Robust control design by scenario optimization

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STOCHASTIC OPERATIONS RESEARCH

Acknowledge Dr Luis G. Crespo, Dr Sean P. Kenny for the kind support

B. Wie and D. S. Bernstein, "*Benchmark Problems for Robust Control Design*," in 1991 American Control Conference, 1991

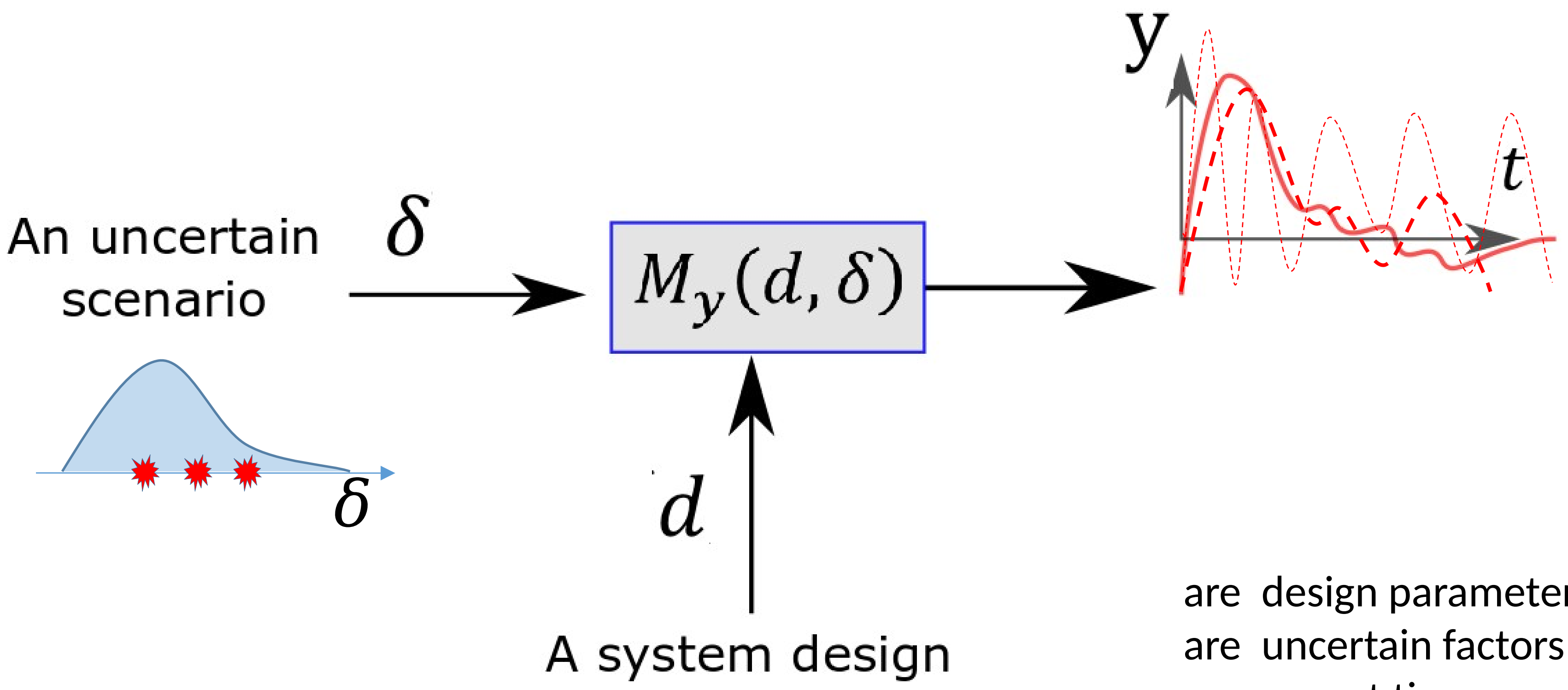
Problem statement

Given a numerical model of a dynamic system a reliability model and a set of of experimental observations

We want to find a reliable design that maximizes the probability of satisfactory performance (reliability) while avoids catastrophic failures (robustness against worst-case scenarios)

In contrast with , the random factors are inherently variable and non-tunable.

