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# COUNTERFACTUAL TRAINING: TEACHING MODELS PLAUSIBLE AND ACTIONABLE EXPLANATIONS

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## ABSTRACT

Counterfactual Explanations (CE) have emerged as a popular method to explain the predictions made by opaque machine learning models in a post-hoc fashion. We propose a novel approach that leverages counterfactuals during the training phase of models.

**Keywords** Counterfactual Explanations • Explainable AI

## 1 Introduction

## 2 Related Literature

### 2.1 Background on Counterfactual Explanations

(Wachter, Mittelstadt, and Russell 2017; Joshi et al. 2019; Altmeyer et al. 2024)

### 2.2 Learning Representations

For example, joint-energy models

### 2.3 Generalization and Robustness

Sauer and Geiger (2021) generate counterfactual images for MNIST and ImageNet through independent mechanisms (IM): each IM learns class-conditional input distributions over a specific lower-dimensional, semantically meaningful factor, such as *texture*, *shape* and *background*. They demonstrate that using these generated counterfactuals during classifier training improves model robustness. Similarly, Abbasnejad et al. (2020) argue that counterfactuals represent potentially useful training data in machine learning, especially in supervised settings where inputs may be reasonably mapped to multiple outputs. They, too, demonstrate that augmenting the training data of image classifiers can improve generalization.

19 Tenev, Abbasnejad, and Hengel (2020) propose an approach using counterfactuals in training that does not rely on  
 20 data augmentation: they argue that counterfactual pairs typically already exist in training datasets. Specifically, their  
 21 approach relies on, firstly, identifying similar input samples with different annotations and, secondly, ensuring that the  
 22 gradient of the classifier aligns with the vector between pairs of counterfactual inputs using the cosine distance as a loss  
 23 function (referred to as *gradient supervision*) (*this might be useful for our task as well*). In the natural language pro-  
 24 cessing (NLP) domain, counterfactuals have similarly been used to improve models through data augmentation: Wu et  
 25 al. (2021), propose POLYJUICE, a general-purpose counterfactual generator for language models. They demonstrate  
 26 empirically that augmenting training data through POLYJUICE counterfactuals improves robustness in a number of  
 27 NLP tasks.

## 28 **2.4 Link to Adversarial Training**

29 Freiesleben (2022) propose two definitional differences between Adversarial Examples (AE) and Counterfactual Ex-  
 30 planations (CE): firstly, and more importantly according to the authors, the term AE implies missclassification, which  
 31 is not the case for CE (*this might be a useful notion for use to distinguish between adversarials and explanations*  
 32 *during training*); secondly, they argue that closeness plays a more critical role in the context of CE but confess that  
 33 even counterfactuals that are not close might be relevant explanations. Pawelczyk et al. (2022) show that CE and AE  
 34 are equivalent under certain conditions and derive upper bounds on the distances between them.

## 35 **2.5 Closely Related**

36 Guo, Nguyen, and Yadav (2023) are the first to propose end-to-end training pipeline that includes counterfactual ex-  
 37 planations as part of the training procedure. In particular, they propose a specific network architecture that includes  
 38 a predictor and CE generator network (*akin a GAN?*), where the parameters of the CE generator network are learn-  
 39 able. Counterfactuals are generated during each training iteration and fed back to the predictor network (*here we are*  
 40 *aligned*). In contrast, we impose no restrictions on the neural network architecture at all. (*to ensure the one-hot en-*  
 41 *coding of categorical features is maintained, they simple use softmax (might be interesting for CE.jl)*) Interestingly,  
 42 the authors find that their approach is sensitive to the choice of the loss function: only MSE seems to lead to good  
 43 performance. They also demonstrate theoretically, that the objective function is difficult to optimize due to divergent  
 44 gradients and suffers from poor adversarial robustness. (*because partial gradients with respect to the classification*  
 45 *loss component and the counterfactual validity component point in opposite directions*). To mitigate these issues,  
 46 the authors use block-wise gradient descent: they first update with respect to classification loss and then use a second  
 47 update with respect to the other loss components (*this might be useful for our task as well*). Ross, Lakkaraju, and  
 48 Bastani (2024) propose a way to train models that are guaranteed to provide recourse for individuals with high proba-  
 49 bility. The approach builds on adversarial training (*here we are aligned*), where in this context adversarial examples  
 50 are actively encouraged to exist, but only target attacks with respect to the positive class. The proposed method allows  
 51 for imposing a set of actionable recourse ex-ante: for example, users can impose mutability constraints for features  
 52 (*here we are aligned*). (*To solve their objective function more efficiently, they use a first-order Taylor approximation*  
 53 *to approximate the recourse loss component (might be applicable in our case)*)

