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ASTRONOMY AND ASTROPHYSICS

Orientation of the JPL Ephemerides, DE 200/LE 200, to the Dynamical Equinox of J 2000

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Summary. The lunar and planetary ephemerides, DE 200/LE 200 have been recently produced at JPL. They will form the basis of the ephemerides in the "Astronomical Almanac" starting in the year 1984. The origin has been referenced to the J 2000 dynamical equinox of the ephemerides themselves. The procedure for this orientation is described here in detail. Analyses of the ephemerides also provide comparisons with the equinox of the FK 4 and with presently adopted values for the Earth's obliquity.

Key words: ephemerides – dynamical equinox

I. Introduction

This paper describes the procedure which was followed in order to orient the coordinate system of the JPL ephemerides, DE 200/LE 200, onto the dynamical equinox of the epoch J 2000. Section II presents some initial comments on the procedure and defines some of the necessary quantities. Section III gives a short summary of the procedure which is then given in more detail in Sect. IV. Some of the results are related to quantities of astronomical interest in Sect. V.

II. Initial Comments

The latest lunar and planetary ephemeris produced at JPL which was fit to observational data is DE 118/LE 62. This ephemeris was referenced to the equator and equinox of 1950, and it was integrated in that reference frame. Since the data included optical observations referred to the FK 4 system, the origin of the ephemeris frame should coincide fairly closely ($\pm 0\farcs05$) to the origin of the FK 4. Also, since the data included lunar laser-ranging observations, the true obliquity of the ecliptic at the mean epoch of these data (1975) should be well represented ($\pm 0\farcs01$) in DE 118/LE 62. This is so because the lunar data is highly sensitive to both the instantaneous Earth's equator and to the of-date orientation of the ecliptic.

It is DE 118/LE 62, exactly, which has been rotated onto the mean equator and dynamical equinox of J 2000 and which, at that time, was renumbered DE 200/LE 200.

Four comments apply to the motivation and procedure for the creation of the 3×3 matrix which is applied to both the positions and velocity of DE 118/LE 62 in order to produce DE 200/LE 200.

1. The attempt was made to put DE 200/LE 200 onto its own dynamical equinox of J 2000. As such, if one were to analyse DE 200/LE 200 in order to find the node of the mean ecliptic upon the J 2000 mean equator (x–y plane of the ephemeris), one should find that $\bar{\Omega}$(J 2000) = 0. Specifically, there was no attempt to relate the origin of DE 200/LE 200 to that of the FK 4 or to that of the FK 5, nor was there any use of Fricke's (1981) determination of the FK 4 equinox offset.

2. The accurate determination of the obliquity of the ecliptic, inherent in DE 118/LE 62, has been retained in DE 200/LE 200. There was no use of any defined value of the obliquity such as that adopted by IAU. To accomplish this, the transformation from DE 118/LE 62 to DE 200/LE 200 involved, other than precession, rotations about the z-axis only.

3. Attention is drawn to the difference between the equinox and obliquity computed in a rotating frame as opposed to those quantities computed in an inertial frame. These differences are given by Standish (1981). In order to be consistent with the conventions used classically, and especially those implied in the determination of Fricke (1981), the attempt made here was to put DE 200/LE 200 onto its own dynamical equinox as defined in the rotating sense.

4. All computations of the equinox and obliquity referred to in this paper were done using analyses of the motion of the Earth-Moon barycenter about the Sun as opposed to inferring these quantities from analyses of the motion of the Moon.

III. Procedure Summary

The transformation of DE 118/LE 62 in order to produce DE 200/LE 200 involved rotating DE 118/LE 62 onto its own dynamical equinox of 1950, precessing to J 2000, and then performing a minor adjustment to the equinox of J 2000. This procedure was done in the following five steps:

1. The 1950 dynamical equinox of DE 118 was determined to be $E^R_{118}(1950) = -\bar{\Omega}^R(1950) = +0\farcs5316$, where the superscript signifies that the "rotating" definition is used.