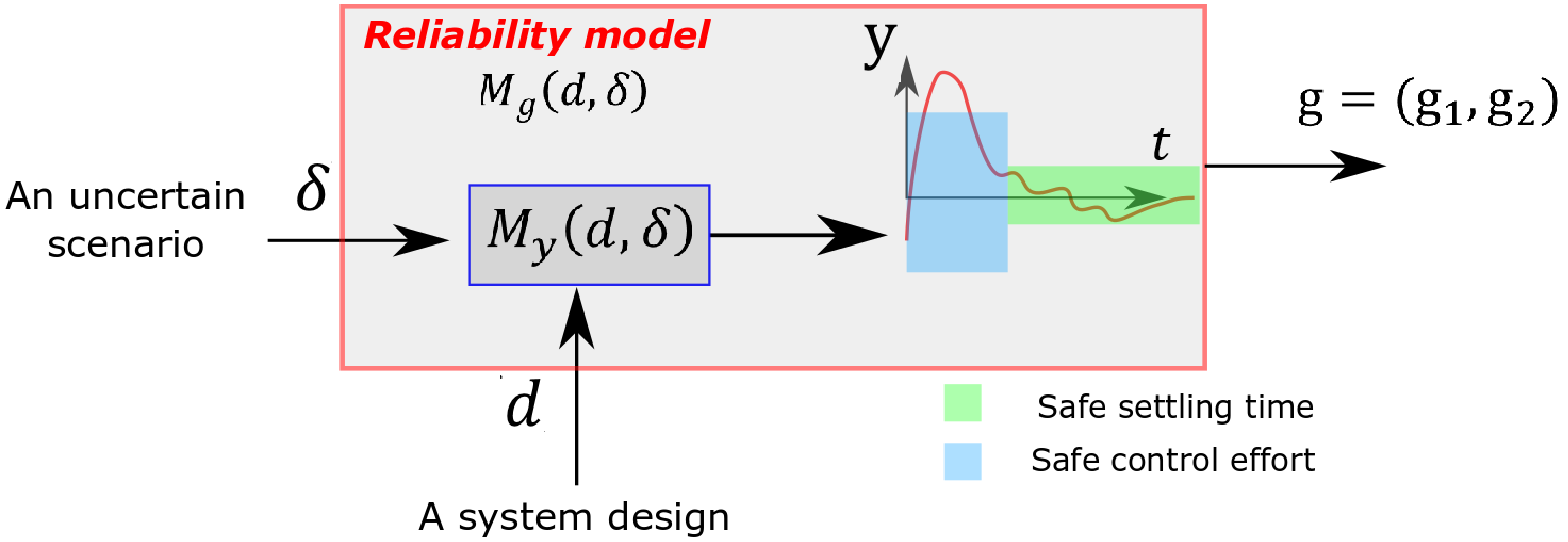
Problem statement



Different uncertain scenarios affect the system's response

are design parameters,
are uncertain factors
response at time

Problem statement



is a vector of performance scores for reliability requirements.

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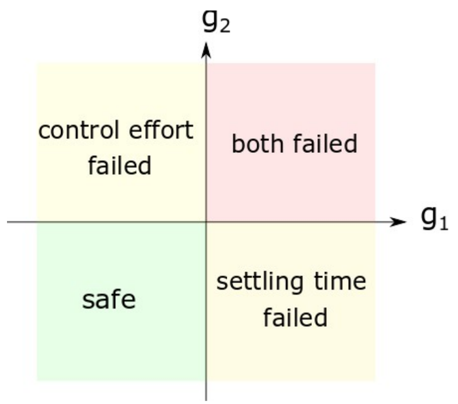

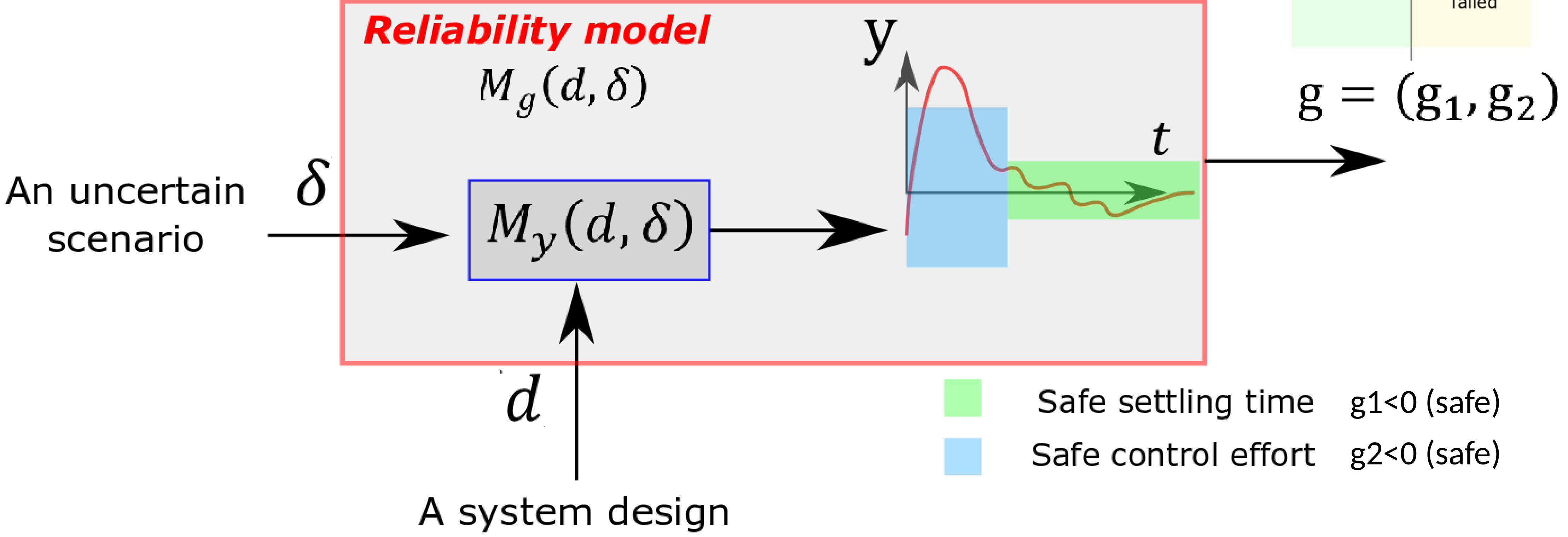

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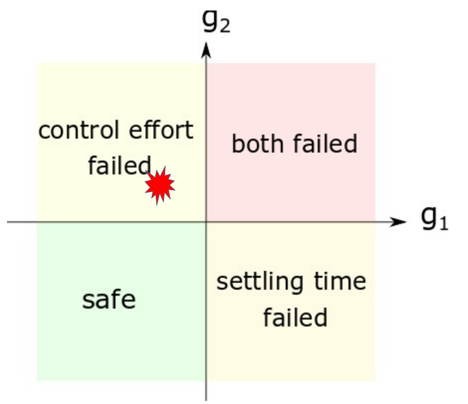