54 Luu and Inoue (2023) introduce Counterfactual Adversarial Training (CAT) with intention of improving generalization  
 55 and robustness of language models. Specifically, they propose to proceed as follows: firstly, identify training samples  
 56 that are subject to high predictive uncertainty (entropy); secondly, generate counterfactual explanations for those  
 57 samples; and, finally, finetune the model on the augmented dataset that includes the generated counterfactuals.

## 58 **3 Counterfactual Training**

## 59 **4 Experiments**

### 60 **4.1 Experimental Setup**

### 61 **4.2 Experimental Results**

## 62 **5 Discussion**

## 63 **6 Conclusion**

## 64 **References**

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102 **A Training Details**

103 **A.1 Initial Grid Search**

104 For the initial round of experiments we

105 **A.1.1 Generator Parameters**

106 The hyperparameter choices are shown in Parameters A.1:

107 **Parameters A.1 (Parameters).**

- 108 • **generator\_params:**
  - 109 – **lambda\_cost**: 0.0, 0.001, 0.1
  - 110 – **lambda\_energy**: 0.01, 0.05, 0.1, 0.5, 1.0, 5.0, 10.0, 15.0
  - 111 – **lr**: 1.0
  - 112 – **maxiter**: 20, 50, 100
  - 113 – **opt**: sgd
- 114 • **generator\_type**: ecco, generic, omni, revise
- 115 • **training\_params**:
  - 116 – **objective**: full, vanilla

117 **A.1.1.1 Linearly Separable**

- 118 • **Energy Penalty** (Table A1): *ECCo* generally does yield better results than *Vanilla* for higher choices of the  
119 energy penalty (10,15) during training. *Generic* performs poorly across the board. *Omni* seems to have an  
120 anchoring effect, in that it never performs terribly but also never as good as the best *ECCo* results. *REVISE*  
121 performs poorly across the board.
- 122 • **Cost (distance penalty)**: Results for all generators (except *Omni*) are quite bad, which can likely be attributed  
123 to extremely bad results for some choices of the **Energy Penalty** (results here are averaged). For *ECCo* and  
124 *Generic*, higher cost values generally lead to worse results.
- 125 • **Maximum Iterations**: No clear patterns recognizable, so it seems that smaller choices are ok.
- 126 • **Validity**: *ECCo* almost always valid except for very low values during training and high values at evaluation  
127 time. *Generic* often has poor validity.
- 128 • **Accuracy**: Seems largely unaffected.

Table A1: Results for Linearly Separable data by energy penalty.

