Verification of Flight Control Algorithms Using Reachability Analysis

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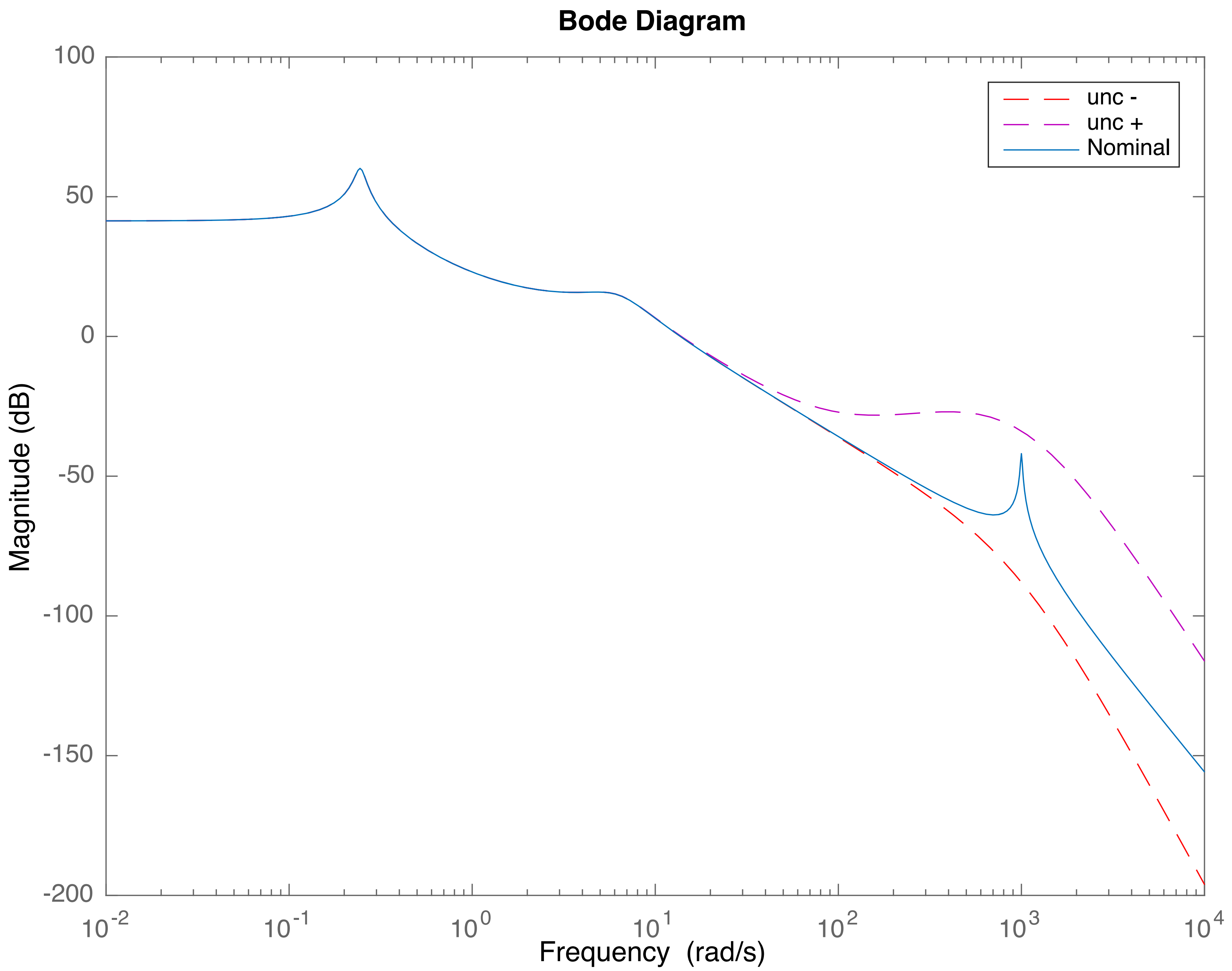
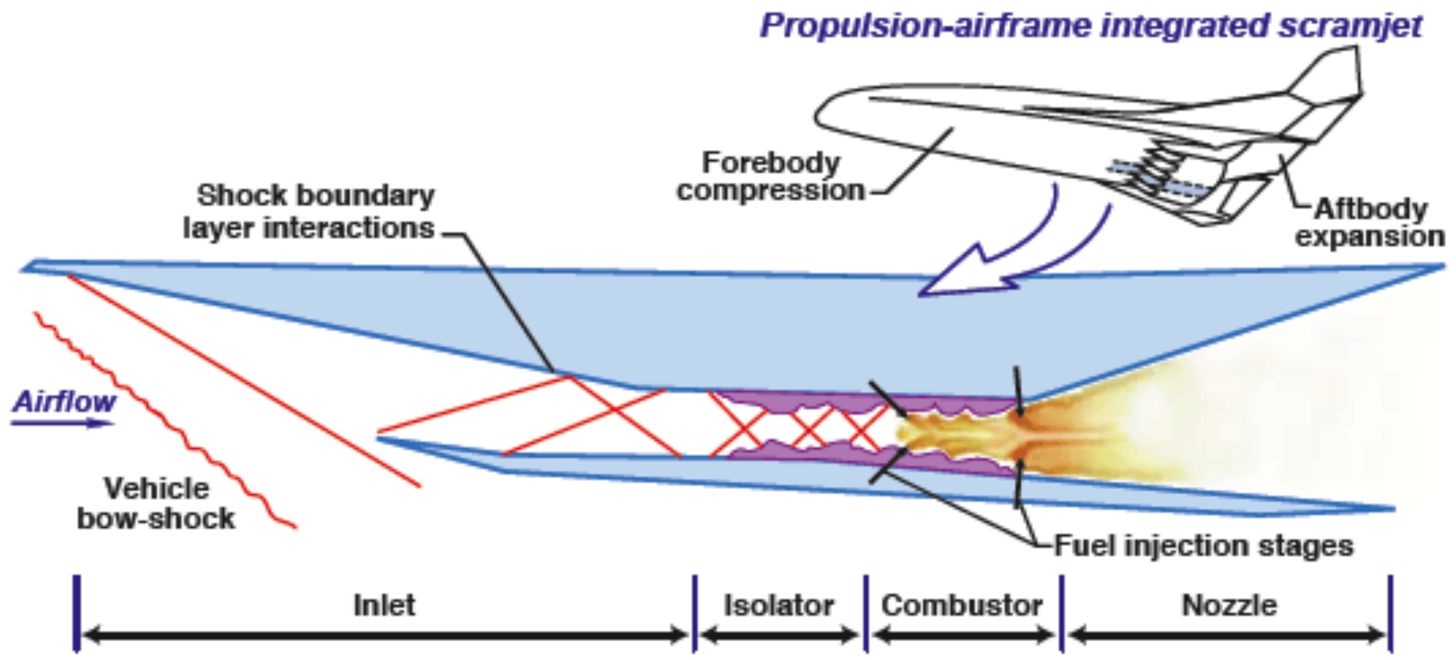
Outline

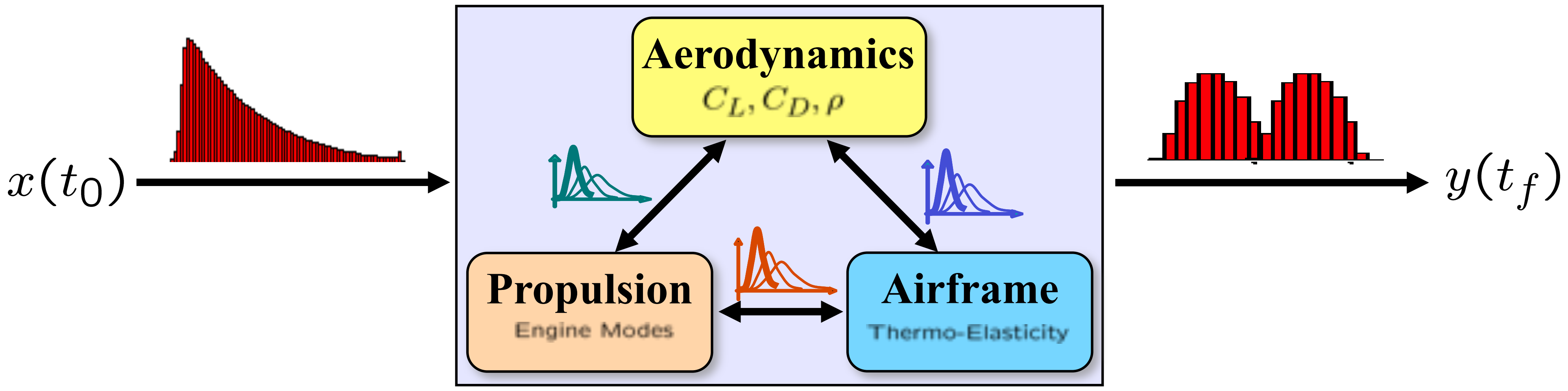
- Flight Certification Problem
- New Approach for Uncertainty & Robustness Quantification
- F16Model.jl
  - Basic usage
  - Control law design
  - Simulation-based verification with linear and nonlinear dynamics
  - Definition of the challenge problem

