Unicycle Platoon Verification Challenge

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Outline

1. Easy: 3-Dimensional Unicycle, Reachability Analysis
2. Difficult: Platoon Model - 12 dimensional
3. Contrast difficulty of Platoon Reachability Analysis with Single Unicycle
4. Verification Problem and Compare Different Algorithms

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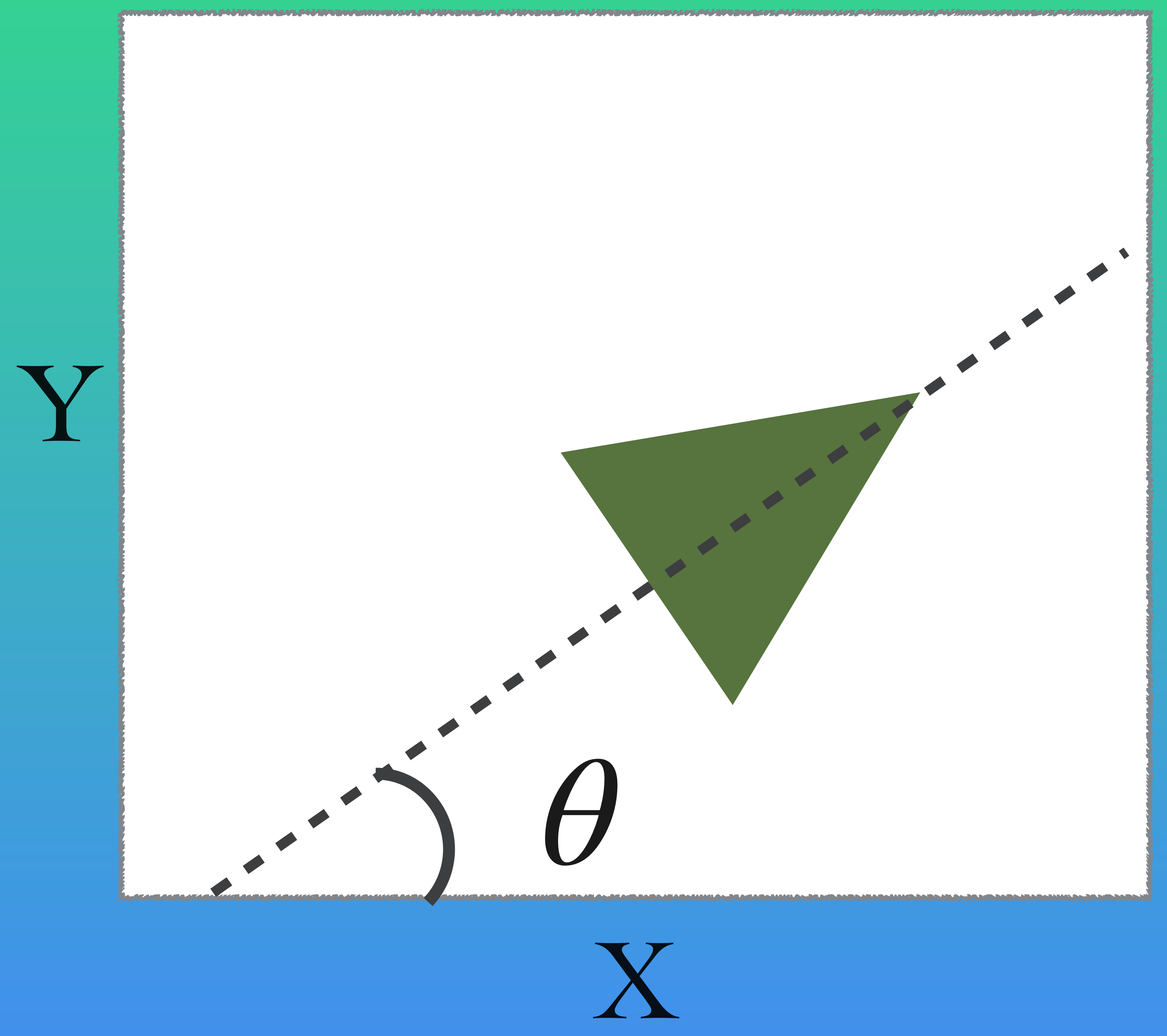
Unicycle model