Objective	$\lambda_{\text{div}}(\text{train})$	Generator	Value	Std
full	0.01	<i>ECCo</i>	$-9.91 \cdot 10^{11}$	$2.25 \cdot 10^{12}$
full	0.01	<i>Generic</i>	$-5.71 \cdot 10^{17}$	$1.3 \cdot 10^{18}$
<b>full</b>	<b>0.01</b>	<b>Omniscient</b>	<b>-2.54</b>	<b>0.116</b>
full	0.01	<i>REVISE</i>	-15.6	13.2
vanilla	0.01	<i>ECCo</i>	-4.28	3.52
vanilla	0.01	<i>Generic</i>	-4.45	3.47
vanilla	0.01	<i>Omniscient</i>	-5.12	4.46
vanilla	0.01	<i>REVISE</i>	-4.91	4.24
full	0.05	<i>ECCo</i>	$-5.63 \cdot 10^5$	$1.28 \cdot 10^6$
full	0.05	<i>Generic</i>	$-8.35 \cdot 10^{17}$	$1.9 \cdot 10^{18}$
<b>full</b>	<b>0.05</b>	<b>Omniscient</b>	<b>-2.53</b>	<b>0.114</b>
full	0.05	<i>REVISE</i>	-15	12.6
vanilla	0.05	<i>ECCo</i>	-4.4	3.66
vanilla	0.05	<i>Generic</i>	-4.38	3.48
vanilla	0.05	<i>Omniscient</i>	-5.25	4.62
vanilla	0.05	<i>REVISE</i>	-4.94	4.22
full	0.1	<i>ECCo</i>	$-6.74 \cdot 10^5$	$1.53 \cdot 10^6$
full	0.1	<i>Generic</i>	$-1.72 \cdot 10^{11}$	$3.9 \cdot 10^{11}$
<b>full</b>	<b>0.1</b>	<b>Omniscient</b>	<b>-2.56</b>	<b>0.124</b>

Continuing table below.

Objective	$\lambda_{\text{div}}(\text{train})$	Generator	Value	Std
full	0.1	<i>REVISE</i>	-15.6	13.2
vanilla	0.1	<i>ECCo</i>	-4.28	3.52
vanilla	0.1	<i>Generic</i>	-4.45	3.48
vanilla	0.1	<i>Omniscient</i>	-5.12	4.46
vanilla	0.1	<i>REVISE</i>	-4.91	4.25
full	0.5	<i>ECCo</i>	-11.8	9.83
full	0.5	<i>Generic</i>	$-1.06 \cdot 10^{18}$	$2.42 \cdot 10^{18}$
<b>full</b>	<b>0.5</b>	<b>Omniscient</b>	<b>-2.54</b>	<b>0.123</b>
full	0.5	<i>REVISE</i>	-15	12.6
vanilla	0.5	<i>ECCo</i>	-4.4	3.65
vanilla	0.5	<i>Generic</i>	-4.38	3.48
vanilla	0.5	<i>Omniscient</i>	-5.25	4.61
vanilla	0.5	<i>REVISE</i>	-4.95	4.22
full	1	<i>ECCo</i>	-11.5	11.1
full	1	<i>Generic</i>	$-1.71 \cdot 10^{11}$	$3.88 \cdot 10^{11}$
<b>full</b>	<b>1</b>	<b>Omniscient</b>	<b>-2.59</b>	<b>0.117</b>
full	1	<i>REVISE</i>	-15.7	13.3
vanilla	1	<i>ECCo</i>	-4.28	3.51
vanilla	1	<i>Generic</i>	-4.44	3.47
vanilla	1	<i>Omniscient</i>	-5.11	4.46
vanilla	1	<i>REVISE</i>	-4.91	4.25
full	5	<i>ECCo</i>	-3.99	3.12
full	5	<i>Generic</i>	$-4.88 \cdot 10^{17}$	$1.11 \cdot 10^{18}$
<b>full</b>	<b>5</b>	<b>Omniscient</b>	<b>-2.53</b>	<b>0.117</b>
full	5	<i>REVISE</i>	-14.6	12.1
vanilla	5	<i>ECCo</i>	-4.4	3.65
vanilla	5	<i>Generic</i>	-4.38	3.48
vanilla	5	<i>Omniscient</i>	-5.25	4.61
vanilla	5	<i>REVISE</i>	-4.95	4.22
<b>full</b>	<b>10</b>	<b>ECCo</b>	<b>-2.31</b>	<b>0.735</b>
full	10	<i>Generic</i>	$-1.7 \cdot 10^{11}$	$3.86 \cdot 10^{11}$
full	10	<i>Omniscient</i>	-2.53	0.117
full	10	<i>REVISE</i>	-15.5	13
vanilla	10	<i>ECCo</i>	-4.28	3.51
vanilla	10	<i>Generic</i>	-4.44	3.47
vanilla	10	<i>Omniscient</i>	-5.12	4.46
vanilla	10	<i>REVISE</i>	-4.91	4.24
<b>full</b>	<b>15</b>	<b>ECCo</b>	<b>-2.01</b>	<b>0.488</b>
full	15	<i>Generic</i>	$-4.91 \cdot 10^{17}$	$1.12 \cdot 10^{18}$
full	15	<i>Omniscient</i>	-2.53	0.116
full	15	<i>REVISE</i>	-14.4	11.7
vanilla	15	<i>ECCo</i>	-4.4	3.65
vanilla	15	<i>Generic</i>	-4.38	3.48
vanilla	15	<i>Omniscient</i>	-5.25	4.6
vanilla	15	<i>REVISE</i>	-4.95	4.23

