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# COUNTERFACTUAL TRAINING: TEACHING MODELS PLAUSIBLE AND ACTIONABLE EXPLANATIONS

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## ABSTRACT

We propose a novel training regime called Counterfactual Training that leverages counterfactual explanations to increase the explanatory capacity of models. Counterfactual explanations have emerged as a popular post-hoc explanation method for opaque machine learning models: they inform how factual inputs would need to change in order for a model to produce some desired output. To be useful in real-word decision-making systems, counterfactuals should be plausible with respect to the underlying data and actionable with respect to stakeholder requirements. Much existing research has therefore focused on developing post-hoc methods to generate counterfactuals that meet these desiderata. In this work, we instead hold models directly accountable for this desired end goal: Counterfactual Training employs counterfactuals ad-hoc during the training phase to minimize the divergence between learned representations and plausible, actionable explanations. We demonstrate empirically and theoretically that our proposed method facilitates training models that deliver inherently desirable explanations while maintaining high predictive performance.

**Keywords** Counterfactual Training • Counterfactual Explanations • Algorithmic Recourse • Explainable AI • Representation Learning

**1 Introduction**

Today's prominence of artificial intelligence (AI) has largely been driven by advances in **representation learning**: instead of relying on features and rules that are carefully hand-crafted by humans, modern machine learning (ML)

18 models are tasked with learning the representations directly from data, guided by narrow objectives such as predictive  
 19 accuracy (I. Goodfellow, Bengio, and Courville 2016). Modern advances in computing have made it possible to  
 20 provide such models with ever growing degrees of freedom to achieve that task, which has often led them to outperform  
 21 traditionally more parsimonious models. Unfortunately, in doing so, the models learn increasingly complex and highly  
 22 sensitive representations that we can no longer easily interpret.

23 The trend towards complexity for the sake of performance has come under serious scrutiny in recent years. At the  
 24 very cusp of the deep learning revolution, Szegedy et al. (2013) showed that artificial neural networks (ANN) are  
 25 sensitive to adversarial examples: counterfactuals of model inputs that yield vastly different model predictions despite  
 26 being “imperceptible” in that they are semantically indifferent from their factual counterparts. Although some partially  
 27 effective mitigation strategies have been proposed, for example **adversarial training** (I. J. Goodfellow, Shlens, and  
 28 Szegedy 2014), truly robust deep learning (DL) remains unattainable even for models that are considered shallow by  
 29 today’s standards (Kolter 2023).

30 Part of the problem is that the high degrees of freedom provide room for many solutions that are locally optimal with  
 31 respect to narrow objectives (Wilson 2020).<sup>1</sup> Indeed, recent work on the so called “lottery tickets” suggests that modern  
 32 neural networks can be pruned by up to 90% while preserving their predictive performance (Frankle and Carbin 2019)  
 33 and generalizability (Morcos et al. 2019). Similarly, Zhang et al. (2021) showed that state-of-the-art neural networks  
 34 are so expressive that they can fit randomly labeled data. Thus, looking at the predictive performance, the solutions may  
 35 seem to provide compelling explanations for the data, when in fact they are based on purely associative, semantically  
 36 meaningless patterns. This poses two related challenges. Firstly, there is no dependable way to verify if such complex  
 37 representations correspond to meaningful and plausible explanations. Secondly, even if we could resolve the first  
 38 challenge, it remains undecided how to ensure that models can *only* learn valuable explanations.

39 The first challenge has attracted an abundance of research on **explainable AI** (XAI) which aims to develop tools to  
 40 derive explanations from complex model representations. This can mitigate a scenario in which we deploy opaque  
 41 models and blindly rely on their predictions. On countless occasions, this scenario has occurred in practice and caused  
 42 real harm to people who were affected adversely and often unfairly by automated decision-making (ADM) systems  
 43 involving opaque models (O’Neil 2016; McGregor 2021). Effective XAI tools can aid us in monitoring models and  
 44 providing recourse to individuals to turn adverse outcomes (e.g., “loan application rejected”) into positive ones (e.g.,  
 45 “application accepted”). Wachter, Mittelstadt, and Russell (2017) propose **counterfactual explanations** (CE) as an  
 46 effective approach to achieve this goal: CEs explain how factual inputs need to change in order for some fitted model  
 47 to produce some desired output, typically involving minimal perturbations.

48 To our surprise, the second challenge has not yet attracted any major consolidated research effort. Specifically, there  
 49 has been no concerted effort towards improving models’ explanatory capacity, which we will henceforth  
 50 simply call “explainability”, defined as the degree to which learned representations correspond to explanations that are  
 51 interpretable and deemed **plausible** by humans (see Definition 3.1). Instead, the choice has typically been to improve  
 52 the ability of XAI tools to identify the subset explanations that are both plausible and valid for any given model,  
 53 independent of whether the learned representations are also compatible with implausible explanations (Altmeyer et  
 54 al. 2024). Fortunately, recent findings indicate that explainability can arise as byproduct of regularization techniques  
 55 aimed at other objectives such as robustness, generalization, and generative capacity (Schut et al. 2021; Augustin,  
 56 Meinke, and Hein 2020; Altmeyer et al. 2024).