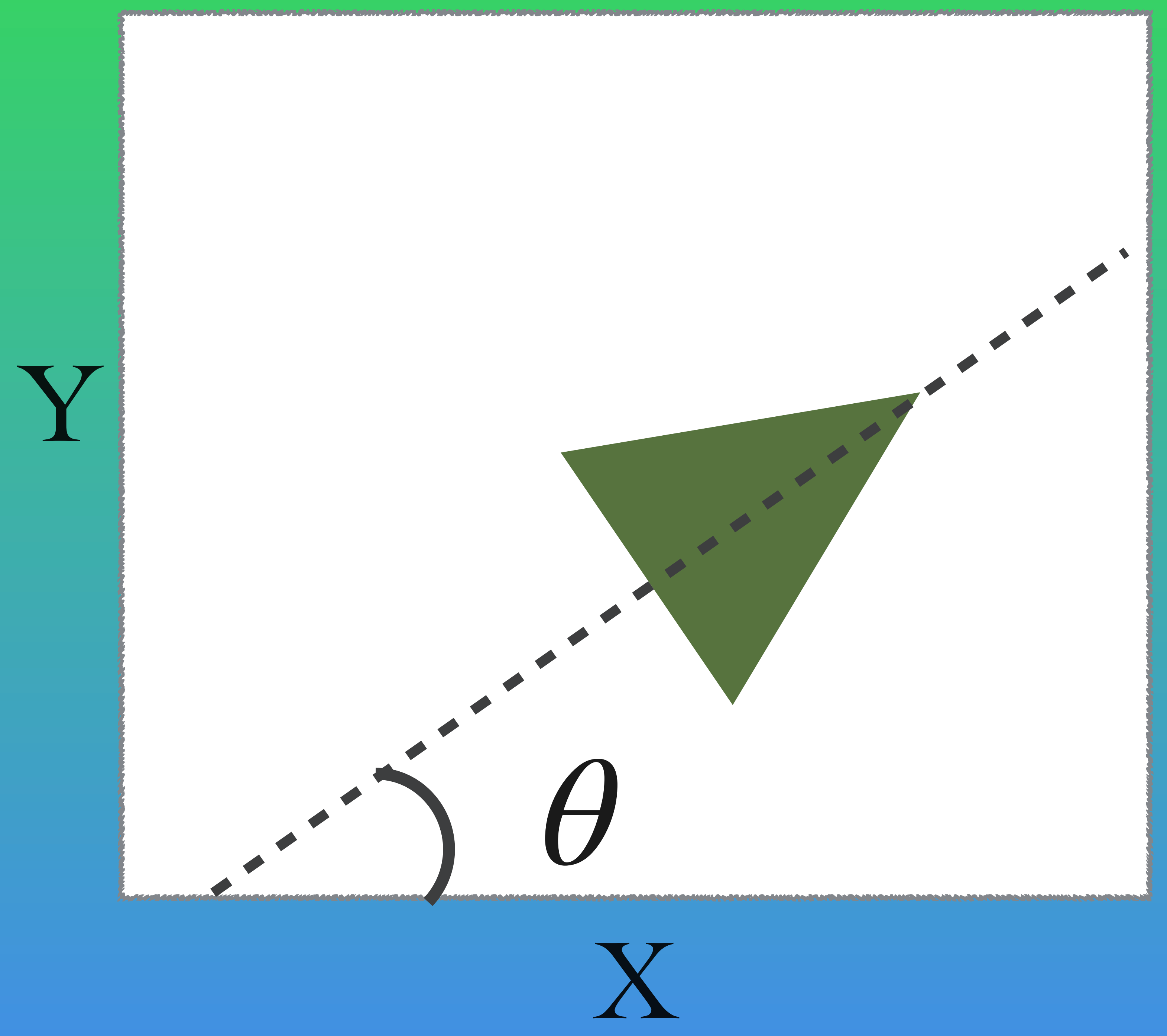
- Approximate robot movement in 2D
- $$\dot{x}(t) = u(t)\ cos\left(\theta(t)\right)$$
  $$\dot{y}(t) = u(t)\ sin\left(\theta(t)\right)$$
  $$\dot{\theta}(t) = w(t)$$



Unicycle Model with Angular Feedback

$$\dot{x}(t) = u(t)\ cos\left(\theta(t)\right)$$
$$\dot{y}(t) = u(t)\ sin\left(\theta(t)\right)$$
$$\dot{\theta}(t) = -\lambda\ \theta(t)$$