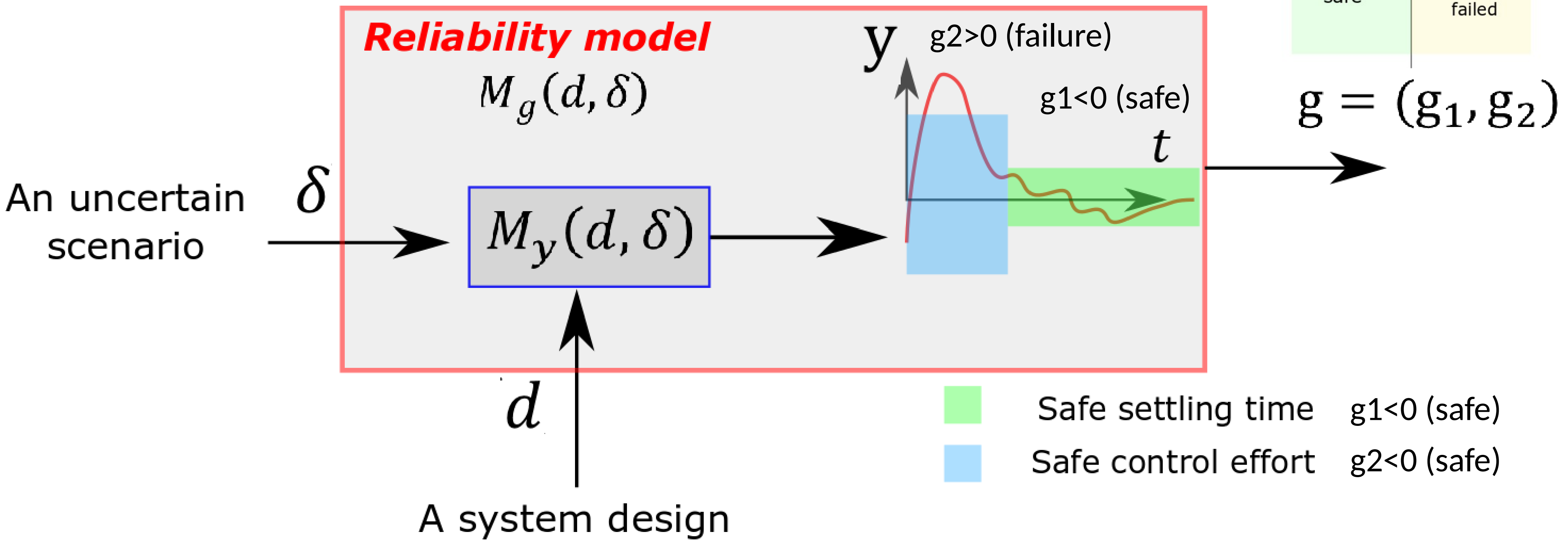

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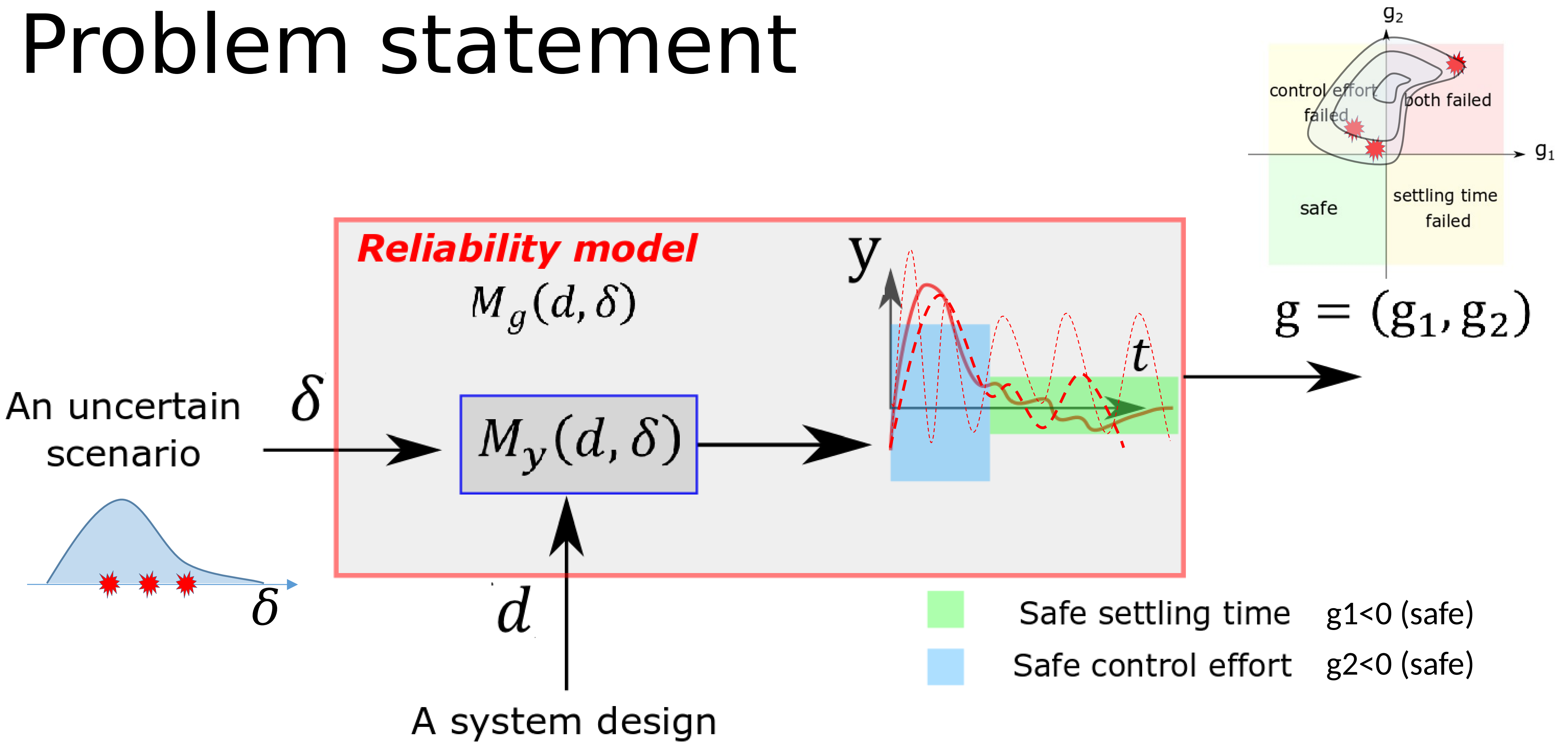