2. DE 118 was rotated onto its own dynamical equinox of 1950, producing DE 119:

$$\boldsymbol{r}_{119} = \boldsymbol{R}_z(-0\farcs5316)\boldsymbol{r}_{118}.$$

3. DE 119 was precessed to the epoch J 2000, using the 3×3 matrix, $\boldsymbol{P}$, given by Lieske (1979):

$$\boldsymbol{r}_{119'} = \boldsymbol{P}\boldsymbol{r}_{119}.$$

4. The dynamical equinox of DE 119′ at J 2000 was determined to be $E^R_{119'} = -0\farcs00073$.

5. DE 119′ was adjusted onto its own dynamical equinox of J 2000, thereby producing DE 200:

$r_{200} = R_z(+0''.00073)\,r_{119'}$.

IV. Detailed Procedure

The five steps followed in producing DE 200 from DE 118 will be explained in detail after the following comments:

The determination of the dynamical equinox from an integrated ephemeris was done by performing a least-squares fit (Fourier series plus a power series in time) to the motion of the instantaneous node of the Earth-Moon barycenter about the Sun. This node is defined by the plane normal to the angular momentum vector. Specifically, the instantaneous node at each point in time is defined by:

$\Omega = \tan^{-1}(h_x/-h_y)$,

where $\boldsymbol{h} = \boldsymbol{r}_B \times \dot{\boldsymbol{r}}_B$, with $\boldsymbol{r}_B$ and $\dot{\boldsymbol{r}}_B$ being the position and velocity of the heliocentric Earth-Moon barycenter. The arguments of the Fourier series represent combinations of the planetary mean motions, found by means of a spectral analysis of $\Omega(t)$ covering a 1400 yr interval centered on the epoch of the fit. The selection of terms, i.e., which to include or exclude, is somewhat arbitrary. In the present case, any term whose period is greater than 1000 yr has been excluded, as well as any term whose coefficient was shown to be less than 0″.004.

The constant term arising from the fit represents the mean node at the epoch of the power series in time and is equal to the negative equinox: $\bar{\Omega}^I = -E^I$. As indicated, this is the definition in the *inertial* sense, and so, a further correction must be applied:

for 1950, $E^{(R)} = E^{(I)} - 0''.09363$, and

for 2000, $E^{(R)} = E^{(I)} - 0''.09366$.

Since a well-determined fit requires an analysis of the motion of the Earth-Moon barycenter over many centuries, it was necessary to use an intermediary ephemeris which has been integrated both forward and backward over a long period of time. For this, the JPL ephemeris DE 102 was chosen. Since DE 102 differs from the more modern ephemerides, mainly by a pure rotation of axes, one may use the determination of $\bar{\Omega}$ from DE 102 and apply a simple correction, $\Delta\Omega$, found by comparing the two ephemerides at epoch.

Appendix A presents a summary of the comparisons of ephemerides which were used in this study, useful as a reference.

The details of the five steps to produce DE 200 starting with DE 118 are as follows:

1. The least-squares fit to DE 102 at 1950 yielded $E^I_{102}(1950) = +0''.22955$, and so $E^R_{102}(1950) = +0''.13592$. The various terms and resulting coefficients from the fit are presented in Appendix B. Comparison of DE 102 and DE 118 at 1950 gives:

$E_{118} - E_{102} = -(\bar{\Omega}_{118} - \bar{\Omega}_{102}) = +0''.39563$.

From these:

$E^R_{118}(1950) = 0''.13592 + 0''.39563 = +0''.53155$,

the dynamical equinox of DE 118 at 1950.

2. DE 119 was produced using:

$r_{119} = R_z(-0''.5316)\,r_{118}$.

(The angle actually used here was rounded slightly from the result in step I, but the difference is compensated for in steps 4 and 5.)

3. DE 119′ is precessed to J 2000:

$r_{119'} = P r_{119}$.

4. For analysing DE 119′, an intermediary, DE 102*, was found from:

$r_{102^*} = P R_z(-0''.1360)\,r_{102}$

which closely approximated the orientation of DE 119′. The least squares fit to DE 102* at J 2000 (detailed in Appendix B) gave:

$E^I_{102^*}(2000) = +0''.09304$

and thus

$E^R_{102^*}(2000) = -0''.00062$.

Finally, a comparison of DE 102* and DE 119′ at J 2000 gives $E_{119'} - E_{102^*} = -0''.00011$ so that

$E^R_{119'}(2000) = -0''.00073$.