## 129 A.1.1.2 Moons

- 130 • **Energy Penalty** (Table A2): *ECCo* consistently yields better results than *Vanilla*, except for very low choices  
 131 of the energy penalty during training for which it performs abysmal. *Generic* performs quite badly across  
 132 the board for high enough choices of the energy penalty at evaluation time. *Omni* has small positive effect.  
 133 *REVISE* performs poorly across the board.
- 134 • **Cost (distance penalty)**: *Generic* generally does better for higher values, while *ECCo* does better for lower  
 135 values.
- 136 • **Maximum Iterations**: No clear patterns recognizable, so it seems that smaller choices are ok.

- 137 • **Validity:** *ECCo* generally achieves full validity except for very low choices the energy penalty during training  
 138 and high choices at evaluation time. *Generic* performs poorly for high choices of the energy penalty during  
 139 evaluation.  
 140 • **Accuracy:** Largely unaffected although *ECCo* suffers a bit for very low choices the energy penalty during  
 141 training. *REVISE* suffers a lot in general (around 10 percentage points).

Table A2: Results for Moons data by energy penalty.

Objective	$\lambda_{\text{div}}(\text{train})$	Generator	Value	Std
full	0.01	<i>ECCo</i>	$-2.8 \cdot 10^{22}$	$6.39 \cdot 10^{22}$
full	0.01	<i>Generic</i>	$-4.89 \cdot 10^{30}$	$1.11 \cdot 10^{31}$
<b>full</b>	<b>0.01</b>	<b>Omniscient</b>	<b>-4.74</b>	<b>5.08</b>
full	0.01	<i>REVISE</i>	-572	$1.25 \cdot 10^3$
vanilla	0.01	<i>ECCo</i>	-15.5	17.3
vanilla	0.01	<i>Generic</i>	-10.9	11.9
vanilla	0.01	<i>Omniscient</i>	-12.7	14.4
vanilla	0.01	<i>REVISE</i>	-11.2	13
full	0.05	<i>ECCo</i>	$-1.55 \cdot 10^{16}$	$3.52 \cdot 10^{16}$
full	0.05	<i>Generic</i>	$-2.22 \cdot 10^{20}$	$5 \cdot 10^{20}$
<b>full</b>	<b>0.05</b>	<b>Omniscient</b>	<b>-4.41</b>	<b>4.48</b>
full	0.05	<i>REVISE</i>	$-1.04 \cdot 10^3$	$2.3 \cdot 10^3$
vanilla	0.05	<i>ECCo</i>	-15.5	17.2
vanilla	0.05	<i>Generic</i>	-11.7	12.8
vanilla	0.05	<i>Omniscient</i>	-12.4	14.1
vanilla	0.05	<i>REVISE</i>	-11.3	13.1
full	0.1	<i>ECCo</i>	$-3.41 \cdot 10^3$	$7.73 \cdot 10^3$