λ : exponent
for rate of convergence



A Reachability Problem for Single Unicycle

$$\dot{x}(t) = u(t)\ cos\left(\theta(t)\right)$$
$$\dot{y}(t) = u(t)\ sin\left(\theta(t)\right)$$
$$\dot{\theta}(t) = -\lambda\ \theta(t)$$

λ : rate of stabilization

Given:

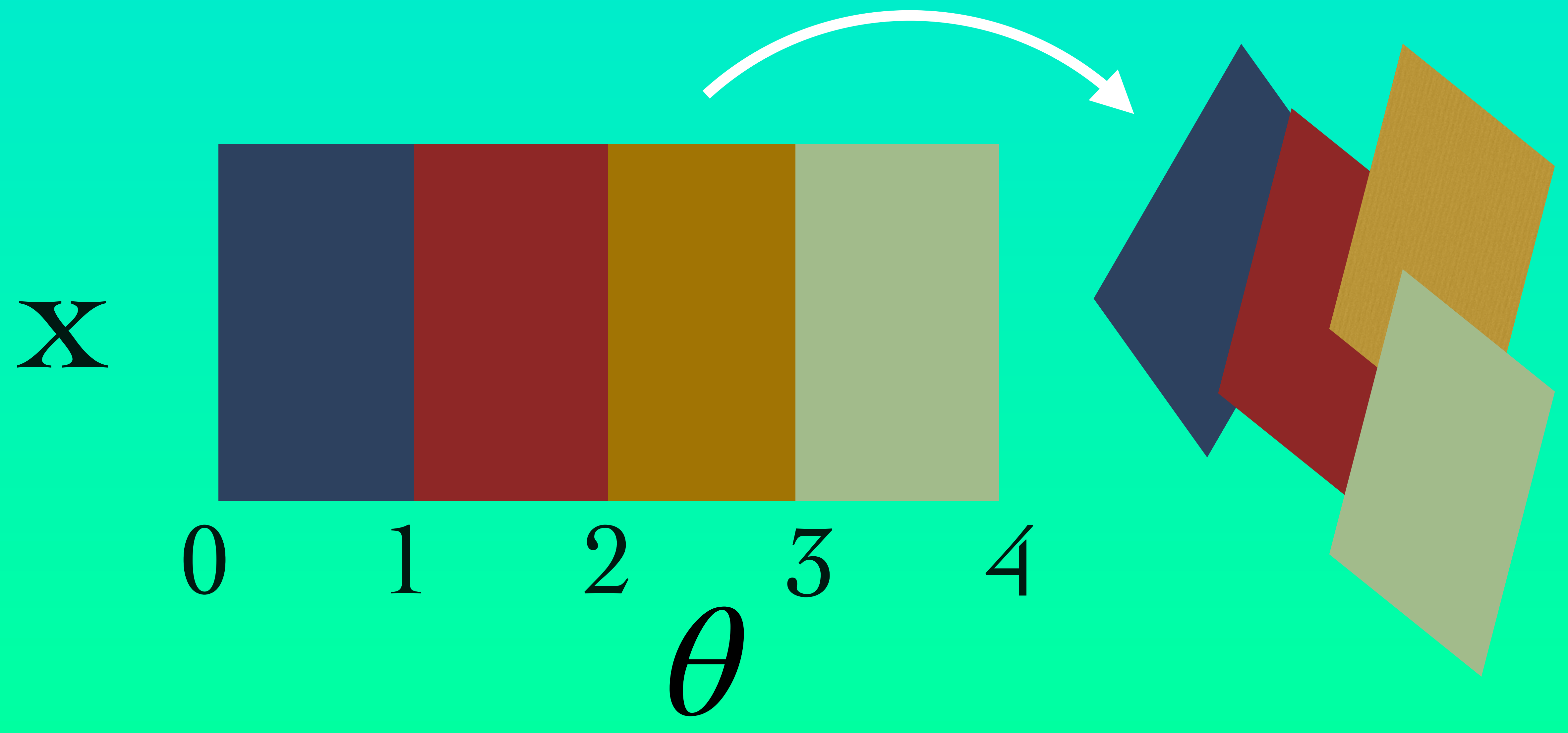
- Initial: $\left(x(0), y(0), \theta(0)\right) \in Box(c, r),\ c, r \in \mathbb{R}^3$
- Input bounds: $u(t) \in [a, b]$
- Direction $p \in \mathbb{R}^2$, time t

