$\underline{\underline{n=2}}$

$$X = [a, b]^T, \quad X^{\otimes 2} = X^{[2]} = [\underset{\begin{pmatrix}2\\0\end{pmatrix}}{a^2}, \underset{\begin{pmatrix}1\\1\end{pmatrix}}{ab}, \underset{\begin{pmatrix}1\\1\end{pmatrix}}{ba}, \underset{\begin{pmatrix}0\\2\end{pmatrix}}{b^2}]^T$$

$I_{n,m}$: set of n-tuples of deg m

Total order : lex $\quad I_{\underset{\text{"variables"}}{2},\underset{\text{"degree"}}{2}} = \{ (0,2) < (1,1) < (2,0) \}$

1st entry, 2nd entry etc.

reduced

$$x^{<2>} = [a^2, ab, b^2]^T$$

$f : \mathrm{Monom}_{n,m} \to \mathcal{I}_{n,m}$

$x_1^m \longrightarrow (m, 0, \ldots, 0)$

$x_1^{m-1} x_2 \longrightarrow (m-1, 1, 0, \ldots, 0)$

$\vdots$

0

$F_1 = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}, \quad F_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & -2.2 & 0 & 0 \end{pmatrix}$

$$A = \begin{pmatrix} F_1 & F_2 \\ 0 & \tilde{F}_1 \end{pmatrix} \begin{matrix} \}\ 2 \\ \}\ 4 \end{matrix}$$

Non-compressed

$\underbrace{}_{X} \quad \underbrace{}_{X \otimes X}$ n

$$\tilde{F}_1 = F_1 \otimes I + I \otimes F_2$$

$N = 1 \to$ Return F_1

~~fn~~ L35, L47

loop invariants

Sum(m) const?

$$\begin{array}{c} 2 \\ 3 \end{array}\left(\begin{array}{cc} \boxed{\bar{F}_1^{\,c}} & \boxed{\bar{F}_2^{\,c}} \\ \boxed{O_{3\times 2}} & \boxed{\tilde{F}_1^{\,c}} \\ \underbrace{\hphantom{O_{3\times 2}}}_{2} & \underbrace{\hphantom{\tilde{F}_1^{\,c}}}_{3} \end{array}\right)\begin{array}{c} 2 \\ 3 \end{array}$$

$$\bar{F}_1 = \bar{F}_1^{\,c} = \begin{pmatrix} 3 & 0 \\ 1 & -1 \end{pmatrix}$$

$$\bar{F}_2^{\,c} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & -2.2 & 0 \end{pmatrix}$$

$$\bar{F}_2 = \;-\,-$$