Example 1

NASA X43

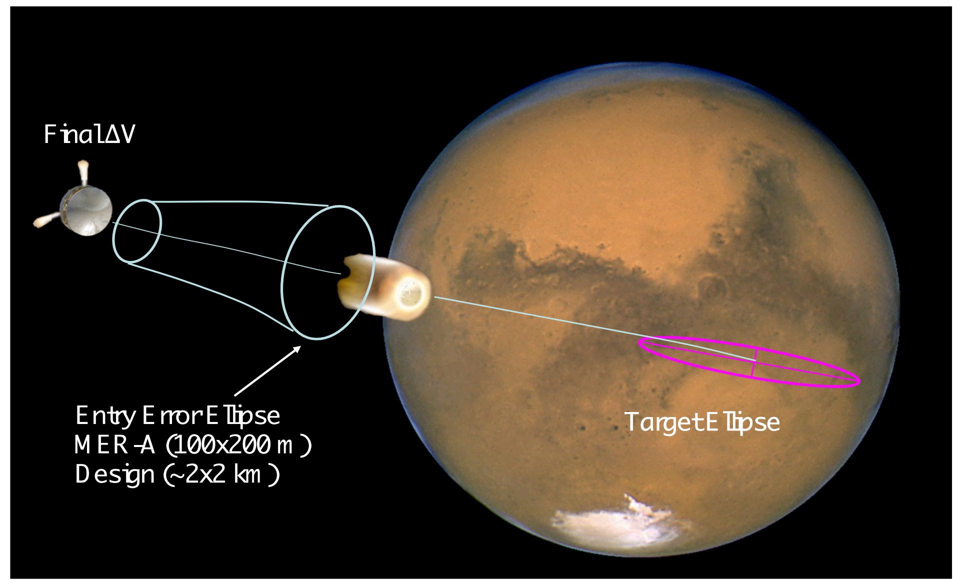


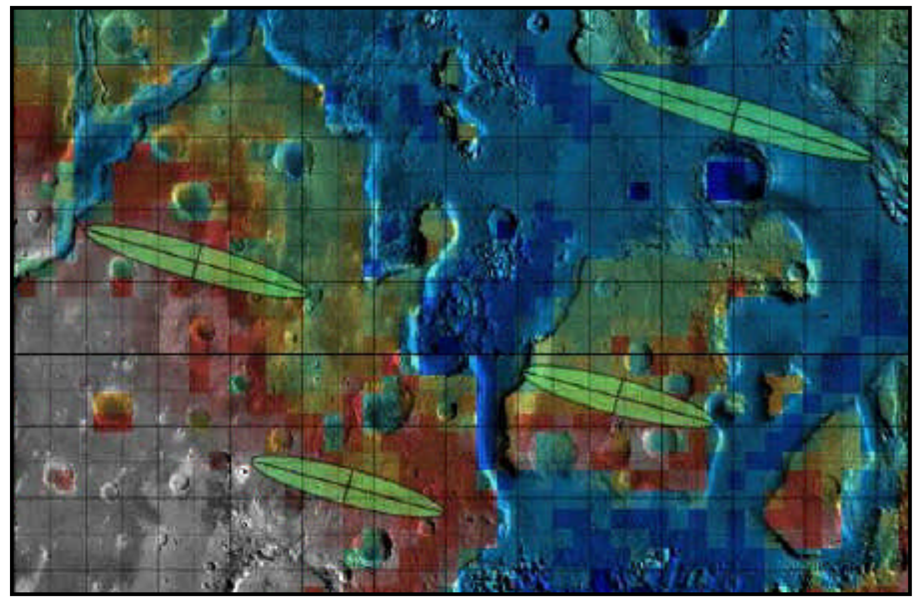




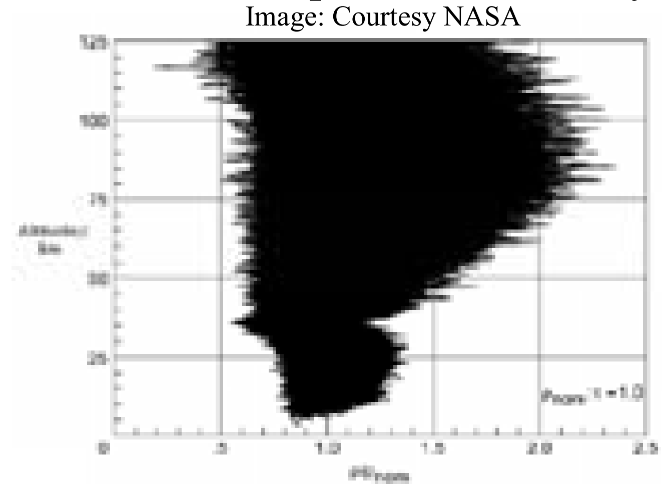
Example 2

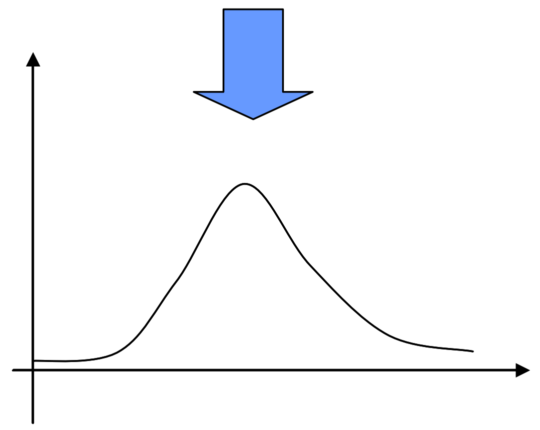
Planetary Reentry