Compute directional bounds:

$$\sup \left\{ p^T \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \mid \left(x, y, \theta\right) : \text{trajectory of unicycle} \right\}$$

Reachability Approach for Easy 3-dimensional model : Break Initial Angle for Accuracy

- Divide initial set along θ-axis and linearize (piecewise)
- Compute reachable set post piecewise linearization (on the fly)



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Unicycle Platoon with Feedback: Coupling different Unicycles

- Feedback controls lateral drift and inter vehicle distance

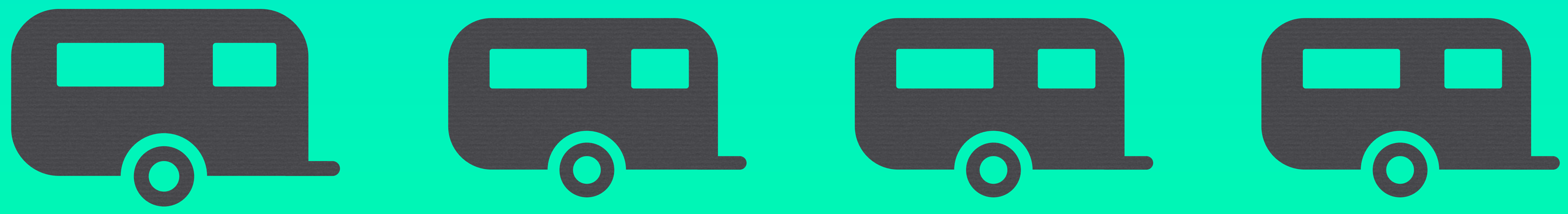
$$\dot{x}_i(t) = u_i(t)\ cos\left(\theta_i(t)\right)$$

$$\dot{y}_i(t) = u_i(t)\ sin\left(\theta_i(t)\right)$$

$$\dot{\theta}_i(t) = g(x_i(t))$$

$$u_i(t) = f\left(x_{i-1}(t), x_i(t), \theta_i(t), v(t)\right)$$

- f, g can be nonlinear feedback
- $i \in \{1, \cdots, N\} \quad N \geq 4$



$x_4(t)$ $x_3(t)$ $x_2(t)$ $x_1(t)$

Unicycle Platoon Model

$$\dot{x}_i(t) = u_i(t)\ cos\left(\theta_i(t)\right)$$
$$\dot{y}_i(t) = u_i(t)\ sin\left(\theta_i(t)\right)$$
$$\dot{\theta}_i(t) = -\ \theta_i(t)\left(0.5 + 2\theta_i(t)^2\right)$$

