Clamped-Free Bar Model for ARCH-COMP 2021 AFF category (draft)

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1 Model description

We consider a uni-dimensional wave propagation problem from [1]. The problem is governed by the partial differential equation

$$EA\frac{\partial^2 u}{\partial x^2}(x,t) - \rho A\frac{\partial^2 u}{\partial t^2}(x,t) = 0, \quad (1)$$

where $u(x,t)$ is the displacement of the point in position x at time t, considering the axis shown in Figure 1. The model consists of a bar of length $L = 200$ and cross-section area $A = 1$. The bar is formed by a linear elastic material with Young modulus $E = 30 \times 10^6$ and density $\rho = 7.3 \times 10^{-4}$.

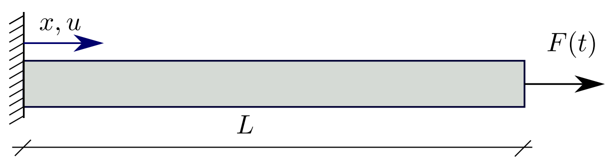

Figure 1: Example 2: diagram of the clamped-free bar excited by end load

The bar is considered to be initially at rest, with $u(x,0) = 0$ and $\frac{\partial u}{\partial t}(x,0) = 0$ for all $x \in [0,L]$. The boundary conditions are $u(0,t) = 0$, corresponding to the fixed end, and $\sigma(L)A = F(t)$, for the free end, where $\sigma(x,t) = E\dfrac{\partial u}{\partial x}(x,t)$. The free end is submitted to a step force $F(t) = 10{,}000H(t)$, where $H(t)$ is the heaviside function.

The analytical solution of this problem in the continuum can be obtained using mode superposition [2] and it is given by:

$$u(x,t) = \frac{8FL}{\pi^2 EA}\sum_{s=1}^{\infty}\left\{\frac{(-1)^{s-1}}{(2s-1)^2}\sin\frac{(2s-1)\pi x}{2L}\left(1-\cos\frac{(2s-1)\pi\mu t}{2L}\right)\right\}, \quad (2)$$

where $\mu = \sqrt{E/\rho}$.

Considering N two-node finite elements with linear interpolation, the following system of linear ODEs is obtained:

$$M\frac{\partial^2 u(t)}{\partial t^2} + Ku(t) = F(t), \qquad t \in [0, T], \tag{3}$$

where the $N \times N$ stiffness and mass matrices are respectively:

$$K = \frac{EA}{\ell}\begin{bmatrix} 2 & -1 & & & & 0 \\ -1 & 2 & -1 & & & \\ & -1 & 2 & \ddots & & \\ & & \ddots & \ddots & -1 & \\ & & & -1 & 2 & -1 \\ 0 & & & & -1 & 1 \end{bmatrix}, \quad M = \frac{\rho A \ell}{2}\begin{bmatrix} 2 & & & & & 0 \\ & 2 & & & & \\ & & 2 & & & \\ & & & \ddots & & \\ & & & & 2 & \\ 0 & & & & & 1 \end{bmatrix},$$

where $\ell = L/N$ is the length of each element. In this problem it is assumed that no damping is present.

2 Reachability settings

References

[1] Mohammad Mahdi Malakiyeh, Saeed Shojaee, and Klaus-Jürgen Bathe. The bathe time integration method revisited for prescribing desired numerical dissipation. *Computers & Structures*, 212:289–298, 2019.

[2] Michel Géradin and Daniel J Rixen. *Mechanical vibrations: theory and application to structural dynamics*. John Wiley & Sons, 2014.