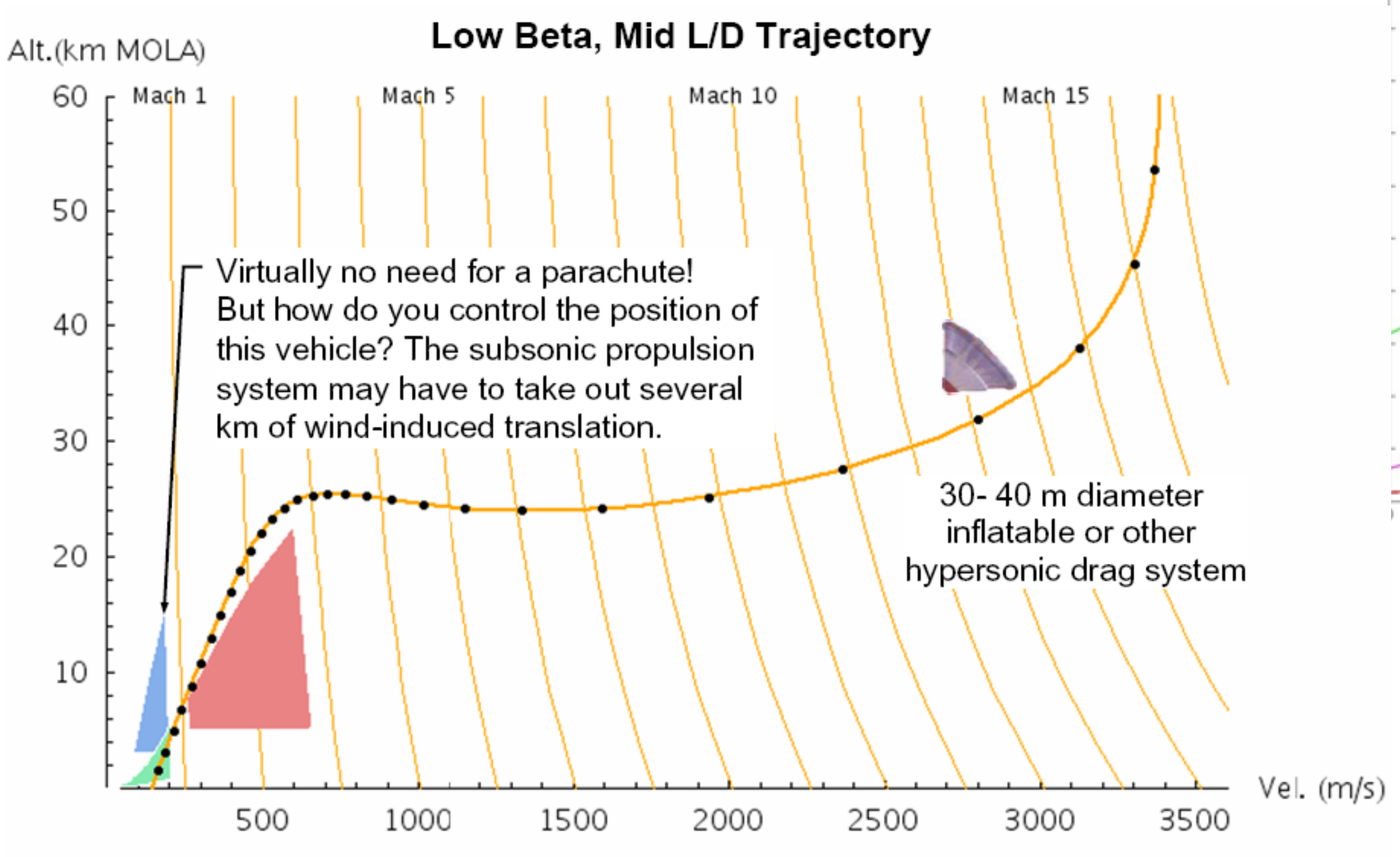


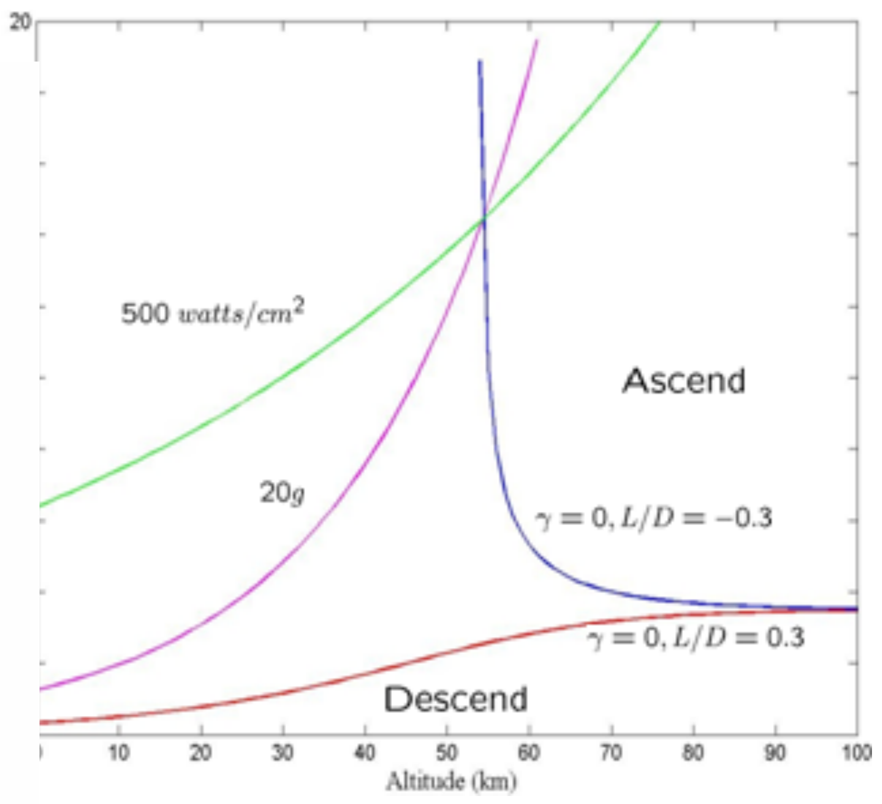
Mars Atmosphere Uncertainty
Image: Courtesy NASA

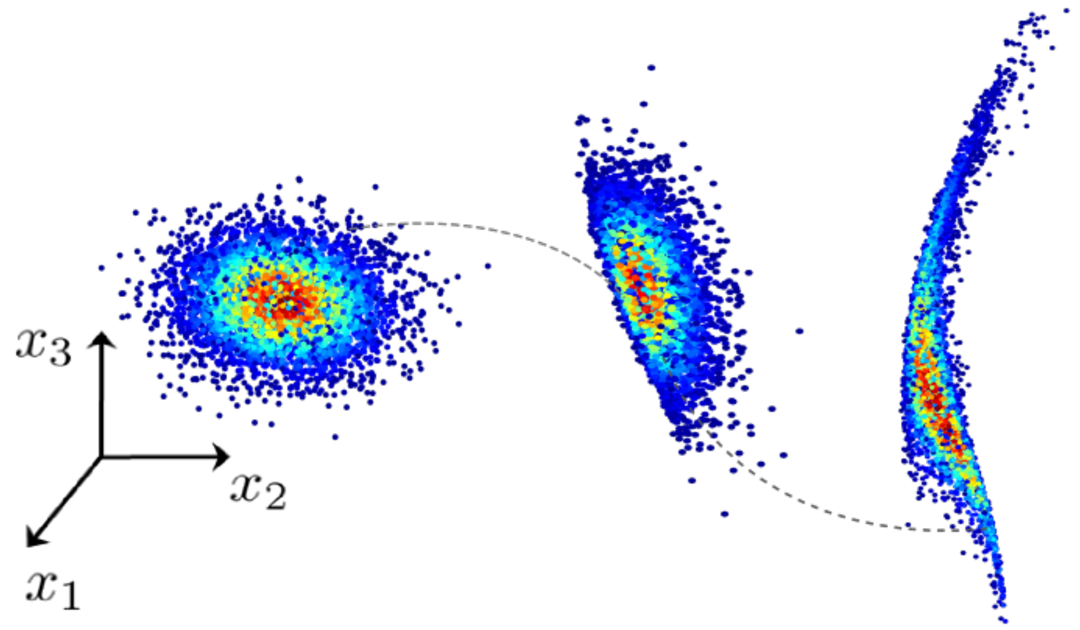




Probability Density Function on System Parameters.