5. DE 200 was produced by rotating DE 119′ onto its own equinox:

$r_{200} = R_z(+0''.00073)\,r_{119'}$.

The full transformation, then, from DE 118 to DE 200 is given by the product of the three matrices:

$r_{200} = R_z(+0''.00073)\,P R_z(-0''.53160)\,r_{118}$

$$= \begin{bmatrix} 0.9999256791774783 & -0.0111815116768724 & -0.0048590038154553 \\ 0.0111815116959975 & 0.9999374845751042 & -0.0000271625775175 \\ 0.0048590037714450 & -0.0000271704492210 & 0.9999881946023742 \end{bmatrix} r_{118}.$$

As a final check, DE 102 was rotated to conform exactly (±0″.00001) to DE 200 at the epoch J 2000 using the following:

$r_{102\dagger} = R_x(-0''.00036)\,R_y(-0''.12483)\,R_z(+0''.28854)\,P$

$\cdot\, R_z(-0''.13600)\,r_{102}$.

Comparison with DE 200 showed

$E_{200} - E_{102\dagger} = 0$

and an analysis of $r_{102\dagger}$ gave

$E^I_{102\dagger}(\mathrm{J\,2000}) = +0''.09366$

and so

$E^R_{102\dagger}(\mathrm{J\,2000}) = 0''.00000$.

V. Astronomical Constants

From the above analyses and comparisons, there are a number of features which merit further discussion:

1. The second adjustment (steps 4 and 5) was necessary for a number of reasons. The value of +0″.00073 arises from:

a) $+0.''00005$ in using only the approximate value of $0.''53160$ instead of $0.''53155$ in step 2.

b) $+0.''00003$ in transforming to the rotating definition at 1950 and then at J 2000:

$$[E^R(1950) - E^I(1950)] - [E^R(2000) - E^I(2000)] = +0.''00003.$$

c) $+0.''00014$ from the drift of the equinox between DE 102 and DE 118 occurring from 1950–2000:

$$[\Omega_{102}(1950) - \Omega_{118}(1950)]$$
$$- [\Omega_{102}(2000) - \Omega_{118}(2000)] = +0.''00014.$$

The remaining part, $+0.''00051$ would seem to be attributable to a discrepancy between the planetary part of the precession matrix, $\boldsymbol{P}$, and the motion of the ecliptic as given by DE 118. The value of $-\Delta p_A = +0.''00065$ from Bretagnon and Chapront (1981, Table II with $t=0.5$, $T=0$) is seen to be in exact agreement with this result (since they fit to DEIDZ).

2. The origin of the reference system of DE 118 should be *approximately* that of the FK 4, since the ephemeris has been fit to data which included transit observations of the U.S. Naval Observatory which have been referenced to the FK 4. As such, the determination of $E^R_{118}(1950)$ may be interpreted as a determination of the FK 4 equinox. The value found in step 1 of $+0.''53155$ agrees remarkably closely with that of Fricke (1981) for the FK 4 ($E_{\rm FK4}(1950) = 0.''525$). The difference of $0.''006$ must be fortuitous, however, for the expected accuracies of the two determinations are nearly an order of magnitude greater.

3. One may calculate the mean obliquity, $\bar{\varepsilon}$, at a given epoch directly from the ephemerides, using an analysis similar to the one used for computing $\bar{\Omega}$ above. This determination should be quite accurate ($\pm 0.''01$) since the data set used in the adjustment of the ephemerides included ten years of lunar laser ranging (see Sect. II). As such, the instantaneous obliquity at the mean epoch of the laser ranging data (1975) is well represented by the ephemerides. The analysis gives the following result:

$$\varepsilon^I_{200}(\text{J }2000) = 23°26'21.''40856$$

and correspondingly,

$$\varepsilon^R_{200}(\text{J }2000) = 23°26'21.''41190.$$

This latter number is then to be compared with IAU (1976) value of

$$\varepsilon_{\rm IAU} = 23°26'21.''448$$

giving $\varepsilon_{\rm IAU} - \bar{\varepsilon}^R_{200} = 0.''0361$, where again, the rotating sense of the definition has been used.

The details of the fits for $\bar{\varepsilon}$ are also given in Appendix B along with the fits for $\bar{\Omega}$.