57 Building on these findings, we introduce **counterfactual training**: a novel training regime explicitly geared towards  
 58 aligning model representations with plausible explanations. Our contributions are as follows:

- 59 • We discuss existing related work on improving models and consolidate it through the lens of counterfactual  
   explanations (Section 2).
- 60 • We present our proposed methodological framework that leverages faithful counterfactual explanations during  
   the training phase of models to achieve the explainability objective (Section 3).
- 61 • Through extensive experiments we demonstrate the counterfactual training improve model explainability  
   while maintaining high predictive performance. We run ablation studies and grid searches to understand  
   how the underlying model components and hyperparameters affect outcomes. (Section 4).

66 Despite some limitations of our approach discussed in Section 5, we conclude in Section 6 that counterfactual training  
 67 provides a practical framework for researchers and practitioners interested in making opaque models more trustworthy.  
 68 We also believe that this work serves as an opportunity for XAI researchers to re-evaluate the trend of improving XAI  
 69 tools without improving the underlying models.

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<sup>1</sup>We follow the standard ML convention, where “degrees of freedom” refer to the number of parameters estimated from data.

## 70 2 Related Literature

71 To the best of our knowledge, our proposed framework of counterfactual training represents the first attempt to use  
 72 counterfactual explanations during training to improve model explainability. In high-level terms, we define model  
 73 explainability as the extent to which valid explanations derived for an opaque model are also deemed plausible with  
 74 respect to the underlying data and stakeholder requirements. To make this more concrete, we follow Augustin, Meinke,  
 75 and Hein (2020) in tying the concept of explainability to the quality of counterfactual explanations that we can generate  
 76 for a given model. The authors show that counterfactual explanations—understood here as minimal input perturbations  
 77 that yield some desired model prediction—are generally more meaningful if the underlying model is more robust to  
 78 adversarial examples. We can make intuitive sense of this finding when looking at adversarial training (AT) through  
 79 the lens of representation learning with high degrees of freedom: by inducing models to “unlearn” representations that  
 80 are susceptible to worst-case counterfactuals (i.e., adversarial examples), AT effectively removes some implausible  
 81 explanations from the solution space.

### 82 2.1 Adversarial Examples are Counterfactual Explanations

83 This interpretation of the link between explainability through counterfactuals on one side, and robustness to adversarial  
 84 examples on the other, is backed by empirical evidence. Sauer and Geiger (2021) demonstrate that using counterfactual  
 85 images during classifier training improves model robustness. Similarly, Abbasnejad et al. (2020) argue that counter-  
 86 factuals represent potentially useful training data in machine learning, especially in supervised settings where inputs  
 87 may be reasonably mapped to multiple outputs. They, too, demonstrate the augmenting the training data of image  
 88 classifiers can improve generalization. Teney, Abbasnejad, and Hengel (2020) propose an approach using counterfac-  
 89 tuals in training that does not rely on data augmentation: they argue that counterfactual pairs typically already exist  
 90 in training datasets. Specifically, their approach relies on identifying similar input samples with different annotations  
 91 and ensuring that the gradient of the classifier aligns with the vector between such pairs of counterfactual inputs using  
 92 the cosine distance as the loss function.

93 In the natural language processing (NLP) domain, counterfactuals have similarly been used to improve models through  
 94 data augmentation: Wu et al. (2021), propose *Polyjuice*, a general-purpose counterfactual generator for language mod-  
 95 els. They demonstrate empirically that augmenting training data through *Polyjuice* counterfactuals improves robust-  
 96 ness in a number of NLP tasks. Balashankar et al. (2023) also use *Polyjuice* to augment NLP datasets through diverse  
 97 counterfactuals and show that classifier robustness improves up to 20%. Finally, Luu and Inoue (2023) introduce  
 98 Counterfactual Adversarial Training (CAT), which also aims at improving generalization and robustness of language  
 99 models. Specifically, they propose to proceed as follows: firstly, they identify training samples that are subject to  
 100 high predictive uncertainty; secondly, they generate counterfactual explanations for those samples; and, finally, they  
 101 fine-tune the given language model on the augmented dataset that includes the generated counterfactuals.

102 There have also been several attempts at formalizing the relationship between counterfactual explanations and adver-  
 103 sarial examples (AE). Pointing to clear similarities in how CE and AE are generated, Freiesleben (2022) makes the  
 104 case for jointly studying the opaqueness and robustness problem in representation learning. Formally, AE can be seen  
 105 as the subset of CE for which misclassification is achieved (Freiesleben 2022). Similarly, Pawelczyk et al. (2022)  
 106 show that CE and AE are equivalent under certain conditions and derive theoretical upper bounds on the distances  
 107 between them.