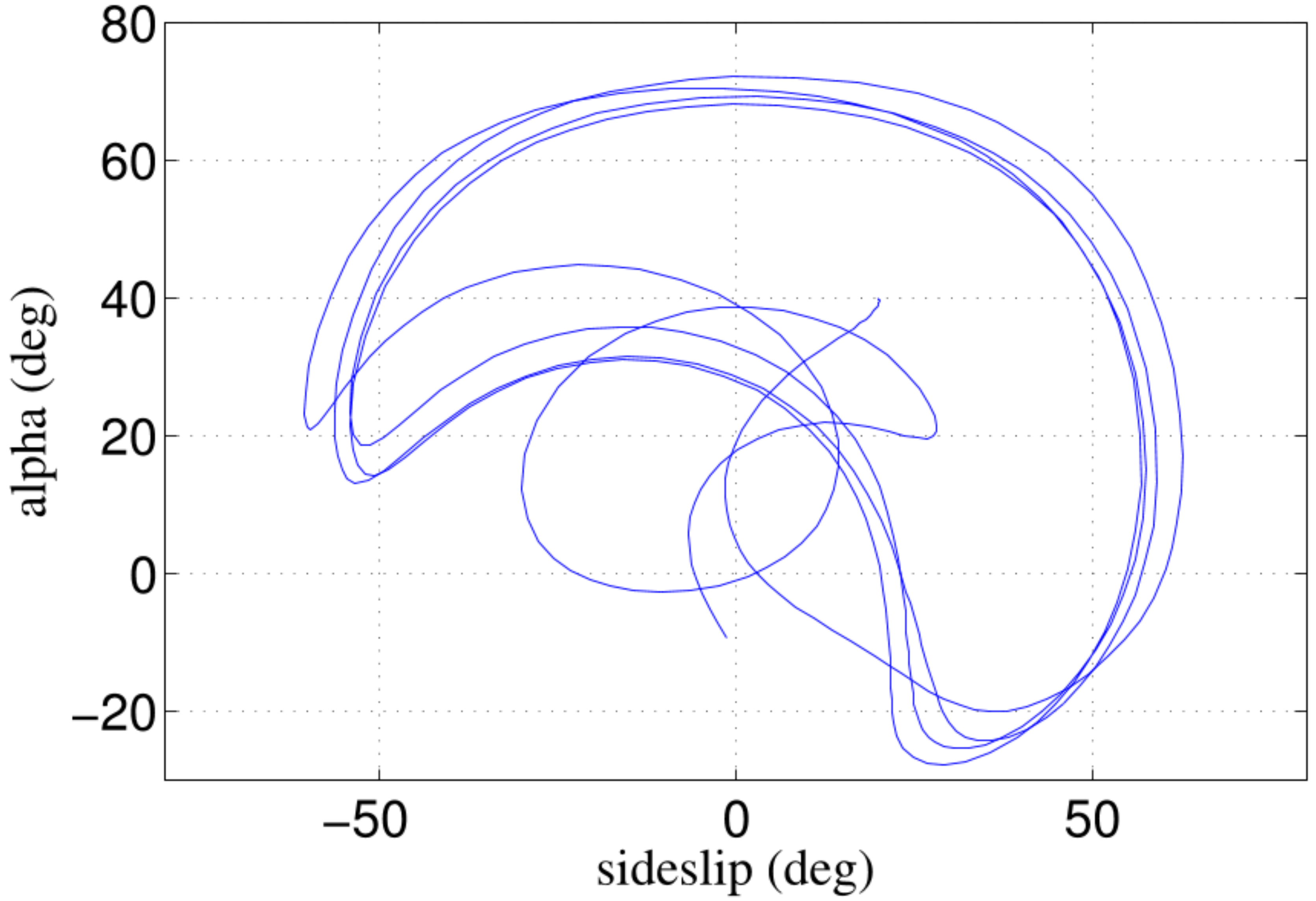






Example 3

F/A 18 Falling Leaf



Out of Control Motion

- High angle of attack and rate
- High side-slip
- Strong coupling between
  - nonlinear rigid body dynamics
  - unsteady aerodynamics
- Loss of vehicles F/A18 A/B/C/D
  - early 1980's onwards
  - about 50 vehicles
- Fixed in F/A18 E/F
  - revised control law by NAVAIR & BOEING
  - started in 2001 — completed 2003
  - able to suppress the falling leaf mode
- Changes
  - upgraded AOA estimator (faulty AOA probe)
  - added $\dot{\beta}, \beta$ to the feedback loop via δ_a and δ_e

Example 3

F/A 18 Falling Leaf

(https://www.youtube.com/watch?v=QSLmblzI_c4)

Flight Control Validation

Some Certification Questions

- How well does the controller work for nonlinear system?
- What is the largest set of initial condition that can be stabilized?
- What is the largest set of parametric uncertainty? (Uncertainty Quantification, Robustness Margins, etc.)

Reachability analysis is a common approach.

New Formulation

- Analyze system with trajectory densities
  - Infinite-dimensional **linear** system
  - Linear PDE — Continuity equation, Fokker-Planck-Kolmogorov Equation

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}) + \Lambda(\boldsymbol{x}, t)\Gamma,$$

$$\frac{\partial \rho(t, \boldsymbol{x})}{\partial t} + \boldsymbol{\nabla}.\left(\boldsymbol{F}(\boldsymbol{x})\rho(t, \boldsymbol{x})\right) - \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2}{\partial x_i \partial x_j}\left(D_{ij}(\boldsymbol{x}, t)\rho(t, \boldsymbol{x})\right) = 0.$$

- Computational Challenges
  - Infinite domain
  - Positivity — PDF is positive
  - Normalization — Integrates to one

- Steady-State Distribution with Reverse-Time Dynamics
  - Characterizes the separatrix
  - Include process-noise, sensor-noise, etc.

$$\lim_{t\to\infty} \rho(\boldsymbol{x}) = N_0 \exp(-\eta(\boldsymbol{x})), \quad \Longrightarrow \quad \sum_{i=1}^{n}\left(\frac{\partial F_i(\boldsymbol{x})}{\partial x_i} - F_i(\boldsymbol{x})\frac{\partial \eta(\boldsymbol{x})}{\partial x_i}\right) - \sum_{i=1}^{n}\sum_{j=1}^{n}\left[D_{ij}\left(\frac{\partial^2 \eta(\boldsymbol{x})}{\partial x_i \partial x_j} + \frac{\partial \eta(\boldsymbol{x})}{\partial x_i}\frac{\partial \eta(\boldsymbol{x})}{\partial x_j}\right)\right] = 0.$$

Uncertainty Propagation

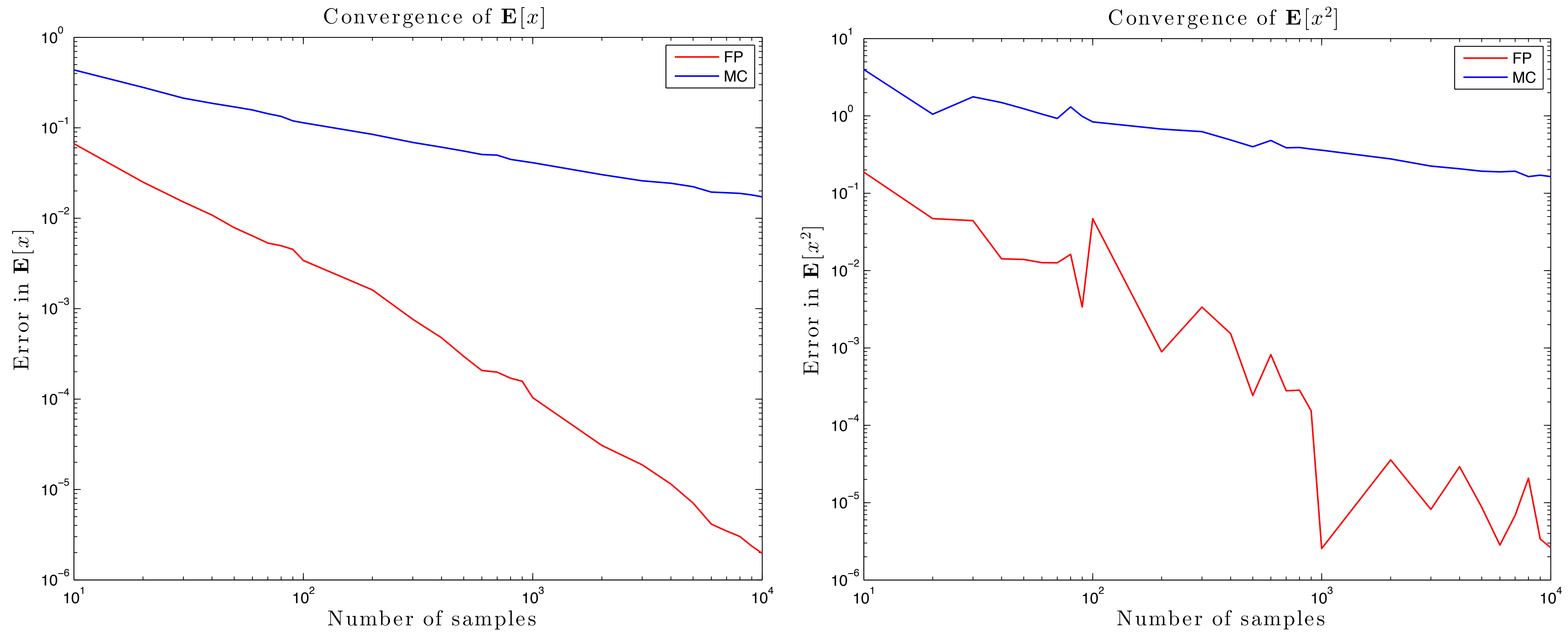
For continuity equation

$$\frac{\partial p}{\partial t} + \sum_{i=1}^{n} \frac{\partial p}{\partial x_i} F_i(t, \boldsymbol{x}) + p \sum_{i=1}^{n} \frac{\partial F_i(t, \boldsymbol{x})}{\partial x_i} = 0$$

