My Title

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Abstract

Here's the abstract

1 Simple case

Consider the problem

$$\text{minimize} \quad \frac{1}{2}\mathbf{x}^T\mathbf{Q}_0\mathbf{x} + \mathbf{g}_0^T\mathbf{x} \tag{1a}$$

$$\text{subject to} \quad \frac{1}{2}\mathbf{x}^T\mathbf{Q}_1\mathbf{x} \leq c \tag{1b}$$

for positive-semidefinite $\mathbf{Q}_1$. Following chapter 4.3 of Nocedal & Wright, consider $\mathbf{Q}_0+\lambda\mathbf{Q}_1$ and assume that λ is big enough to make this sum positive-definite. We seek a λ such that Eq. (1b) is satisfied at

$$\mathbf{x} = (\mathbf{Q}_0 + \lambda\mathbf{Q}_1)^{-1}(-\mathbf{g}_0). \tag{2}$$

Consider

$$\phi_2(\lambda) = \frac{1}{c} - \frac{2}{\mathbf{x}^T\mathbf{Q}_1\mathbf{x}}, \tag{3}$$

which exhibits a root at λ such that Eq. (1b) becomes an equality. We can therefore update λ by Newton's method

$$\lambda^{(l+1)} = \lambda^{(l)} - \frac{\phi_2(\lambda)}{\phi_2'(\lambda)}. \tag{4}$$

Since

$$\phi_2'(\lambda) = \frac{4\mathbf{x}^T\mathbf{Q}_1\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\lambda}}{\left(\mathbf{x}^T\mathbf{Q}_1\mathbf{x}\right)^2}, \tag{5}$$

we have

$$\frac{\phi_2(\lambda)}{\phi_2'(\lambda)} = \frac{\mathbf{x}^T\mathbf{Q}_1\mathbf{x}}{2\mathbf{x}^T\mathbf{Q}_1\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\lambda}}\frac{\frac{1}{2}\mathbf{x}^T\mathbf{Q}_1\mathbf{x} - c}{c}. \tag{6}$$

From Eq. (2), we have

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}\lambda} = -(\mathbf{Q}_0 + \lambda\mathbf{Q}_1)^{-1}\mathbf{Q}_1(\mathbf{Q}_0 + \lambda\mathbf{Q}_1)^{-1}(-\mathbf{g}_0) \tag{7}$$

$$= -(\mathbf{Q}_0 + \lambda\mathbf{Q}_1)^{-1}\mathbf{Q}_1\mathbf{x}. \tag{8}$$

2 Cholesky factorization of symmetric tridiagonal matrices

In this case the factorization is simple:

$$A_{ii} = L_{i,i-1}^2 + L_{ii}^2 \tag{9a}$$

$$A_{i+1,i} = L_{i+1,i}L_{ii}. \tag{9b}$$

We initiate it with $L_{11} = \sqrt{A_{11}}$, and then iterate

$$L_{i+1,i} = \frac{A_{i+1,i}}{L_{ii}}; \tag{10a}$$

$$L_{i+1,i+1} = \sqrt{A_{i+1,i+1} - L_{i+1,i}^2}. \tag{10b}$$

`http://www.mat.uc.pt/~cmf/papers/tri_positive_definite_revisited.pdf` contains a number of potential conditions that could be leveraged for guessing an initial λ, for example

$$4\cos^2\left(\frac{\pi}{n+1}\right)b_i^2 < a_i a_{i+1}, \tag{11}$$

where b_i are the off-diagonal values and a_i the diagonals. This results in a quadratic inequality for λ.

Figure 1: What a figure!

Table 1: Who needs tables?