108 Two recent works are closely related to ours in that they use counterfactuals during training with the explicit goal  
 109 of affecting certain properties of post-hoc counterfactual explanations. Firstly, Ross, Lakkaraju, and Bastani (2024)  
 110 propose a way to train models that are guaranteed to provide recourse for individuals to move from an adverse outcome  
 111 to some positive target class with high probability. Their approach builds on adversarial training, where in this context  
 112 susceptibility to targeted adversarial examples for the positive class is explicitly induced. The proposed method allows  
 113 for imposing a set of actionability constraints ex-ante: for example, users can specify that certain features (e.g., *age*,  
 114 *gender*, ...) are immutable. Secondly, Guo, Nguyen, and Yadav (2023) are the first to propose an end-to-end training  
 115 pipeline that includes counterfactual explanations as part of the training procedure. In particular, they propose a  
 116 specific network architecture that includes a predictor and CE generator network, where the parameters of the CE  
 117 generator network are learnable. Counterfactuals are generated during each training iteration and fed back to the  
 118 predictor network. In contrast to Guo, Nguyen, and Yadav (2023), we impose no restrictions on the neural network  
 119 architecture at all.

### 120 2.2 Beyond Robustness

121 Improving the adversarial robustness of models is not the only path towards aligning representations with plausible  
 122 explanations. In a work closely related to this one, Altmeyer et al. (2024) show that explainability can be improved  
 123 through model averaging and refined model objectives. The authors propose a way to generate counterfactuals that  
 124 are maximally **faithful** to the model in that they are consistent with what the model has learned about the underlying

125 data. Formally, they rely on tools from energy-based modelling to minimize the divergence between the distribution  
 126 of counterfactuals and the conditional posterior over inputs learned by the model. Their proposed counterfactual  
 127 explainer, *ECCo*, yields plausible explanations if and only if the underlying model has learned representations that  
 128 align with them. They find that both deep ensembles (Lakshminarayanan, Pritzel, and Blundell 2017) and joint energy-  
 129 based models (JEMs) (Grathwohl et al. 2020) tend to do well in this regard.

130 Once again it helps to look at these findings through the lens of representation learning with high degrees of freedom.  
 131 Deep ensembles are approximate Bayesian model averages, which are most called for when models are underspecified  
 132 by the available data (Wilson 2020). Averaging across solutions mitigates the aforementioned risk of relying on a  
 133 single locally optimal representations that corresponds to semantically meaningless explanations for the data. Previous  
 134 work by Schut et al. (2021) similarly found that generating plausible (“interpretable”) counterfactual explanations is  
 135 almost trivial for deep ensembles that have also undergone adversarial training. The case for JEMs is even clearer:  
 136 they involve a hybrid objective that induces both high predictive performance and generative capacity (Grathwohl et al.  
 137 2020). This is closely related to the idea of aligning models with plausible explanations and has inspired our proposed  
 138 counterfactual training objective, as we explain in Section 3.

### 139 3 Counterfactual Training

140 Counterfactual training combines ideas from adversarial training, energy-based modelling and counterfactuals expla-  
 141 nitions with the explicit objective of aligning representations with plausible explanations that comply with user re-  
 142 quirements. In the context of CEs, plausibility has broadly been defined as the degree to which counterfactuals comply  
 143 with the underlying data generating process (Poyiadzi et al. 2020; Guidotti 2022; Altmeyer et al. 2024). Plausibility  
 144 is a necessary but insufficient condition for using CEs to provide algorithmic recourse (AR) to individuals affected  
 145 by opaque models in practice. This is because for recourse recommendations to be **actionable**, they need to not only  
 146 result in plausible counterfactuals but also be attainable. A plausible CE for a rejected 20-year-old loan applicant, for  
 147 example, might reveal that their application would have been accepted, if only they were 20 years older. Ignoring  
 148 all other features, this would comply with the definition of plausibility if 40-year-old individuals were in fact more  
 149 credit-worthy on average than young adults. But of course this CE does not qualify for providing actionable recourse  
 150 to the applicant since *age* is not a (directly) mutable feature. For our intents and purposes, counterfactual training aims  
 151 to improve model explainability by aligning models with counterfactuals that meet both desiderata, plausibility and  
 152 actionability. Formally, we define explainability as follows:

153 **Definition 3.1** (Model Explainability). Let  $M_\theta : \mathcal{X} \mapsto \mathcal{Y}$  denote a supervised classification model that maps from the  
 154  $D$ -dimensional input space  $\mathcal{X}$  to representations  $\phi(\mathbf{x}; \theta)$  and finally to the  $K$ -dimensional output space  $\mathcal{Y}$ . Assume  
 155 that for any given input-output pair  $\{\mathbf{x}, \mathbf{y}\}_i$  there exists a counterfactual  $\mathbf{x}' = \mathbf{x} + \Delta : M_\theta(\mathbf{x}') = \mathbf{y}^+ \neq \mathbf{y} = M_\theta(\mathbf{x})$   
 156 where  $\arg \max_y \mathbf{y}^+ = y^+$  and  $y^+$  denotes the index of the target class.

157 We say that  $M_\theta$  is **explainable** to the extent that faithfully generated counterfactuals are plausible (i.e. consistent with  
 158 the data) and actionable. Formally, we define these properties as follows:

- 159 1. (Plausibility)  $\int^A p(\mathbf{x}' | \mathbf{y}^+) d\mathbf{x} \rightarrow 1$  where  $A$  is some small region around  $\mathbf{x}'$ .  
 160 2. (Actionability) Permutations  $\Delta$  are subject to some actionability constraints.