$\dot{\boldsymbol{x}} = \boldsymbol{F}(t, \boldsymbol{x})$ evolution of $\boldsymbol{x}(t)$

$\dot{p} = -p(\boldsymbol{\nabla} \cdot \boldsymbol{F})$ evolution of p along $\boldsymbol{x}(t)$

Better Accuracy & Faster Convergence than MC



(a) First Moment (b) Second Moment

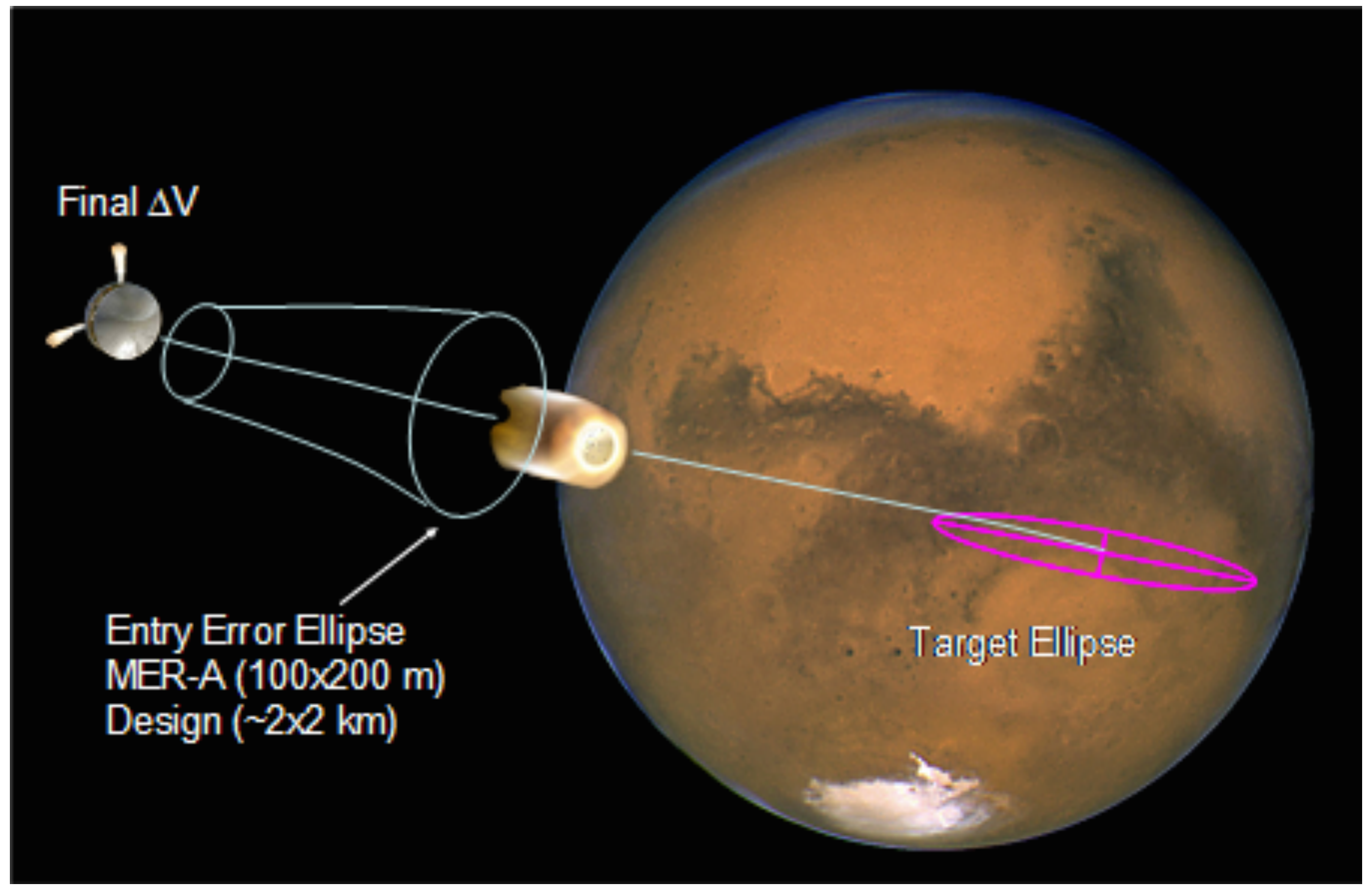
- Data generated from univariate normal distribution
- MC: PDF from kernel density estimation
- FP: PDF from spline interpolation
- Samples generated 1000 times for a given size. Plots show average error vs sample size

Requires $\frac{\partial F_i(\boldsymbol{x})}{\partial x_i}$.

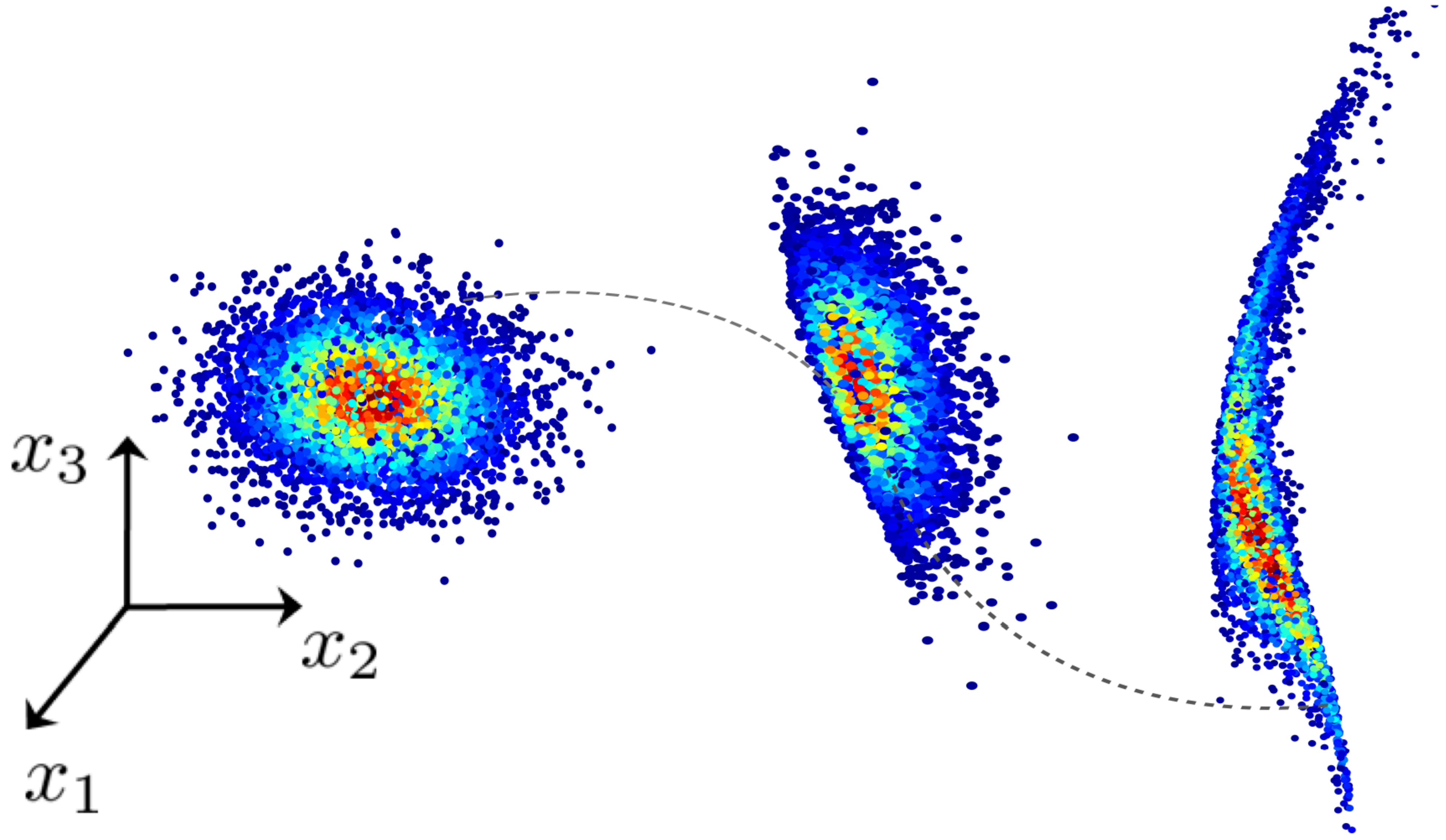
Uncertainty Propagation — Cartesian Coordinates