Ideally, an optimal design pushes the realization of g in the safe region, (and far away from severe failure)

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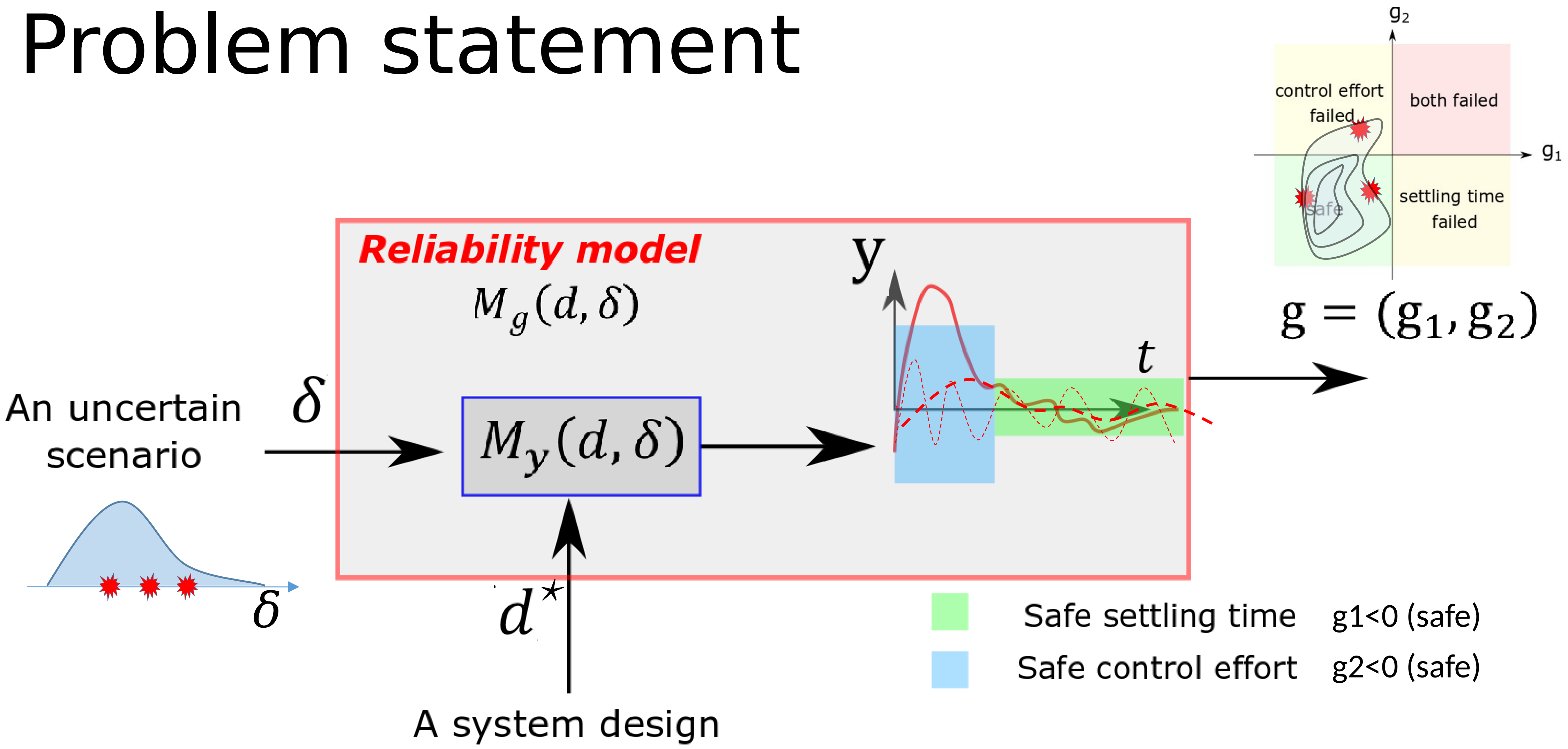


Ideally, an optimal design pushes the realization of g in the safe region, (and far away from severe failure)