161 We consider counterfactuals as faithful to the extent that they are consistent with what the model has learned about the  
 162 input data. Let  $p_\theta(\mathbf{x} | \mathbf{y}^+)$  denote the conditional posterior over inputs, then formally:

- 163 3. (Faithfulness)  $\int^A p_\theta(\mathbf{x}' | \mathbf{y}^+) d\mathbf{x} \rightarrow 1$  where  $A$  is defined as above.

164 The definitions of faithfulness and plausibility in Definition 3.1 are the same as in Altmeyer et al. (2024), with adapted  
 165 notation. Actionability constraints in Definition 3.1 vary and depend on the context in which  $M_\theta$  is deployed. In this  
 166 work, we focus on domain and mutability constraints for individual features  $x_d$  for  $d = 1, \dots, D$ . We limit ourselves  
 167 to classification tasks for reasons discussed in Section 5.

#### 168 3.1 Our Proposed Objective

169 Let  $\mathbf{x}'_t$  for  $t = 0, \dots, T$  denote a counterfactual explanation generated through gradient descent over  $T$  iterations  
 170 as initially proposed by Wachter, Mittelstadt, and Russell (2017). For our purposes, we let  $T$  vary and consider the  
 171 counterfactual search as converged as soon as the predicted probability for the target class has reached a pre-determined  
 172 threshold,  $\tau : \mathcal{S}(M_\theta(\mathbf{x}'))[y^+] \geq \tau$ , where  $\mathcal{S}$  is the softmax function.<sup>2</sup>

<sup>2</sup>For detailed background information on gradient-based counterfactual search and convergence see ?@sec-app-ce.

173 To train models with high explainability as defined in Definition 3.1, we propose to leverage counterfactuals in the  
 174 following objective:

$$\min_{\theta} \text{yloss}(\mathbf{M}_{\theta}(\mathbf{x}), \mathbf{y}) + \lambda_{\text{div}} \text{div}(\mathbf{x}, \mathbf{x}'_T, y; \theta) + \lambda_{\text{adv}} \text{advloss}(\mathbf{M}_{\theta}(\mathbf{x}'_{t \leq T}), \mathbf{y}) + \lambda_{\text{reg}} \text{ridge}(\mathbf{x}, \mathbf{x}'_T, y; \theta) \quad (1)$$

175 where  $\text{yloss}(\cdot)$  is any conventional classification loss that induces discriminative performance (e.g., cross-entropy).  
 176 The second and third terms in Equation 1 are explained in more detail below. For now, they can be sufficiently de-  
 177 scribed as inducing explainability directly and indirectly by penalizing: (1) the contrastive divergence,  $\text{div}(\cdot)$ , between  
 178 mature counterfactuals  $\mathbf{x}'_T$  and observed samples  $x$  and, (2) the adversarial loss,  $\text{advloss}(\cdot)$ , with respect to nascent  
 179 counterfactuals  $\mathbf{x}'_{t \leq T}$ . Finally,  $\text{ridge}(\cdot)$  denotes a Ridge penalty ( $\ell_2$ -norm) that regularises the magnitude of the energy  
 180 terms involved in  $\text{div}(\cdot)$  (Du and Mordatch 2020). The tradeoff between the different components can be governed by  
 181 adjusting the strengths of the penalties  $\lambda_{\text{div}}$ ,  $\lambda_{\text{adv}}$  and  $\lambda_{\text{reg}}$ .

### 182 3.1.1 Directly Inducing Explainability through Contrastive Divergence

183 Grathwohl et al. (2020) observe that any classifier can be re-interpreted as a joint energy-based model (JEM) that  
 184 learns to discriminate output classes conditional on the observed (training) samples from  $p(\mathbf{x})$  and the generated  
 185 samples from  $p_{\theta}(\mathbf{x})$ . They show that JEMs can be trained to perform well at both tasks by directly maximizing the  
 186 joint log-likelihood factorized as  $\log p_{\theta}(\mathbf{x}, \mathbf{y}) = \log p_{\theta}(\mathbf{y}|\mathbf{x}) + \log p_{\theta}(\mathbf{x})$ . The first factor can be optimized using  
 187 conventional cross-entropy as in Equation 1. Then, to optimize  $\log p_{\theta}(\mathbf{x})$  Grathwohl et al. (2020) minimize the  
 188 contrastive divergence between these observed samples from  $p(\mathbf{x})$  and generated samples from  $p_{\theta}(\mathbf{x})$ .