4. From the analyses of $\bar{\Omega}$ and $\bar{\varepsilon}$, one may also derive the time derivatives of these quantities.

For DE 102, there resulted, for 1950,

$$\dot{\bar{\Omega}}_{102} = 11.''5001/\text{cty}, \quad \dot{\bar{\varepsilon}}_{102} = -46.''8088/\text{cty}$$

and for J 2000

$$\dot{\bar{\Omega}}_{102} = 10.''5562/\text{cty}, \quad \dot{\bar{\varepsilon}}_{102} = -46.''8105/\text{cty}.$$

Since DE 118 (or DE 200) represents an improvement over DE 102, one should add to the above numbers, the centennial drift of Ω and ε between DE 118 and DE 102, found by the intercomparisons of the two ephemerides. One then gets

$$\dot{\bar{\Omega}}(1950) = +11.''5004/\text{cty}, \quad \dot{\bar{\varepsilon}}(1950) = -46.''8087/\text{cty}$$

and

$$\dot{\bar{\Omega}}(2000) = +10.''5565/\text{cty}, \quad \dot{\bar{\varepsilon}}(2000) = -46.''8104/\text{cty}.$$

Recently, Bretagnon and Chapront (1982) have made an independent analysis of DE 200/LE 200 itself, using their analytical planetary and lunar theories (Bretagnon, 1980, 1981; Chapront-Touzé, 1980; Chapront-Touzé and Chapront, 1980). The analysis, covering 100 yr, produces a value for the dynamical equinox (rotating sense) of $+0.''00068$. One may conclude that DE 200/LE 200 is on its own dynamical equinox of J 2000.0 to within an accuracy of $\pm 0.''001$.

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Appendix A: Ephemeris Comparisons

Using the position and velocity of the Earth-Moon barycenter at a given epoch, T, the following pair of vector equations was solved in the least-squares sense for the rotational angles, $\theta_x, \theta_y, \theta_z$, which relate ephemeris DE(A) to ephemeris DE(B):

$$\boldsymbol{r}_A = \boldsymbol{R}_x(\theta_x)\,\boldsymbol{R}_y(\theta_y)\,\boldsymbol{R}_z(\theta_z)\,\boldsymbol{r}_B$$

and similarly for $\dot{\boldsymbol{r}}_A$ and $\dot{\boldsymbol{r}}_B$.

From these angles, one may find the differences in the nodes and obliquities from:

$$E_A - E_B = -(\Omega_A - \Omega_B) = \theta_z + \theta_y \operatorname{ctn} \varepsilon,$$

and

$$\varepsilon_A - \varepsilon_B = -\theta_x.$$

The results of these comparisons are given in Table A1

Table A1

A	B	Epoch	θ_x	θ_y	θ_z	ΔE	$\Delta\varepsilon$
118	102	1950	$-0.''00029$	$-0.''11718$	$+0.''66583$	$+0.''39563$	$+0.''00029$
119′	102*	1950	−0.00029	−0.11719	+0.27023	+0.00001	+0.00029
200	102*	1950	−0.00029	−0.11719	+0.27096	+0.00073	+0.00029
118	102	2000	−0.00036	−0.12483	+0.68341	+0.39549	+0.00036
119′	102*	2000	−0.00036	−0.12483	+0.28781	−0.00011	+0.00036
200	102*	2000	−0.00036	−0.12483	+0.28854	+0.00062	+0.00036

Appendix B: Least Squares Fits

In this study there were three least-squares fits to the instantaneous node and obliquity. For each, the following two functions were computed at 16,384 points in time covering 1435 yr (32-d intervals), centered at the epoch in question:

$$\Omega(t) = \tan^{-1}(h_x / -h_y)$$

and

$$\varepsilon(t) = \cos^{-1}(h_z / h),$$

where $\boldsymbol{h} = \boldsymbol{r}_B \times \dot{\boldsymbol{r}}_B$. These were then fit with a function of the form,

$$\sum_{i=0}^{3} a_i T^i + \sum_{i=1}^{57} (C_i' \cos \omega_i T + S_i \sin \omega_i T)$$

$$+ \sum_{i=1}^{7} (C_i \; T \cos \omega_i T + S_i' \; T \sin \omega_i T).$$

The frequencies, ω_i, are various combinations of seven basic frequencies which are the mean motions of Mercury through Saturn plus the lunar node, given in Table B1. The values are the original ones of Newcomb. The results using more modern values ($|\Delta\omega| < 20''$/cty) do not show any differences.