3 DOF Vinh's Equation Models motion of spacecraft during planetary entry

$$\begin{aligned}
\dot{h} &= V\sin(\gamma) \\
\dot{V} &= -\frac{\rho R_0}{2B_c}V^2 - \frac{gR_0}{v_c^2}\sin(\gamma) \\
\dot{\gamma} &= \frac{\rho R_0}{2B_c}\frac{C_L}{C_D}V + \frac{gR_0}{v_c^2}\cos(\gamma)\left(\frac{V}{R_0+h} - \frac{1}{V}\right).
\end{aligned}$$



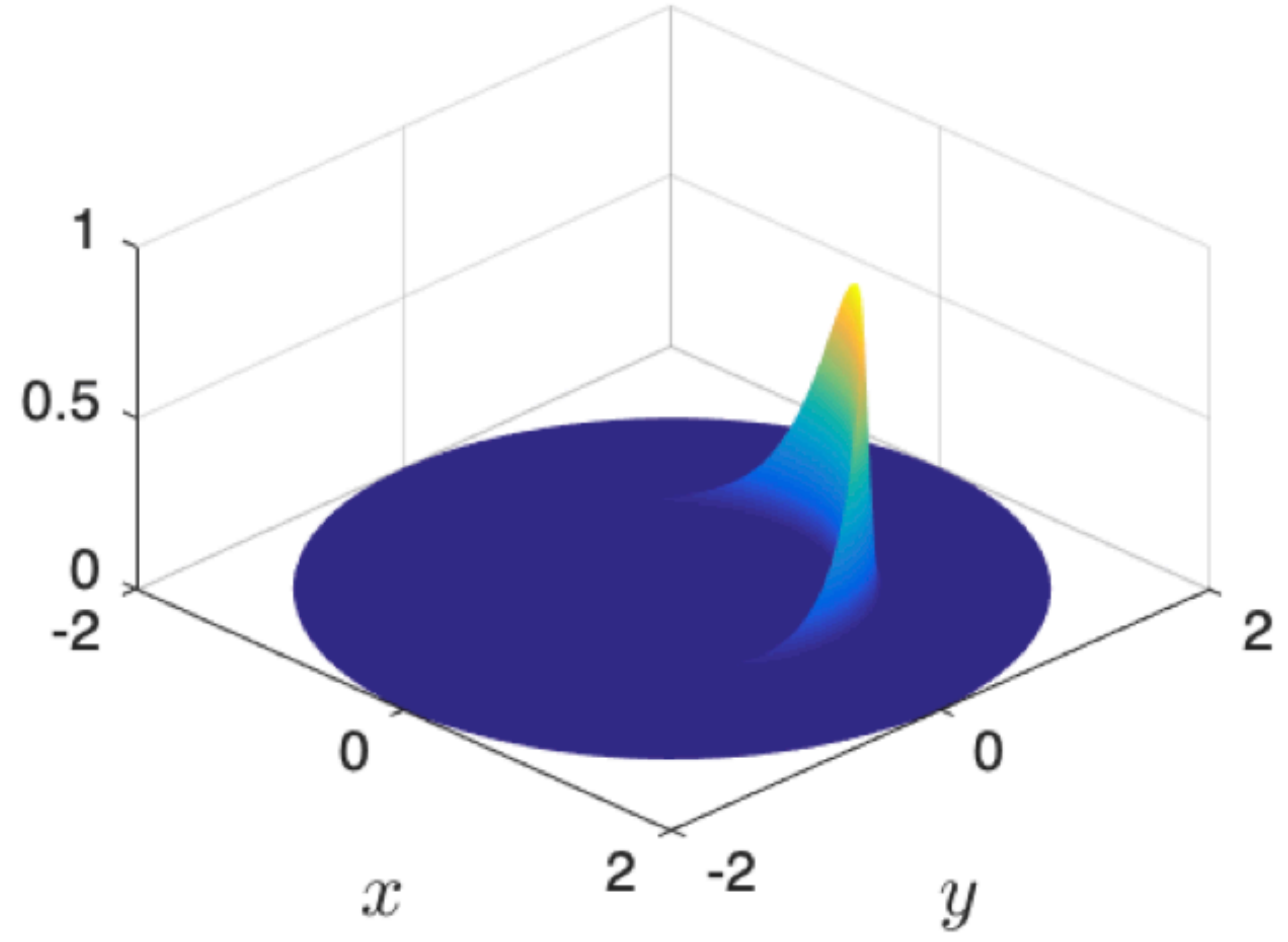
R_0 – radius of Mars
ρ – atmospheric density
v_c – escape velocity
$\frac{C_L}{C_D}$ – lift over drag
B_c – ballistic coefficient
h – height
V – velocity
γ – flight path angle