189 A key empirical finding in Altmeyer et al. (2024) was that JEMs tend to do well with respect to the plausibility ob-  
 190 jective in Definition 3.1. If we consider samples drawn from  $p_{\theta}(\mathbf{x})$  as counterfactuals, this is an expected finding,  
 191 because the JEM objective effectively minimizes the divergence between the conditional posterior and  $p(\mathbf{x}|y^+)$ . To  
 192 generate samples, Grathwohl et al. (2020) rely on Stochastic Gradient Langevin Dynamics (SGLD) using an uninfor-  
 193 mative prior for initialization. This is where we depart from their methodology: instead of SGLD, we propose to use  
 194 counterfactual explainers to generate counterfactuals of observed training samples. Specifically, we have:

$$\text{div}(\mathbf{x}, \mathbf{x}'_T, y; \theta) = \mathcal{E}_{\theta}(\mathbf{x}, y) - \mathcal{E}_{\theta}(\mathbf{x}'_T, y) \quad (2)$$

195 where  $\mathcal{E}_{\theta}(\cdot)$  denotes the energy function. In particular, we set  $\mathcal{E}_{\theta}(\mathbf{x}, y) = -\mathbf{M}_{\theta}(\mathbf{x}^+)[y^+]$  where  $y^+$  denotes the index  
 196 of the randomly drawn target class,  $y^+ \sim p(y)$ , and  $\mathbf{x}^+$  denotes an observed data point sampled from target domain:  
 197  $\mathbf{X}^+ = \{\mathbf{x} : y = y^+\}$ . Conditional on the target class  $y^+$ ,  $\mathbf{x}'_T$  denotes a mature counterfactual for a randomly sampled  
 198 factual from a non-target class generated through a gradient-based counterfactual generator for at most  $T$  iterations.  
 199 We define mature counterfactuals as those that have either exhausted  $T$  or reached convergence in terms of the pre-  
 200 determined decision threshold earlier.

201 Intuitively, the gradient of Equation 2 decreases the energy of observed training samples (positive samples) while at  
 202 same time increasing the energy of counterfactuals (negative samples) (Du and Mordatch 2020). As the generated  
 203 counterfactuals get more plausible (Definition 3.1) over the course of training, these two opposing effects gradually  
 204 balance each out (Lippe 2024).

205 The departure from SGLD allows us to tap into the vast repertoire of explainers that have been proposed in the literature  
 206 to meet different desiderata. Typically, these methods facilitate the imposition of domain and mutability constraints,  
 207 for example. In principle, any existing approach for generating counterfactual explanations is viable, so long as it does  
 208 not violate the faithfulness condition. Like JEMs (Murphy 2022), counterfactual training can be considered as a form  
 209 of contrastive representation learning.

### 210 3.1.2 Indirectly Inducing Explainability through Adversarial Robustness

211 Based on our analysis in Section 2, counterfactuals  $\mathbf{x}'$  can be repurposed as additional training samples (Luu and Inoue  
 212 2023; Balashankar et al. 2023) or adversarial examples (Freiesleben 2022; Pawelczyk et al. 2022). This leaves some  
 213 flexibility with respect to the exact choice for  $\text{advloss}(\cdot)$  in Equation 1. An intuitive functional form to use, though  
 214 likely not the only reasonable choice, is inspired by adversarial training:

$$\begin{aligned} \text{advloss}(\mathbf{M}_{\theta}(\mathbf{x}'_{t \leq T}), \mathbf{y}; \varepsilon) &= \text{yloss}(\mathbf{M}_{\theta}(\mathbf{x}'_{t_{\varepsilon}}), \mathbf{y}) \\ t_{\varepsilon} &= \max_t \{t : \|\Delta_t\|_{\infty} < \varepsilon\} \end{aligned} \quad (3)$$

215 Under this choice, we consider nascent counterfactuals  $\mathbf{x}'_{t \leq T}$  as adversarial examples as long as the magnitude of the  
 216 perturbation to any individual feature is at most  $\varepsilon$ . This is closely aligned with Szegedy et al. (2013), who define an

adversarial attack as an “imperceptible non-random perturbation”. Thus, we choose to work with a different distinction between CE and AE than Freiesleben (2022), who considers misclassification as the key distinguishing feature of AE. One of the key observations in this work is that we can leverage counterfactual explanations during training and get adversarial examples, essentially for free.

### 3.2 Encoding Actionability Constraints

Many existing counterfactual explainers support domain and mutability constraints out-of-the-box. In fact, both types of constraints can be implemented for any counterfactual explainer that relies on gradient descent in the feature space for optimization (Altmeyer, Deursen, et al. 2023). In this context, domain constraints can be imposed by simply projecting counterfactuals back to the specified domain, if the previous gradient step resulted in updated feature values that were out-of-domain. Mutability constraints can similarly be enforced by setting partial derivatives to zero to ensure that features are only mutated in the allowed direction, if at all.

Since actionability constraints are binding at test time, we should also impose them when generating  $\mathbf{x}'$  during each training iteration to align model representations with user requirements. Through their effect on  $\mathbf{x}'$ , both types of constraints influence model outcomes through Equation 2. Here it is crucial that we avoid penalizing implausibility that arises due to mutability constraints. For any mutability-constrained feature  $d$  this can be achieved by enforcing  $\mathbf{x}[d] - \mathbf{x}'[d] := 0$  whenever perturbing  $\mathbf{x}'[d]$  in the direction of  $\mathbf{x}[d]$  would violate mutability constraints. Specifically, we set  $\mathbf{x}[d] := \mathbf{x}'[d]$  if:

1. Feature  $d$  is strictly immutable in practice.
2. We have  $\mathbf{x}[d] > \mathbf{x}'[d]$  but feature  $d$  can only be decreased in practice.
3. We have  $\mathbf{x}[d] < \mathbf{x}'[d]$  but feature  $d$  can only be increased in practice.