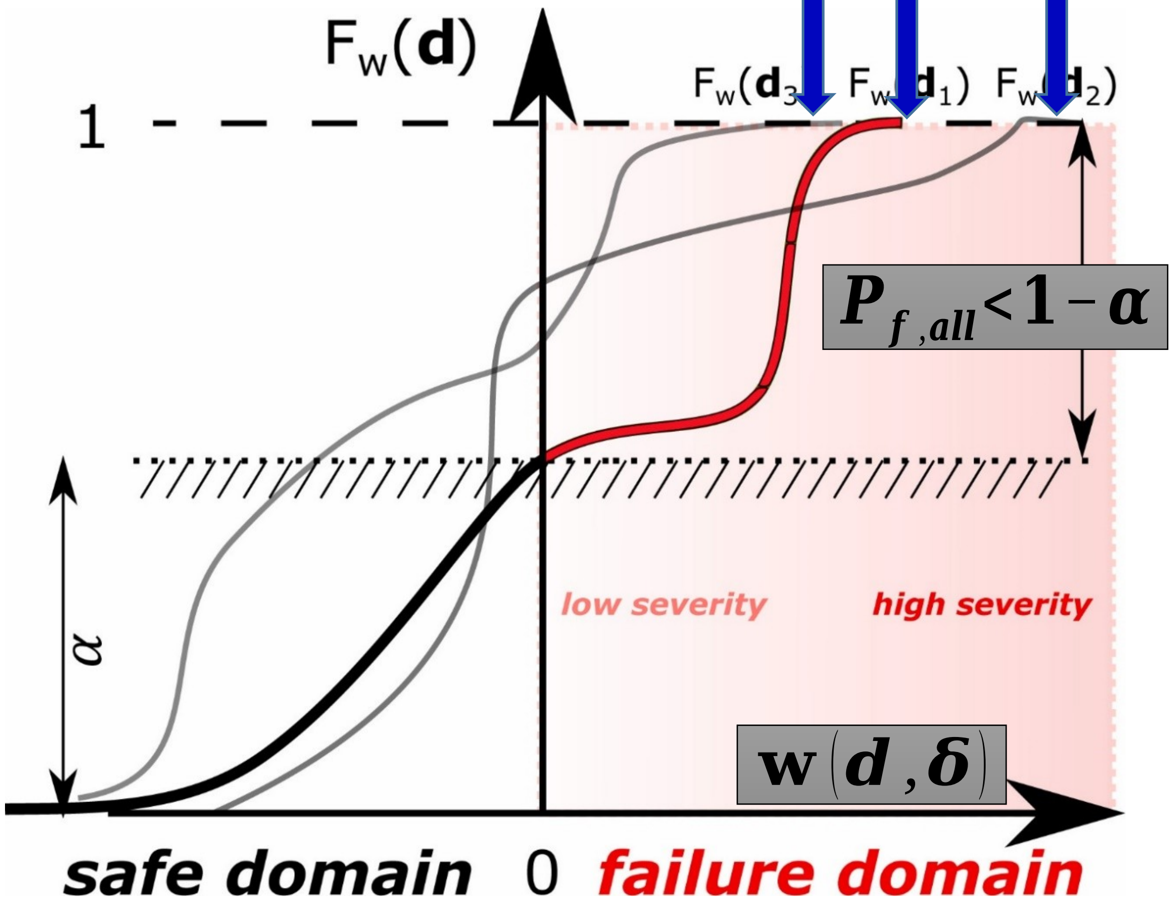
Robustness &Reliability Metrics
(to compare different designs)

Worst-case performance score:

;

Worst-case scenario score:

Empirical cumulative distribution function



Note: and require numerical integration, e.g., via sampling-based methods (but we only have a few samples here)

Robustness &Reliability Metrics
(to compare different designs)

Worst-case performance score:

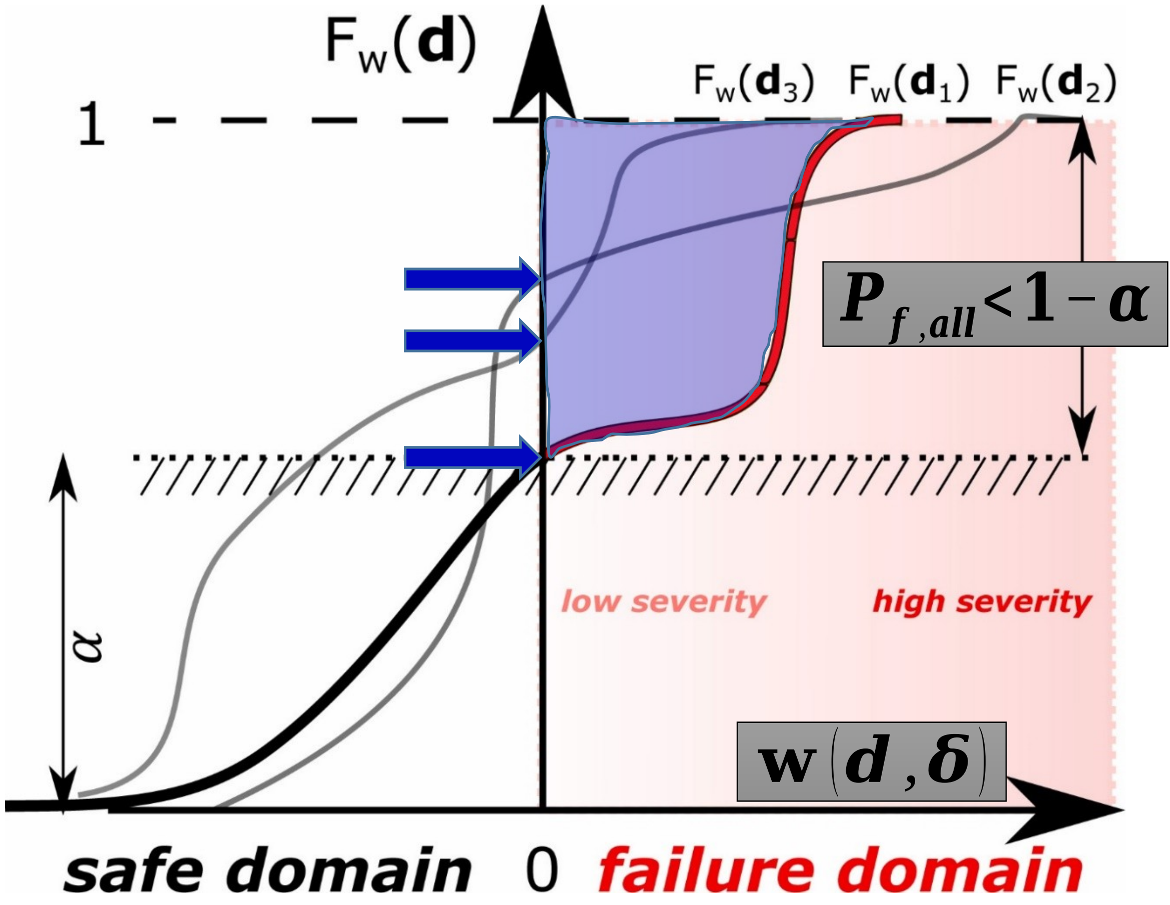
;

Worst-case scenario score:

Failure probability for individual and all requirements:

Severity for individual and all requirements:

Empirical cumulative distribution function



Note: *and require numerical integration, e.g., via sampling-based methods (but we only have a few samples here)*

The challenge problem: Reliability-based design and reliability assessment of a controller

Goals:

- Optimize the controller design for a two-mass spring system from given data.
- Verify system reliability and robustness, e.g., reachability of failure domains and “severe” failure regions.