if $i = 1$, then

$$u_i(t) = \frac{v(t)}{1 + \theta_i(t)^2}$$

if $2 \leq i \leq N$, then

$$u_i(t) = \frac{0.3\left(x_{i-1} - x_i + 10\right)}{1 + \theta_i(t)^2}$$

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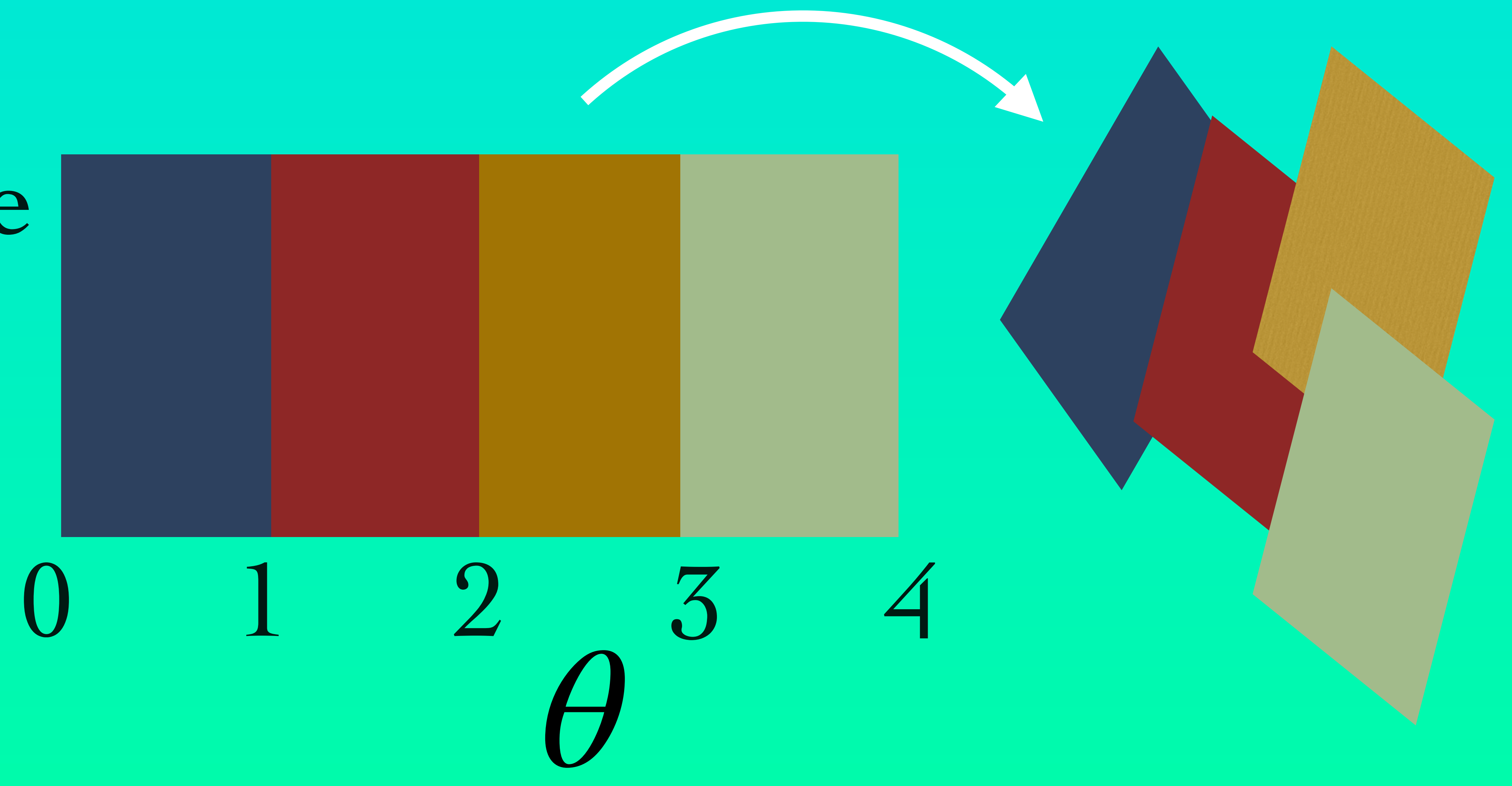
Contrast with single unicycle case

- Appropriate division of initial is difficult to find.
- Division along all axes - *computationally intractable !!*

Single unicycle case

x

Difficult for platoon



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Unicycle Platoon Verification Challenge: Initial Conditions and Input Bounds

$x_1(0) \in [60,70]$ $y_1(0) \in [0,0]$ $\theta_1(0) \in [-0.5,0.5]$ v = 5 m/s

$x_2(0) \in [40,50]$ $y_2(0) \in [0,0]$ $\theta_2(0) \in [-0.5,0.5]$

$x_3(0) \in [20,30]$ $y_3(0) \in [0,0]$ $\theta_3(0) \in [-0.5,0.5]$

$x_4(0) \in [0,10]$ $y_4(0) \in [0,0]$ $\theta_4(0) \in [-0.5,0.5]$

Num robots = 4

Unicycle Platoon Verification Challenge: Property 1

Inter-robot horizontal displacement

Find maximum d, e

1. $\forall t \in [0,5] \forall i \in \{1,2,3\} \left(x_i(t) - x_{i+1}(t)\right) \geq d$ (bounded time)

2. $\forall t \in [0,\infty) \forall i \in \{1,2,3\} \left(x_i(t) - x_{i+1}(t)\right) \geq e$ (unbounded time)

Unicycle Platoon Verification Challenge: Property 2

Lateral drift

Find minimum d, e

1. $\forall t \in [0,5] \forall i \in \{1,2,3,4\} \ \left| y_i(t) \right| \leq d$ (bounded time)

2. $\forall t \in [0,\infty) \forall i \in \{1,2,3,4\} \ \left| y_i(t) \right| \leq e$ (unbounded time)