From a Bayesian perspective, setting  $\mathbf{x}[d] := \mathbf{x}'[d]$  can be understood as assuming a point mass prior for  $p(\mathbf{x})$  with respect to feature  $d$ . Intuitively, we think of this simply in terms ignoring implausibility costs with respect to immutable features, which effectively forces the model to instead seek plausibility with respect to the remaining features. This in turn results in lower overall sensitivity to immutable features, which we demonstrate empirically for different classifiers in Section 4. Under certain conditions, this results holds theoretically.<sup>3</sup>

**Proposition 3.1** (Protecting Immutable Features). *Let  $f_\theta(\mathbf{x}) = \mathcal{S}(\mathbf{M}_\theta(\mathbf{x})) = \mathcal{S}(\Theta\mathbf{x})$  denote a linear classifier with softmax activation  $\mathcal{S}$  (i.e., multinomial logistic regression) where  $y \in \{1, \dots, K\} = \mathcal{K}$  and  $\mathbf{x} \in \mathbb{R}^D$ . If we assume multivariate Gaussian class densities with common diagonal covariance matrix  $\Sigma_k = \Sigma$  for all  $k \in \mathcal{K}$ , then protecting an immutable feature from the contrastive divergence penalty (Equation 2) will result in lower classifier sensitivity to that feature relative to the remaining features, provided that at least one of those is discriminative and mutable.*

It is worth highlighting that Proposition 3.1 assumes independence of features. This raises a valid concern about the effect of protecting immutable features in the presence of proxy features that remain unprotected. We discuss this limitation in Section 5.

### 3.3 Illustration

To better convey the intuition underlying our proposed method, we illustrate different model outcomes in Example 3.1.

**Example 3.1** (Prediction of Consumer Credit Default). Suppose we are interested in predicting the likelihood that loan applicants default on their credit. We have access to historical data on previous loan takers comprised of a binary outcome variable ( $y \in \{1 = \text{default}, 2 = \text{no default}\}$ ) two input features: (1) the subjects’ age, which we define as immutable, and (2) the subjects’ existing level of debt, which we define as mutable.

We have simulated this scenario using synthetic data with independent features and Gaussian class-conditional densities in Figure 1. The four panels in Figure 1 show the outcomes for different training procedures using the same model architecture each time (a linear classifier). In each case, we show the linear decision boundary (green) and the training data colored according to their ground-truth label: orange points belong to the target class,  $y^+ = 2$ , blue points belong to the non-target class,  $y^- = 1$ . Stars indicate counterfactuals in the target class generated at test time using generic gradient descent until convergence.

In panel (a), we have trained our model conventionally, and we do not impose mutability constraints at test time. The generated counterfactuals are all valid, but not plausible: they are clearly distinguishable from the ground-truth data. In panel (b), we have trained our model with counterfactual training, once again not imposing mutability constraints at test time. We observe that the counterfactuals are clearly plausible, therefore meeting the first objective of Definition 3.1.

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<sup>3</sup>For the proof, see the supplementary appendix.

266 In panel (c), we have used conventional training again, this time imposing the mutability constraint on *age* at test time.  
 267 Counterfactuals are valid but involve some substantial reductions in *debt* for some individuals (very young applicants).  
 268 By comparison, counterfactual paths are shorter on average in panel (d), where we have used counterfactual training  
 269 and protected immutable features as described in Section 3.2. In particular, we observe that due to the classifier's  
 270 lower sensitivity to *age*, recourse recommendations with respect to *debt* are much more homogenous, in that they do  
 271 not disproportionately punish younger individuals. The counterfactuals are also plausible with respect to the mutable  
 272 feature. Thus, we consider the model in panel (d) as the most explainable according to Definition 3.1.

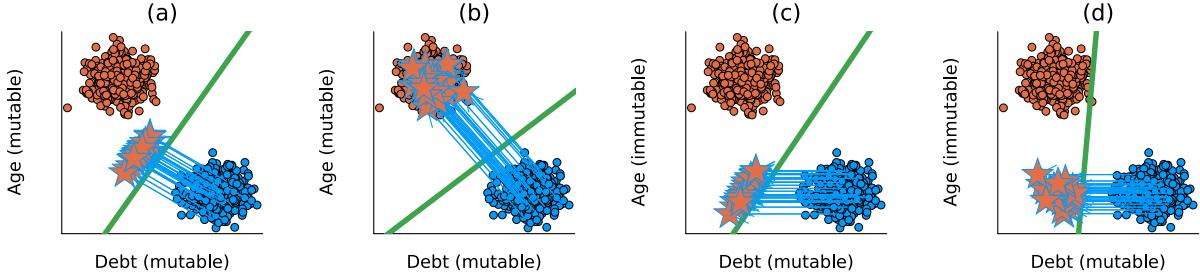


Figure 1: Visual illustration of how counterfactual training improves explainability. See Example 3.1 for details.