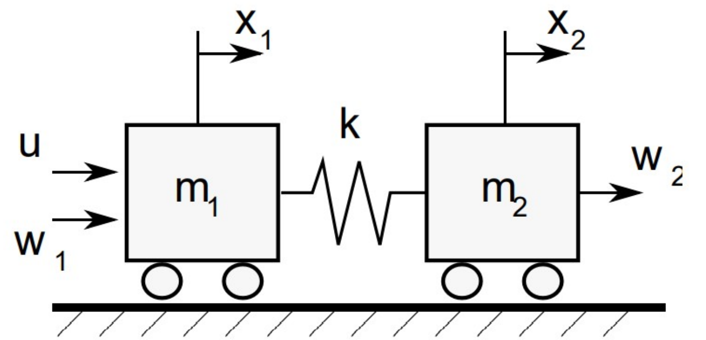


FIGURE 2: THE TWO-MASS SPRING SYSTEM BENCHMARK FOR ROBUST CONTROL DESING.

https://github.com/Roberock/ControllerRobust

The challenge problem: Reliability and system model

Reliability model

: Stability.
: Settling time. position of the mass 2 must fall between ±0.1 from the original state after 15 seconds;
: Control effort. The control signal, u, must fall between ±1.

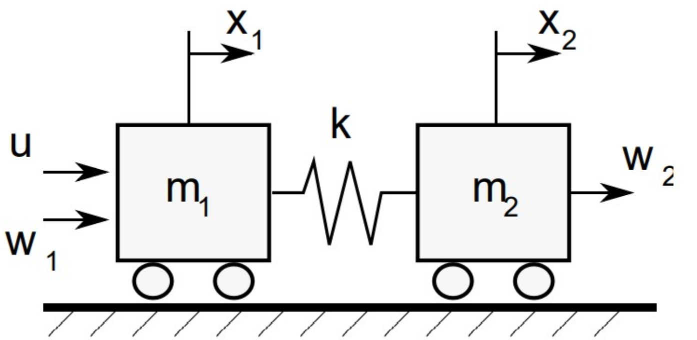


FIGURE 2: THE TWO-MASS SPRING SYSTEM BENCHMARK FOR ROBUST CONTROL DESING.

Note that the time domain requirements require simulating the time response of the system by numerical integration.

Dynamic model

$$\dot{x_c} = Ax_c + By$$
$$u_c = Cx_c + Dy$$

$\xrightarrow[\text{transfer-function}]{\text{From canonical form to}}$

$$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

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The challenge problem:
Design parameters and uncertain factors

Design parameters : $d = [b_0, b_1, b_2, b_3, a_0, a_1, a_2, a_3, a_4] \in \mathbb{R}^9$;

(coefficient of the system transfer function)

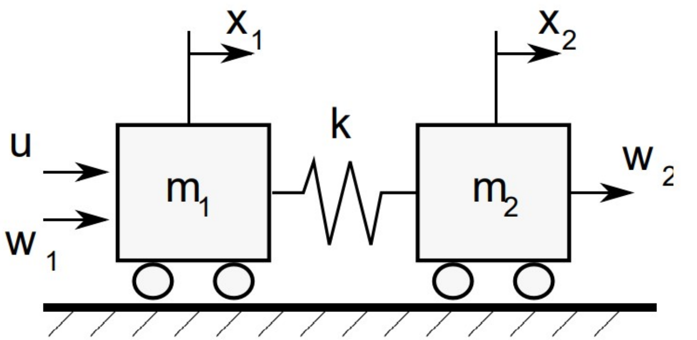

FIGURE 2: THE TWO-MASS SPRING SYSTEM BENCHMARK FOR ROBUST CONTROL DESING.

Uncertain parameters : ;

(masses, springs, time delay, control loop lag)

The system-state matrices depend on the uncertain factor:

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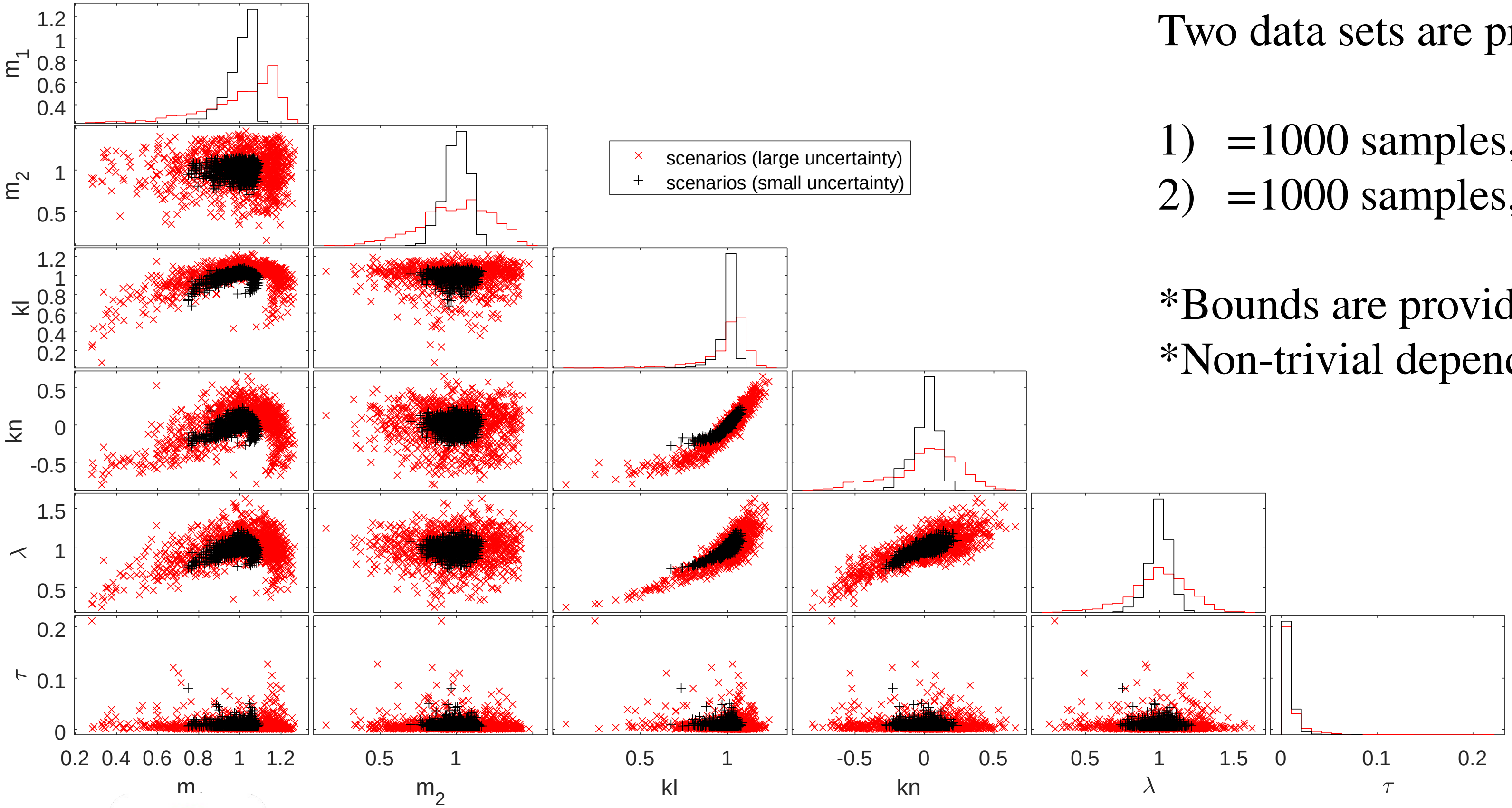
The data set:



Two data sets are provided:

1) =1000 samples, large uncertainty
2) =1000 samples, small uncertainty

*Bounds are provided
*Non-trivial dependency structures

```
δnames=["mass1" "mass2" "kl" "kn" "λ" "τ" ]
# admissible ranges
δlims=[0.1     2;          0.1     2;
       0.05    1.75;       -1      1;
       0.2     1.8;        0.0001 0.3];
```

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Four designs from [1]:

Minimizes the worst-case scenario score,

Minimizes 95th quantile of the distribution

Minimize

A baseline design

Compromise between severity and reliability optimization! Competing reliability requirements!

Designs Names

	$\mathcal{SP}_1(\mathcal{D},0)$	$\mathcal{SP}_1(\mathcal{D},0.05)$	$\mathcal{SP}_2(\mathcal{D})$	Nominal
$\hat{P}_f$	0.981	0.057	0.175	0.734
$\overline{g}_1$	−0.0272	−0.013	0.069	$7e-4$
$\overline{g}_2$	0.5925	1.576	3.054	1.974
$\overline{g}_3$	0.5925	0.193	1.406	0.202
a_4	0.2238	0.5375	0.7600	0.5503
a_3	0.6811	1.3346	1.9491	1.4175
a_2	3.1275	2.4206	3.0497	2.6531
a_1	2.3615	2.1689	2.7344	2.4802
a_0	1.1833	0.8084	1.0594	1.0000
b_3	−0.0982	2.4802	−0.0831	−0.1324
b_2	0.4702	0.6146	0.6358	0.3533
b_1	0.5886	0.5265	0.7752	0.6005
b_0	0.0777	0.0716	0.0981	0.0728