The three fits were performed using: a) DE 102 at B 1950.0, b) DE 102* at J 2000, and c) DE 102† at J 2000. The results for the first two are given in Tables B2 and B3. The results for the third are nearly identical to those of Table B3 except for the values of a_0.

From the fits, one obtains the results for the equinox, $E(=-\bar{\Omega})$, and mean obliquity, $\bar{\varepsilon}$, from the values of the constant term, a_0. These are given in Table B4.

Table B1. Basic frequencies of the least squares fits

	Period (d)	Rate (radians/cty)
1 Mercury	87.96925600	2608.79031812
2 Venus	224.70080000	1021.32855488
3 Earth-Moon	365.25636300	628.30758501
4 Mars	686.97971000	334.06131215
5 Jupiter	4332.58723000	52.96912241
6 Saturn	10759.19810000	21.32996727
7 Lunar node	6793.46000000	33.78151095

Table B2

DE102 (B1950.0)

********* NODE ********	******* OBLIQUITY ******	
-.229554	44.812768	1
11.500047	-46.808774	T
.492708	.052696	T**2
-.000310	.000524	T**3

COS	SIN	COS	SIN	1	2	3	4	5	6	7	MEAN MOTION	PERIOD
.036282	.005217	-.003344	.013369	0	0	0	0	2	-5	0	-.711592	322507.128950
-.018001	-.015757	.006019	-.007027	0	8	-13	0	0	0	0	2.629834	87265.335364
-.005917	-.008339	-.010616	-.005172	0	0	0	0	0	1	0	21.329967	10759.198100
-.010409	.048143	-.019191	-.004115	0	0	0	0	0	0	1	33.781511	6793.460000
-.020993	.023283	.008870	-.010651	0	0	1	-2	0	0	0	-39.815039	5763.986358
.032776	.076270	.029939	-.012871	0	0	0	0	0	2	0	42.659935	5379.599050
-.004442	-.016114	.033389	-.023056	0	0	0	0	1	0	0	52.969122	4332.587230
.010569	-.000021	-.000043	-.004135	0	0	0	0	0	3	0	63.989902	3586.399367
-.022632	-.500825	.193347	-.010791	0	3	-5	0	0	0	0	-77.552260	2959.208954
.013629	.010854	-.007740	.003357	0	0	2	-4	0	0	0	-79.630079	2881.993179
-.030981	.035363	-.013877	-.011754	0	5	-8	0	0	0	0	80.182094	2862.152020
.000379	-.000983	-.000399	-.000201	0	0	0	0	0	4	0	85.319869	2689.799525
-.050770	.376743	.151445	.020576	0	0	0	0	2	0	0	105.938245	2166.293615
.001128	-.002029	-.000796	-.000462	0	0	0	0	0	5	0	106.649836	2151.839620
-.000659	-.006992	.003102	.000722	0	0	3	-6	0	0	0	-119.445118	1921.328786
-.010962	-.000999	.016902	-.013399	0	2	-3	0	0	0	0	157.734355	1454.935697
-.039149	.020288	.008203	.015483	0	0	0	0	3	0	0	158.907367	1444.195743
.001977	-.001767	-.007692	.009107	0	1	-2	0	0	0	0	-235.286615	975.377810
.005094	-.001664	-.002340	-.000118	0	0	2	-3	0	0	0	254.431234	901.985736
-.005627	.006779	.003015	-.002784	0	0	1	-1	0	0	0	294.246273	779.936280
.005510	-.012759	.004975	.002038	0	4	-7	0	0	0	0	-312.838876	733.583200
-.025084	.092166	-.035520	-.009204	0	4	-6	0	0	0	0	315.468709	727.467848
.035651	.074640	.035003	-.091574	0	1	-1	0	0	0	0	393.020970	583.921370
.004939	.000164	-.000670	.002081	0	0	1	-3	0	0	0	-373.876351	613.821501
-.002565	-.004957	.002105	-.000950	0	0	1	0	-4	0	0	416.431095	551.095598
.008553	-.017913	.007455	.003983	0	0	1	0	-3	0	0	469.400218	488.907620
-.036536	-.098545	.038055	-.014606	0	2	-4	0	0	0	0	-470.573230	487.688905
-.005794	.003513	-.001407	-.002203	0	6	-9	0	0	0	0	473.203064	484.978566
-.003631	-.001854	.005833	-.002526	0	3	-4	0	0	0	0	550.755325	416.688379
-.006328	.003152	-.008402	.011900	0	0	1	0	-1	0	0	575.338463	398.884063
.011933	-.012126	-.004901	-.004752	0	0	1	0	1	0	0	681.276707	336.857757
.000657	-.006837	.002728	.000167	0	3	-6	0	0	0	0	-705.859845	325.125937
-.017496	.027369	-.010539	-.006776	0	5	-7	0	0	0	0	708.489679	323.919106
-.046478	.012621	.032582	-.051031	0	2	-2	0	0	0	0	786.041940	291.960685
-.044076	-.056133	.021721	-.017370	0	1	-3	0	0	0	0	-863.594200	265.742108
.005941	.001612	.000625	-.002564	0	0	2	0	-3	0	0	1097.707803	209.065967
-.012415	.010066	-.003871	-.004640	0	6	-8	0	0	0	0	1101.510649	208.344189
.016127	.029302	.013141	-.006623	0	0	2	0	-2	0	0	1150.676925	199.442031
-.027409	-.002936	.025071	-.021088	0	3	-3	0	0	0	0	1179.062910	194.640457
-.084915	.041105	.015979	.033541	0	0	2	0	0	0	0	1256.615170	182.628181
-.008312	.003232	-.001250	-.003096	0	7	-9	0	0	0	0	1494.531619	153.555362
-.015267	-.007360	.017976	-.007491	0	4	-4	0	0	0	0	1572.083880	145.980343
-.022579	.006747	.002454	.008728	0	1	1	0	0	0	0	1649.636140	139.117553