## 273 4 Experiments

274 In this section, we present experiments that we have conducted in order to answer the following research questions:

275 **Research Question 4.1** (Plausibility). *Does our proposed counterfactual training objective (Equation 1) induce mod-  
 276 els to learn plausible explanations?*

277 **Research Question 4.2** (Actionability). *Does our proposed counterfactual training objective (Equation 1) yield more  
 278 favorable algorithmic recourse outcomes in the presence of actionability constraints?*

279 Beyond this, we are also interested in understanding how robust our answers to RQ 4.1 and RQ 4.2 are:

280 **Research Question 4.3** (Hyperparameters). *What are the effects of different hyperparameter choices with respect to  
 281 Equation 1?*

### 282 4.1 Experimental Setup

#### 283 4.1.1 Evaluation

284 Our key outcome of interest is how well models perform with respect to explainability (Definition 3.1): to this end, we  
 285 focus primarily on the plausibility and cost of faithfully generated counterfactuals at test time. To measure the cost of  
 286 counterfactuals, we follow the standard convention of using distances ( $\ell_1$ -norm) between factuals and counterfactuals  
 287 as a proxy. For plausibility, we assess how similar counterfactuals are to observed samples in the target domain. We  
 288 rely on the distance-based metric used by Altmeyer et al. (2024),

$$\text{implaus}_{\text{dist}}(\mathbf{x}', \mathbf{X}^+) = \frac{1}{|\mathbf{X}^+|} \sum_{\mathbf{x} \in \mathbf{X}^+} \text{dist}(\mathbf{x}', \mathbf{x}) \quad (4)$$

289 and introduce a novel divergence metric,

$$\text{implaus}_{\text{div}}(\mathbf{X}', \mathbf{X}^+) = \text{MMD}(\mathbf{X}', \mathbf{X}^+) \quad (5)$$

290 where  $\mathbf{X}'$  denotes a set of multiple counterfactuals and  $\text{MMD}(\cdot)$  is an unbiased estimate of the squared population  
 291 maximum mean discrepancy (Gretton et al. 2012). The metric in Equation 5 is equal to zero iff  $\mathbf{X}' = \mathbf{X}^+$ .

292 In addition to cost and plausibility, we also compute other standard metrics to evaluate counterfactuals at test time in-  
 293 cluding validity and redundancy. Finally, we also assess the predictive performance of models using standard metrics.

294 We run the experiments with three CE generators: *Generic* of Wachter, Mittelstadt, and Russell (2017) as a simple  
 295 baseline approach, *REVISE* (Joshi et al. 2019) that aims to generate plausible counterfactuals using a surrogate Vari-  
 296 ational Autoencoder (VAE), and *ECCo*—the generator of Altmeyer et al. (2023) but without the conformal prediction  
 297 component—as a method that directly targets both faithfulness and plausibility of the CEs.

298 **4.2 Experimental Results**299 **4.2.1 Plausibility**300 **4.2.2 Actionability**301 **4.2.3 Impact of hyperparameter settings**

302 We extensively test the impact of three types of hyperparameters on the proposed training regime. Our complete results  
 303 are available in the technical appendix; this section focuses on the main findings.

304 **Hyperparameters of the CE generators.** First, we observe that CT is highly sensitive to hyperparameter settings but  
 305 (a) there are manageable patterns and (b) we can typically identify settings that improve either plausibility or cost, and  
 306 commonly both of them at the same time. Second, we note that the choice of a CE generator has a major impact on  
 307 the results. For example, *REVISE* tends to perform the worst, most likely because it uses a surrogate VAE to generate  
 308 counterfactuals which impedes faithfulness (Altmeyer et al. 2024). Third, increasing  $T$ , the maximum number of  
 309 steps, generally yields better outcomes because more CEs can mature in each training epoch. Fourth, the impact of  $\tau$ ,  
 310 the required decision threshold is more difficult to predict. On “harder” datasets it may be difficult to satisfy high  $\tau$  for  
 311 any given sample (i.e., also factuals) and so increasing this threshold does not seem to correlate with better outcomes.  
 312 In fact, we have generally found that a choice of  $\tau = 0.5$  leads to optimal results because it is associated with high  
 313 proportions of mature counterfactuals.

314 **Hyperparameters for penalties.** We find that the strength of the energy regularization,  $\lambda_{\text{reg}}$  is highly impactful; energy  
 315 must be sufficiently regularized to avoid poor performance in terms of decreased plausibility and increased costs. The  
 316 sensitivity with respect to  $\lambda_{\text{div}}$  and  $\lambda_{\text{adv}}$  is much less evident. While high values of  $\lambda_{\text{reg}}$  may increase the variability  
 317 in outcomes when combined with high values for any of the other penalties in Equation 1, this effect is not very  
 318 pronounced.