Scores

The design parameters

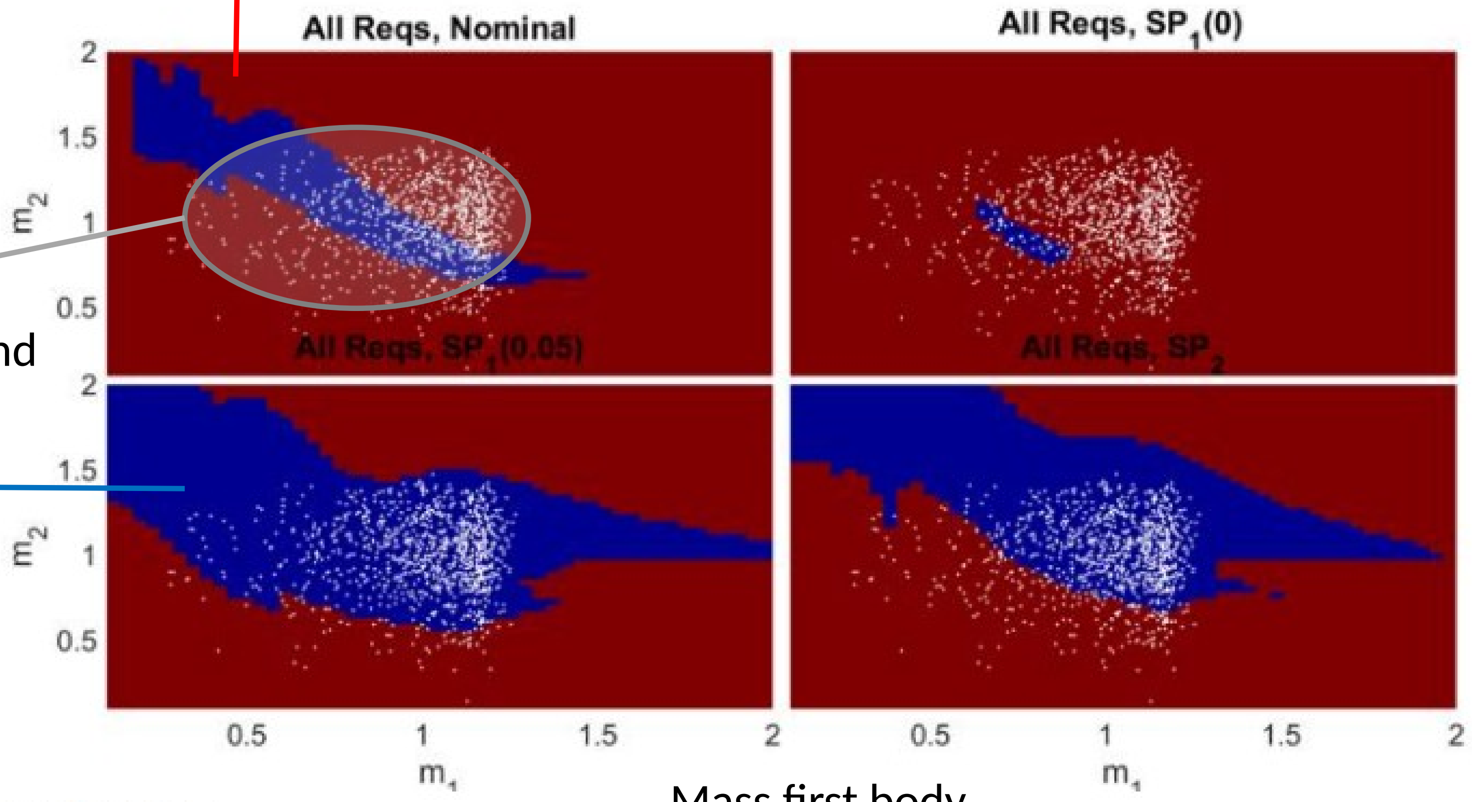
https://github.com/Roberock/ControllerRobust

[1] Rocchetta, R., Crespo, L. G., & Kenny, S. P. (2019). Solution of the benchmark control problem by scenario optimization. In Dynamic Systems and Control Conference, USA.

Design resulting from previous works [1]

Failure domain,

Also note that *and are non-convex functions of and*



Samples

Mass second body

Safe domain

Mass first body

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[1] Rocchetta, R., Crespo, L. G., & Kenny, S. P. (2019). Solution of the benchmark control problem by scenario optimization. In Dynamic Systems and Control Conference, USA.

Concluding discussion on challenges a few remarks

Value of information & uncertainty characterization challenges:

How to use the available data in the best way? Shall we prescribe a model for the uncertainty?
How to keep track of the value of information in the initial data set (only 1k samples) ?
The data present some complex dependency structures. Can we account for this?

Reliability challenges:

- ***Competing reliability requirements.***
- ***Robustness and reliability (competing)***
- ***Estimation of failure probability from set-based characterization (reachability of failure sets)***

Robust vs Reliable design optimization challenges:

- ***Need to define an appropriate optimization program (minimize or constrain severity/reliability/worst-cases?)***
- ***Probabilities and expectations are step-wise discontinuous (gradient based optimizer are inapplicable).***

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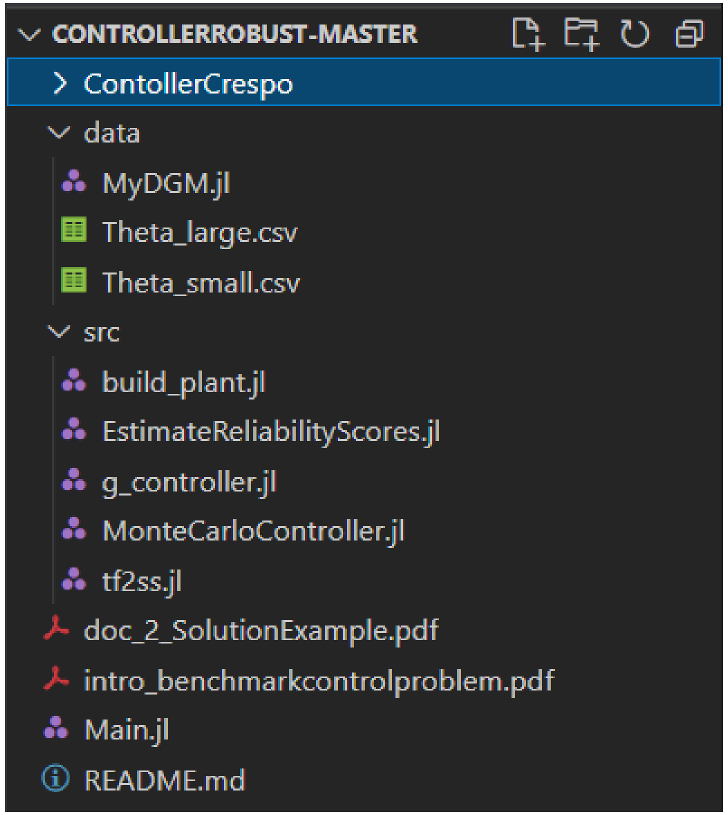
Thank you for listening!



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[2] B. Wie and D. S. Bernstein, "*Benchmark Problems for Robust Control Design*," in 1991 American Control Conference, 1991

[3] Roberto Rocchetta, Luis G. Crespo, Sean P. Kenny, “*A scenario optimization approach to reliability-based design*”, Reliability Engineering & System Safety, Volume 196, 2020, https://doi.org/10.1016/j.ress.2019.106755

[4] R. Rocchetta, Luis G. Crespo, *“A scenario optimization approach to reliability-based and risk-based design: soft-constrained modulation of failure probability bounds”*, Reliability Engineering & System Safety, https://doi.org/10.1016/j.ress.2021.107900