Unicycle Platoon Verification Challenge: Property 3

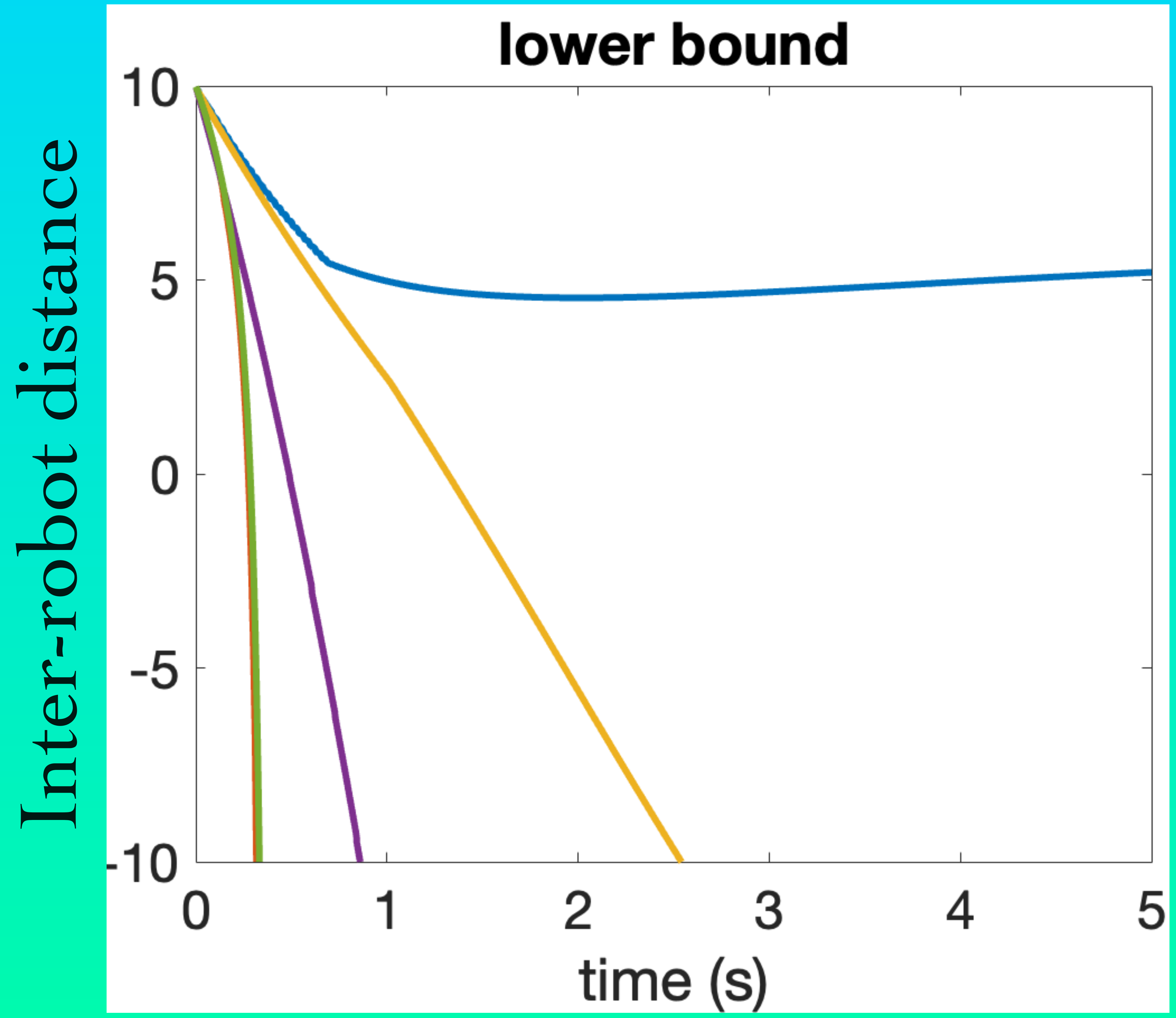
Convergence of robots

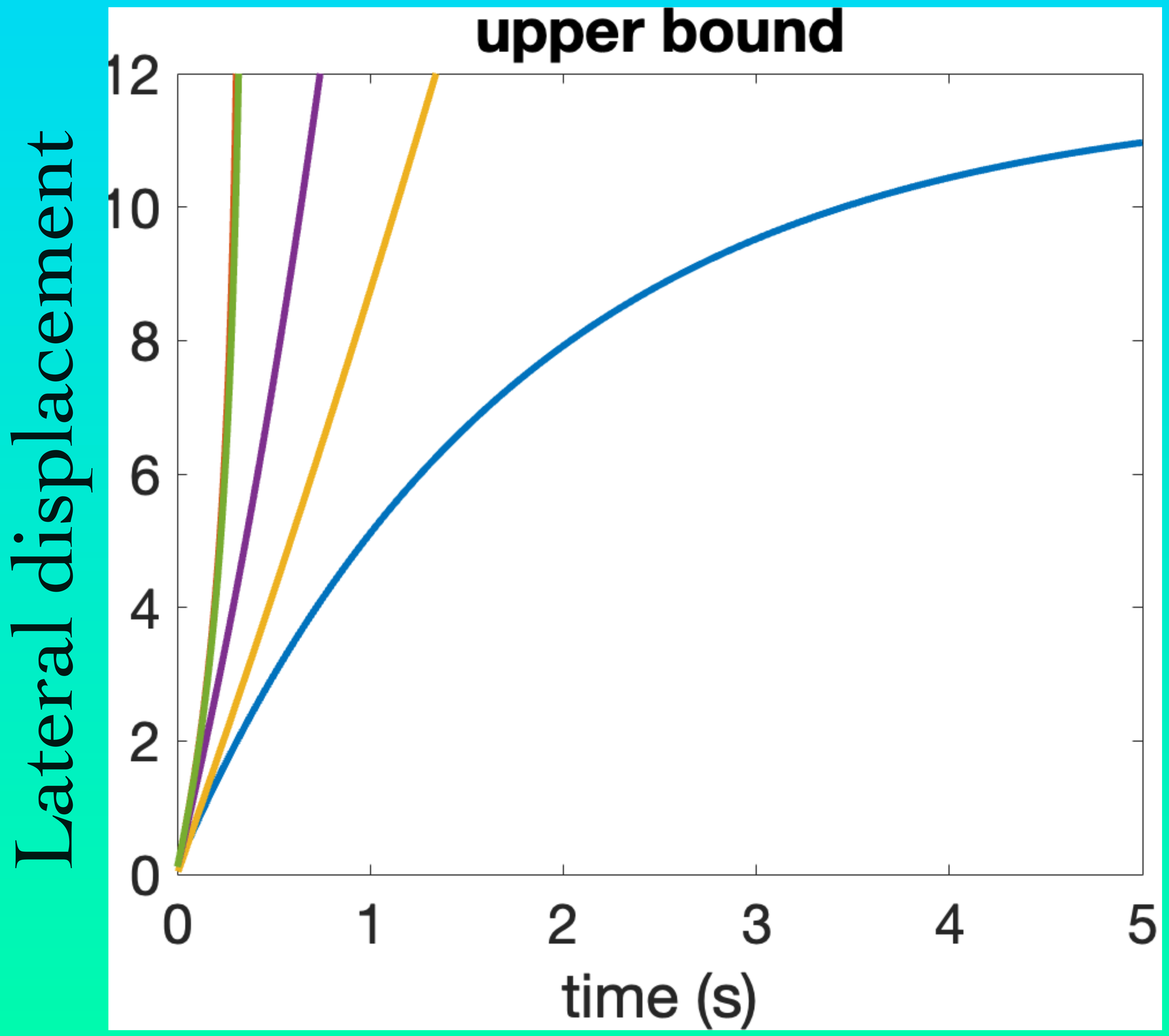
Find minimum d, e

1. $\forall t \in [5,10] \forall i \in \{1,2,3\} \left(x_i(t) - x_{i+1}(t) \right) \leq d$ (bounded time)

2. $\forall t \in [5,\infty] \forall i \in \{1,2,3\} \left(x_i(t) - x_{i+1}(t) \right) \leq e$ (unbounded time)

Comparison of Different Algorithms





Taylor model

iou zonotope

polynomial zonotope

single objective optimization

conservative linearization

Summary

- Proposed Unicycle Platoon Model - 12 dimensional, tight coupling between robots
- Three properties to verify (or compute bounds)
- Compared different algorithms in bounded time case.

Possible Extensions of Challenge

- Increase Number of Robots

  (model is easily scalable)

- Compute Initial Conditions