319 **Other hyperparameters.** We observe that the effectiveness and stability of CT is positively associated with the number  
 320 of counterfactuals generated during each training epoch. We also confirm that a higher number of training epochs is  
 321 beneficial. Interestingly, we find that it is not necessary to employ CT during the entire training phase to achieve the  
 322 desired improvements in explainability. We have tested training models conventionally during the first half of training  
 323 before switching to CT after this initial “burn-in” period and observed positive results. Put differently, CT may be a  
 324 way to improve the explainability of trained models in a fine-tuning manner.

325 **5 Discussion**

326 We begin the discussion by addressing the direct extensions of the counterfactual training approach in Section 5.1.  
 327 Then, we look at its broader limitations and challenges in Section 5.2.

328 **5.1 Future research**

329 **CT is defined only for classification settings.** Our formulation relies on the distinction between non-target class(es)  
 330  $y^-$  and target class(es)  $y^+$  to generate counterfactuals through Equation 1. While  $y^-$  and  $y^+$  can be arbitrarily defined  
 331 by the user, CT requires the output space  $\mathcal{Y}$  to be discrete. Thus, it applies to binary and multi-class classification but  
 332 it is not well-defined for other ML tasks where the change in outcome with respect to a decision threshold  $\tau$  cannot  
 333 be readily quantified. In fact, this is a common restriction in research on CEs and AR that predominantly focuses on  
 334 classification models. Although other settings have attracted some interest (e.g., regression in Spooner et al. 2021;  
 335 Zhao, Broelemann, and Kasneci 2023), there is still no consensus on what constitutes a counterfactual in such settings.

336 **CT is subject to training instabilities.** Joint energy-based models are susceptible to instabilities during training (Grath-  
 337 wohl et al. 2020) and even though we depart from the SGLD-based sampling, we still encounter major variability in  
 338 the outcomes. CT is exposed to two potential sources of instabilities: (1) the energy-based contrastive divergence term  
 339 in Equation 2, and (2) the underlying counterfactual explainers. For example, Altmeyer et al. (2023) recognize this  
 340 to be a challenge for *ECCCo* and so it may have downstream impacts on our proposed method. Still, we find that  
 341 training instabilities can be successfully mitigated by regularizing energy ( $\lambda_{\text{reg}}$ ), generating a sufficiently large number  
 342 of counterfactuals during each training epoch and including only mature counterfactuals for contrastive divergence.

343 **CT is sensitive to hyperparameter selection.** As discussed in Section 4.2.3, our method benefits from tuning certain  
 344 key hyperparameters. In this work, we have relied exclusively on grid search for this task. Future work on CT could  
 345 benefit from investigating more sophisticated approaches towards hyperparameter tuning. Notably, counterfactual  
 346 training is iterative which makes a variety of methods applicable, including Bayesian (e.g., Snoek, Larochelle, and  
 347 Adams 2012) or gradient-based (e.g., Franceschi et al. 2017) optimization.

348 **5.2 Limitations and challenges**

349 **CT increases the training time of models.** Counterfactual training promotes explainability through CEs and robustness  
 350 through AEs at the cost of longer training times compared to conventional training regimes. While higher numbers  
 351 of iterations and counterfactuals per iteration positively impact the quality of found solutions, they also increase  
 352 the required amount of computations. We find that relatively small grids with 270 settings can take almost four  
 353 hours for more demanding datasets on a high-performance computing cluster with 34 2GB CPUs (see details in  
 354 [?@sec-hardware](#)). However, there are three factors that attenuate the impact of this limitation. First, CT provides  
 355 counterfactual explanations for the training samples essentially for free, which may be beneficial in many ADM  
 356 systems. Second, we find that CT can retain its value when used as a “fine-tuning” training regime for conventionally-  
 357 trained models. Third, in principle, CT yields itself to parallel execution, which we have leveraged for our own  
 358 experiments.

359 **Immutable features may have proxies.** In Proposition 3.1 we define an approach to protect immutable features and  
 360 thus increase the actionability of the generated counterfactuals. However, this approach requires that model owners  
 361 define the mutability constraints for (all) features considered by the model. Even with sufficient domain knowledge  
 362 to protect all immutable features—ones that cannot be changed at all and ones that cannot be reasonably expected  
 363 to change—there may exist proxies that are theoretically mutable (and hence should not be protected) but preserve  
 364 enough information about the principals to counteract the protections. As one example, consider the Adult dataset  
 365 used in our experiments where the mutable education status is a proxy for the immutable age, in that the attainment of  
 366 degrees is correlated with age. Delineating actionability is a major undecided challenge in the AR literature (see, e.g.,  
 367 [Venkatasubramanian and Alfano 2020](#)) impacting the capacity of CT to increase the explainability of the model.

368 **Interventions on features may have downstream impacts on fairness.**

- 369 - Equality of opportunity?
- 370 - Social segregation?
- 371 - Supporting AR within one context/system may still unfairly target recourse-affected individuals within other con-  
 372 texts/systems? In Example 3.1 younger individuals gain access to a loan but their age could still lead to, e.g., enforcing  
 373 stronger supervisory mechanisms?

374 **6 Conclusion**

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