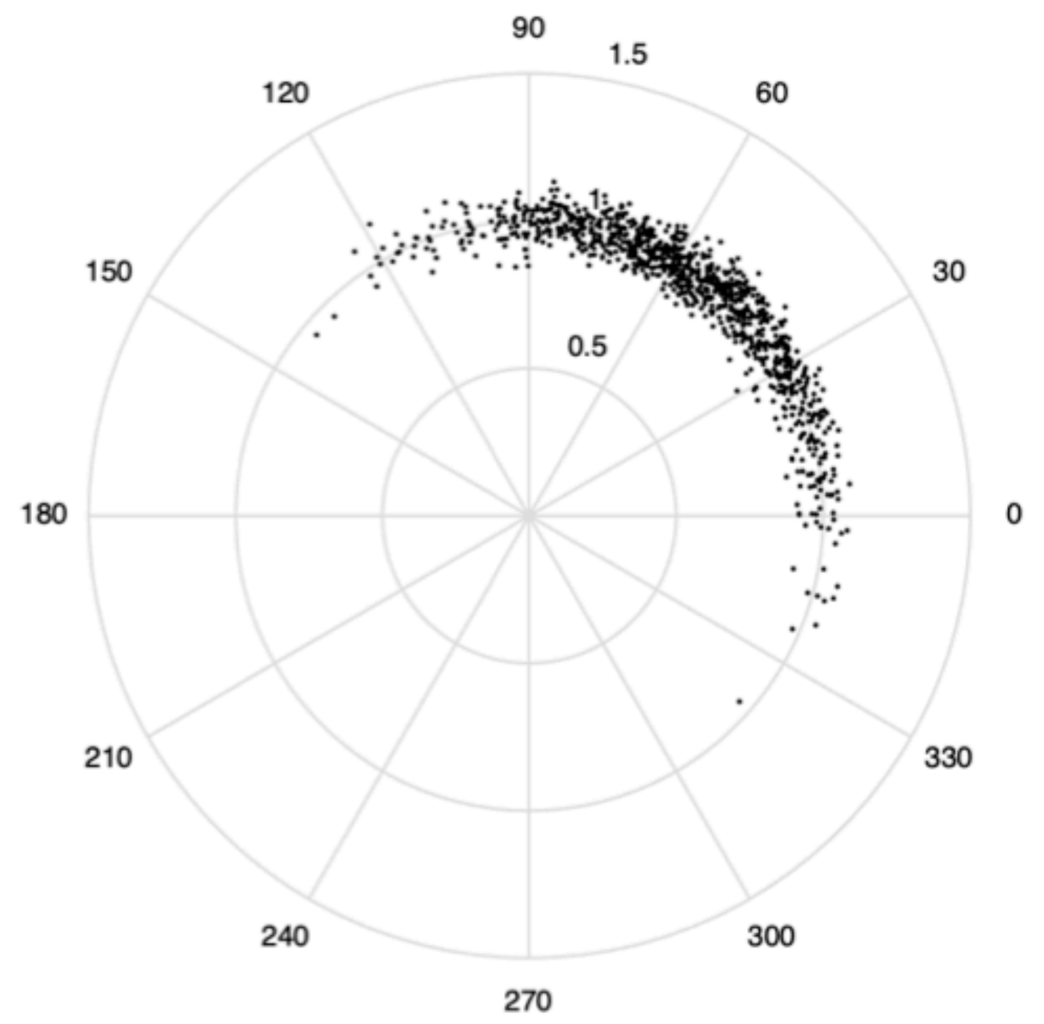


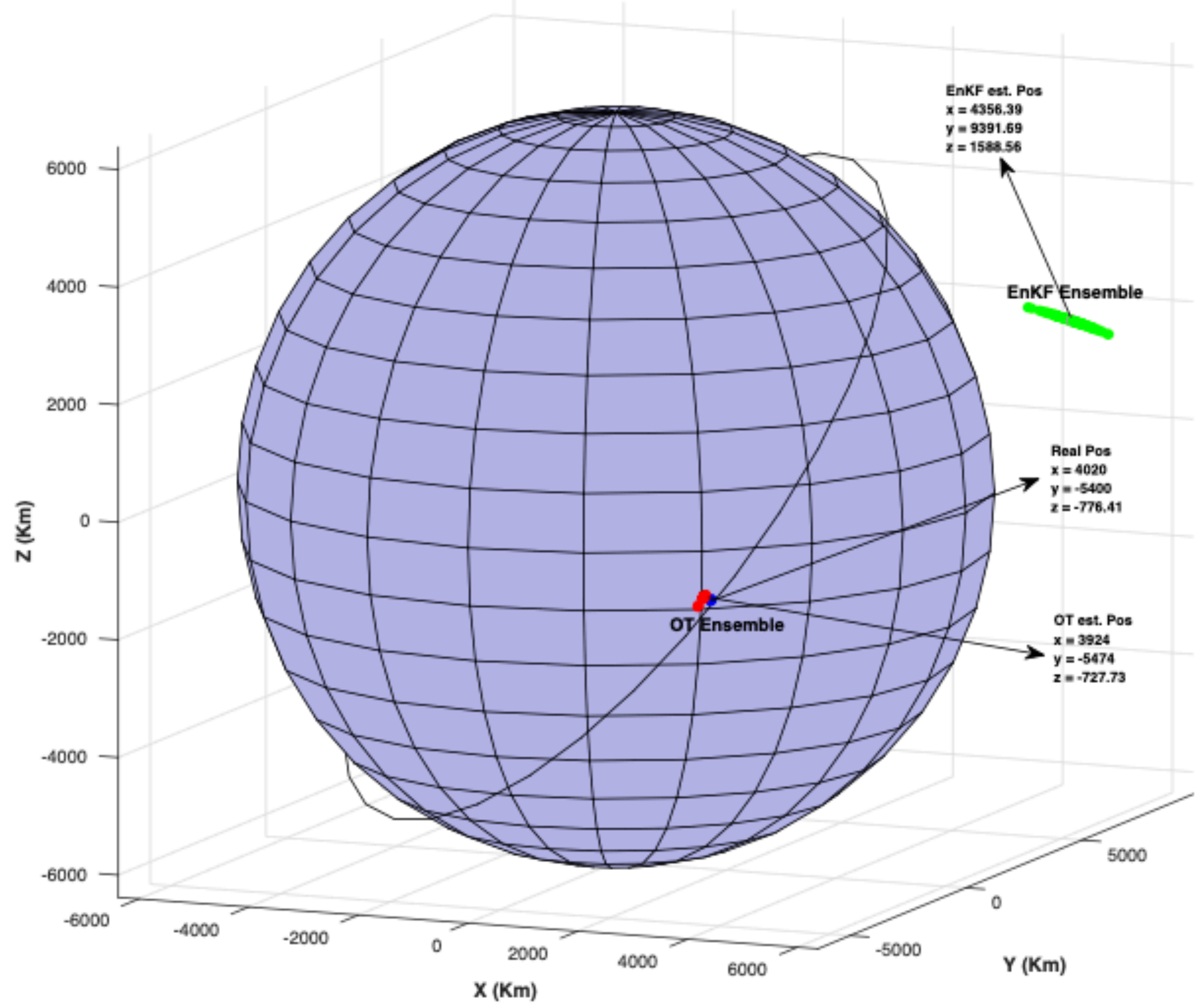
- Gaussian initial condition uncertainty in (h, V, γ)