Table B2 (continued)

.000671	-.005314	-.002221	-.000354	0	0	3	0	-3	0	0	1726.015388	132.961354
.004760	.002333	.000929	-.001895	0	0	3	0	-1	0	0	1831.953633	125.272463
-.005301	.000307	-.000143	-.001966	0	8	-10	0	0	0	0	1887.552589	121.582490
-.007685	-.007640	.012094	-.001075	0	5	-5	0	0	0	0	1965.104849	116.784274
.009041	.003985	-.001418	.003695	1	0	-1	0	0	0	0	1980.482733	115.877477
-.036484	-.000813	-.000434	.014261	0	2	0	0	0	0	0	2042.657110	112.350400
-.003160	-.006375	.007599	.001694	0	6	-6	0	0	0	0	2358.125819	97.320228
-.024068	-.008268	-.003319	.009354	0	3	-1	0	0	0	0	2435.678080	94.221542
-.000642	-.004687	.004403	.002561	0	7	-7	0	0	0	0	2751.146789	83.417339
-.014162	-.010515	-.004149	.005461	0	4	-2	0	0	0	0	2828.699050	81.130350
-.007135	-.005910	-.003881	.002714	0	5	-3	0	0	0	0	3221.720019	71.233174
.006062	.000246	.000091	-.002418	1	0	1	0	0	0	0	3237.097903	70.894780
-.002629	-.008050	-.003127	.000964	0	6	-4	0	0	0	0	3614.740989	63.488185
-.000041	-.005880	-.002265	-.000026	0	7	-5	0	0	0	0	4007.761959	57.262219

T*COS	T*SIN	T*COS	T*SIN	1	2	3	4	5	6	7	MEAN MOTION	PERIOD
.000606	-.000439	-.000180	-.000235	0	0	0	0	0	2	0	42.659935	5379.599050
.004410	-.000602	.000199	.001687	0	3	-5	0	0	0	0	-77.552260	2959.208954
.002645	-.001107	-.000516	-.001054	0	0	0	0	2	0	0	105.938245	2166.293615
-.000847	-.000149	.000058	-.000320	0	4	-6	0	0	0	0	315.468709	727.467848
-.000734	.002074	.000134	.000233	0	1	-1	0	0	0	0	393.020970	583.921370
.000863	-.000454	.000120	.000314	0	2	-4	0	0	0	0	-470.573230	487.688905
.000431	.000642	.000273	-.000180	0	0	2	0	0	0	0	1256.615170	182.628181

