin{bmatrix} 2 & k \\ -2 & 1 \end{bmatrix} \quad k = ?, \det A = \det A^{-1}$$

$$\det A = 2 - (-2k) = 2 + 2k$$

$$\det A, \det A^{-1} = 1$$

$$4k^2 + 8k + 4 - 1 = 0$$

$$(2 + 2k)(2 + 2k) = 1$$

$$4k^2 + 8k + 3 = 0$$

$$4 + 4k + 4k + 4k^2 = 1$$

$$\Delta = 64 - 4 \cdot 4 \cdot 3$$

$$k^I: \frac{-8 + 4}{8} = \frac{-4}{8} = -\frac{1}{2}$$

$$k^{II}: \frac{-8 - 4}{8} = \frac{-12}{8} = -\frac{3}{2}$$

$$\Delta = 64 - 48 = 16$$

$$k = \frac{-1}{2} - \frac{3}{2} = \frac{-4}{2} = -2 \quad \text{Resposta: alternativa b}$$

g. $A_{2 \times 2}, B_{2 \times 2}, \det A \neq 0, \det B \neq 0$

a. $(A+B)(A-B) = A^2 - AB + BA - B^2$ ($AB \neq BA$)

b. $(A+B)^2 = A^2 + 2AB + B^2 \rightarrow AB = BA$

c. $\det(A)$

$\det(-A) \quad \det(-A) = (-1)^2 \det A = \det A \quad \det A = 1$
 $\det(-A)$

d. $B = A^{-1} \quad \det A, \det B = 1 \rightarrow \det B = \frac{1}{\det A}$