Uncertainty Propagation — Cylindrical Coordinates

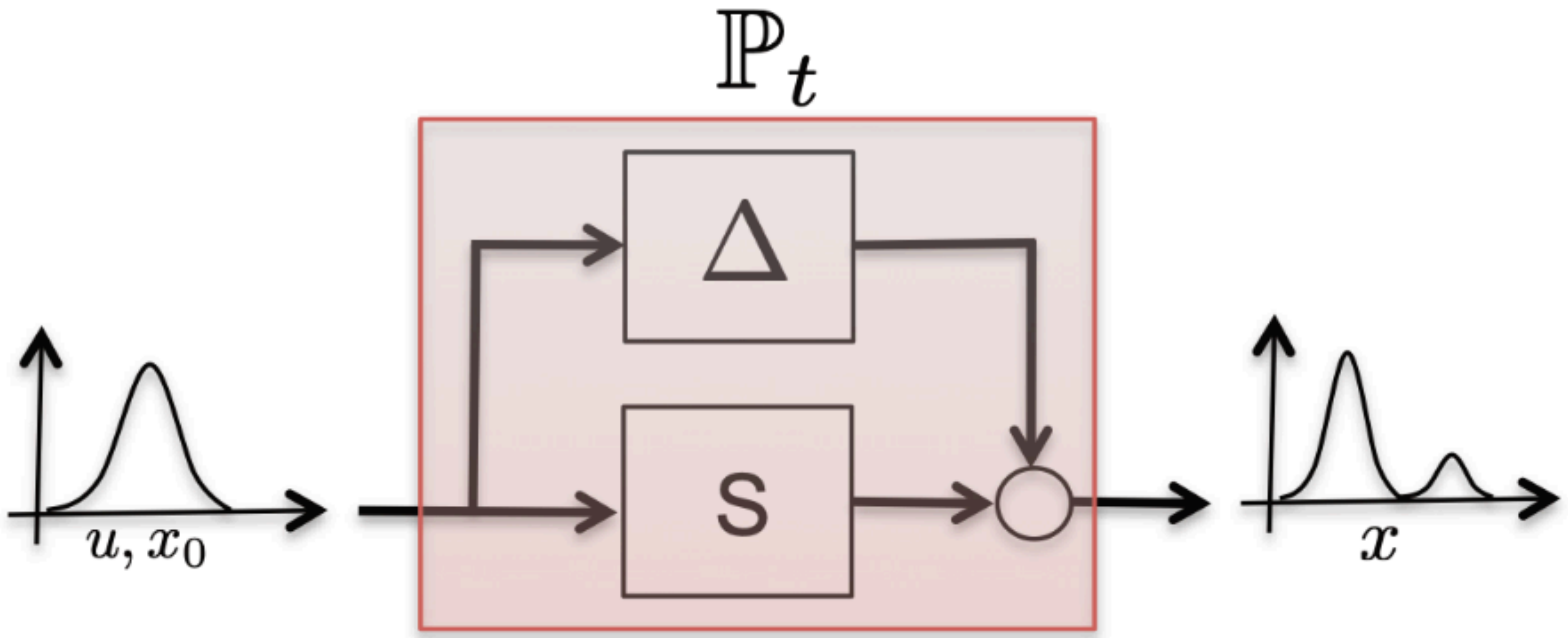
Space-Object Tracking



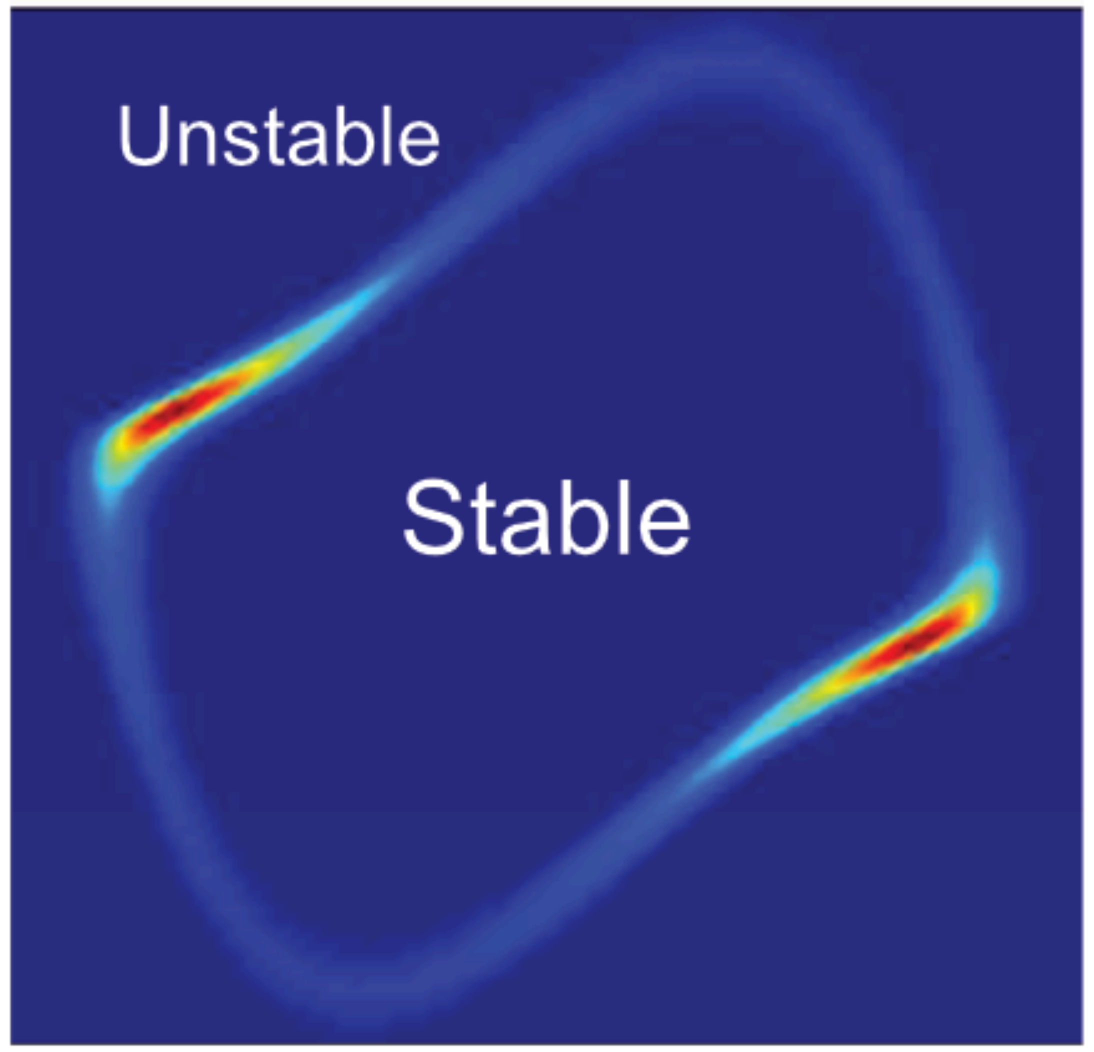




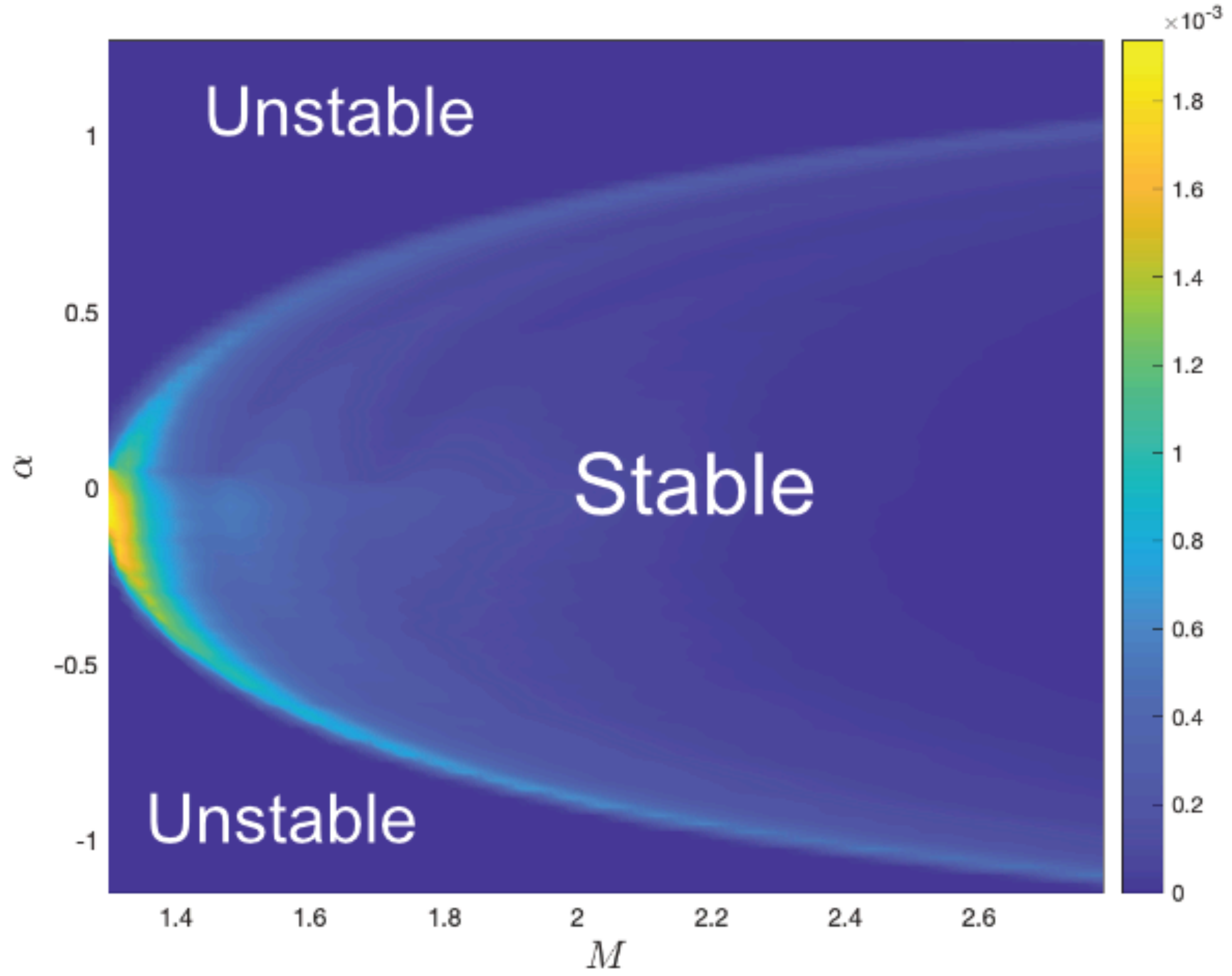
Region of Attraction



Flight control certification using Koopman and Frobenius-Perron operators enabled by scientific machine learning.



Region of attraction of the reverse-time Van der Pol oscillator with white-noise excitation.



Region of attraction of an LQR-based autopilot for a nonlinear missile model, trimmed at Mach 1.2 and 0 deg angle of attack.

Solve Steady-State FPKE

$$\sum_{i=1}^{n}\left(\frac{\partial F_i(\boldsymbol{x})}{\partial x_i} - F_i(\boldsymbol{x})\frac{\partial \eta(\boldsymbol{x})}{\partial x_i}\right) - \sum_{i=1}^{n}\sum_{j=1}^{n}\left[D_{ij}\left(\frac{\partial^2 \eta(\boldsymbol{x})}{\partial x_i \partial x_j} + \frac{\partial \eta(\boldsymbol{x})}{\partial x_i}\frac{\partial \eta(\boldsymbol{x})}{\partial x_j}\right)\right] = 0.$$

SciML Approach

- Neural-Network
- RBF-Net
- Nonlinear least-squares

F16Model.jl

Papers: https://isrlab.github.io/publications.html

Website: https://isrlab.github.io/index.html