Table B3

DE102* (J2000)

********* NODE ********	******* OBLIQUITY ******	
-.093045	21.408204	1
10.556211	-46.810530	T
.493115	.051096	T**2
-.000308	.000528	T**3

COS	SIN	COS	SIN	1	2	3	4	5	6	7	MEAN MOTION	PERIOD
.032189	.018825	-.007814	.011162	0	0	0	0	2	-5	0	-.711592	322507.128950
-.019903	.012921	-.005126	-.007696	0	8	-13	0	0	0	0	2.629834	87265.335364
.010108	-.002888	.008312	-.008293	0	0	0	0	0	1	0	21.329967	10759.198100
-.040343	-.028304	.011264	-.016041	0	0	0	0	0	0	1	33.781511	6793.460000
-.030272	-.006737	.013753	.002489	0	0	1	-2	0	0	0	-39.815039	5763.986358
.019799	-.080588	-.031649	-.007753	0	0	0	0	0	2	0	42.659935	5379.599050
-.016814	.000045	-.015137	-.037697	0	0	0	0	1	0	0	52.969122	4332.587230
.008758	-.005718	-.002337	-.003412	0	0	0	0	0	3	0	63.989902	3586.399367
.434602	-.250277	.098339	.166868	0	3	-5	0	0	0	0	-77.552260	2959.208954
-.016242	.005731	.001212	-.008399	0	0	2	-4	0	0	0	-79.630079	2881.993179
.046775	-.004262	.001896	.018067	0	5	-8	0	0	0	0	80.182094	2862.152020
.001084	.000203	.000080	-.000403	0	0	0	0	0	4	0	85.319869	2689.799525
.200235	-.322339	-.129415	-.080629	0	0	0	0	2	0	0	105.938245	2166.293615
-.001371	.001832	.000737	.000579	0	0	0	0	0	5	0	106.649836	2151.839620
.000318	.006948	-.003088	-.000830	0	0	3	-6	0	0	0	-119.445118	1921.328786
.010412	-.001998	-.011487	.018098	0	2	-3	0	0	0	0	157.734355	1454.935697
.007260	-.043595	-.017539	-.002688	0	0	0	0	3	0	0	158.907367	1444.195743
-.001855	-.001511	.010242	.006003	0	1	-2	0	0	0	0	-235.286615	975.377810
-.001574	-.005008	-.000137	.002363	0	0	2	-3	0	0	0	254.431234	901.985736
.008185	-.002989	-.004029	.000881	0	0	1	-1	0	0	0	294.246273	779.936280
-.003312	-.013532	.005194	-.001344	0	4	-7	0	0	0	0	-312.838876	733.583200
.034837	.089039	-.034052	.013724	0	4	-6	0	0	0	0	315.468709	727.467848
.066121	-.047563	-.096217	-.019728	0	1	-1	0	0	0	0	393.020970	583.921370
.000323	-.004900	.002068	.000729	0	0	1	-3	0	0	0	-373.876351	613.821501
-.005533	-.001386	.000652	-.002178	0	0	1	0	-4	0	0	416.431095	551.095598
-.019473	.003736	-.001226	-.008375	0	0	1	0	-3	0	0	469.400218	488.907620
.065486	.082347	-.031650	.025673	0	2	-4	0	0	0	0	-470.573230	487.688905
.000423	-.006775	.002612	.000128	0	6	-9	0	0	0	0	473.203064	484.978566
-.000075	-.004016	.005009	.003974	0	3	-4	0	0	0	0	550.755325	416.688379
-.004714	-.005652	-.013365	-.005665	0	0	1	0	-1	0	0	575.338463	398.884063
-.009392	-.014165	-.005658	.003799	0	0	1	0	1	0	0	681.276707	336.857757
.006327	-.002734	.001064	.002531	0	3	-6	0	0	0	0	-705.859845	325.125937
.031663	-.007456	.002797	.012201	0	5	-7	0	0	0	0	708.489679	323.919106
.039839	-.025531	-.014889	.058791	0	2	-2	0	0	0	0	786.041940	291.960685
-.048642	.052266	-.020527	-.018799	0	1	-3	0	0	0	0	-863.594200	265.742108
-.002556	-.005953	-.002503	.001138	0	0	2	0	-3	0	0	1097.707803	209.065967
-.001135	-.015907	.006038	-.000519	0	6	-8	0	0	0	0	1101.510649	208.344189
-.026924	-.019691	-.009107	.011509	0	0	2	0	-2	0	0	1150.676925	199.442031
-.009531	-.025318	.030365	.012474	0	3	-3	0	0	0	0	1179.062910	194.640457
-.084636	.041548	.016164	.033426	0	0	2	0	0	0	0	1256.615170	182.628181
-.008883	-.000699	.000225	-.003341	0	7	-9	0	0	0	0	1494.531619	153.555362
-.016431	.002959	.009941	-.016797	0	4	-4	0	0	0	0	1572.083880	145.980343

Table B3 (continued)

.010403	.021152	.008202	-.003872	0	1	1	0	0	0	0	1649.636140	139.117553
-.004607	.002661	.001024	.002059	0	0	3	0	-3	0	0	1726.015388	132.961354
-.001248	.005140	.002046	.000498	0	0	3	0	-1	0	0	1831.953633	125.272463
-.001218	.005167	-.001928	-.000427	0	8	-10	0	0	0	0	1887.552589	121.582490
.000068	.010695	-.009461	-.007646	0	5	-5	0	0	0	0	1965.104849	116.784274
-.009667	.002107	-.001028	-.003814	1	0	-1	0	0	0	0	1980.482733	115.877477
.034810	-.010967	-.004176	-.013643	0	2	0	0	0	0	0	2042.657110	112.350400
.006929	.001130	-.005734	.005289	0	6	-6	0	0	0	0	2358.125819	97.320228
-.004001	-.025119	-.009812	.001463	0	3	-1	0	0	0	0	2435.678080	94.221542
.001482	-.004440	.002874	.004219	0	7	-7	0	0	0	0	2751.146789	83.417339
-.017639	.000187	-.000002	.006861	0	4	-2	0	0	0	0	2828.699050	81.130350
-.001664	.012102	.004681	.000703	0	5	-3	0	0	0	0	3221.720019	71.233174
-.004985	.003436	.001378	.001982	1	0	1	0	0	0	0	3237.097903	70.894780
.008135	.002375	.000960	-.003127	0	6	-4	0	0	0	0	3614.740989	63.488185
.002462	-.005348	-.002043	-.000977	0	7	-5	0	0	0	0	4007.761959	57.262219
T*COS	T*SIN	T*COS	T*SIN	1	2	3	4	5	6	7	MEAN MOTION	PERIOD
-.000758	-.000115	-.000033	.000296	0	0	0	0	0	2	0	42.659935	5379.599050
.002586	.003636	-.001412	.000943	0	3	-5	0	0	0	0	-77.552260	2959.208954
-.002822	-.000091	.000028	.001157	0	0	0	0	2	0	0	105.938245	2166.293615
-.000765	.000387	-.000139	-.000293	0	4	-6	0	0	0	0	315.468709	727.467848
.002152	.000403	.000205	-.000174	0	1	-1	0	0	0	0	393.020970	583.921370
-.000652	.000702	-.000213	-.000269	0	2	-4	0	0	0	0	-470.573230	487.688905
.000400	.000668	.000263	-.000191	0	0	2	0	0	0	0	1256.615170	182.628181

Table B4. Least squares results for the equinox and mean obliquity

Ephemeris	Epoch	E^I	E^R	$\bar{\varepsilon}^I$	$\bar{\varepsilon}^R$
DE 102	B 1950.0	+0.″22955	+0.″13592	23°26′44.″81277	23°26′44.″81641
DE 102*	J 2000	−0.09304	−0.00062	23 26 21.40820	23 26 21.41154
DE 102†	J 2000	+0.09366	0.00000	23 26 21.40856	23 26 21.41190

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