full	0.1	<i>Generic</i>	$-5.22 \cdot 10^{30}$	$1.19 \cdot 10^{31}$
<b>full</b>	<b>0.1</b>	<b>Omniscient</b>	<b>-4.78</b>	<b>5.12</b>
full	0.1	<i>REVISE</i>	-288	594
vanilla	0.1	<i>ECCo</i>	-15.5	17.2
vanilla	0.1	<i>Generic</i>	-10.9	11.9
vanilla	0.1	<i>Omniscient</i>	-12.7	14.4
vanilla	0.1	<i>REVISE</i>	-11.3	13.1
full	0.5	<i>ECCo</i>	-7.09	7.51
full	0.5	<i>Generic</i>	$-1.11 \cdot 10^{31}$	$2.53 \cdot 10^{31}$
<b>full</b>	<b>0.5</b>	<b>Omniscient</b>	<b>-4.58</b>	<b>4.83</b>
full	0.5	<i>REVISE</i>	$-1.19 \cdot 10^3$	$2.64 \cdot 10^3$
vanilla	0.5	<i>ECCo</i>	-15.5	17.2
vanilla	0.5	<i>Generic</i>	-11.7	12.8
vanilla	0.5	<i>Omniscient</i>	-12.4	14.1
vanilla	0.5	<i>REVISE</i>	-11.3	13.1
full	1	<i>ECCo</i>	-6.06	6.33
full	1	<i>Generic</i>	$-1.58 \cdot 10^{33}$	$3.59 \cdot 10^{33}$
<b>full</b>	<b>1</b>	<b>Omniscient</b>	<b>-4.66</b>	<b>4.89</b>
full	1	<i>REVISE</i>	$-1.16 \cdot 10^3$	$2.59 \cdot 10^3$
vanilla	1	<i>ECCo</i>	-15.5	17.3
vanilla	1	<i>Generic</i>	-10.9	11.9
vanilla	1	<i>Omniscient</i>	-12.7	14.4
vanilla	1	<i>REVISE</i>	-11.3	13.1
<b>full</b>	<b>5</b>	<b>ECCo</b>	<b>-2.57</b>	<b>2.07</b>
full	5	<i>Generic</i>	$-1.17 \cdot 10^{28}$	$2.66 \cdot 10^{28}$
full	5	<i>Omniscient</i>	-4.29	4.31
full	5	<i>REVISE</i>	-530	$1.16 \cdot 10^3$
vanilla	5	<i>ECCo</i>	-15.5	17.2
vanilla	5	<i>Generic</i>	-11.7	12.7
vanilla	5	<i>Omniscient</i>	-12.4	14.1

Continuing table below.

Objective	$\lambda_{\text{div}}(\text{train})$	Generator	Value	Std
vanilla	5	<i>REVISE</i>	-11.3	13.1
<b>full</b>	<b>10</b>	<b>ECCo</b>	<b>-1.76</b>	<b>0.974</b>
full	10	<i>Generic</i>	$-1.54 \cdot 10^{33}$	$3.51 \cdot 10^{33}$
full	10	<i>Omniscient</i>	-4.44	4.56
full	10	<i>REVISE</i>	$-1.52 \cdot 10^3$	$3.4 \cdot 10^3$
vanilla	10	<i>ECCo</i>	-15.5	17.3
vanilla	10	<i>Generic</i>	-10.9	11.9
vanilla	10	<i>Omniscient</i>	-12.7	14.4
vanilla	10	<i>REVISE</i>	-11.3	13.1
<b>full</b>	<b>15</b>	<b>ECCo</b>	<b>-1.37</b>	<b>0.365</b>
full	15	<i>Generic</i>	$-5.32 \cdot 10^{28}$	$1.21 \cdot 10^{29}$
full	15	<i>Omniscient</i>	-4.34	4.38
full	15	<i>REVISE</i>	-473	$1.03 \cdot 10^3$
vanilla	15	<i>ECCo</i>	-15.5	17.2
vanilla	15	<i>Generic</i>	-11.7	12.8
vanilla	15	<i>Omniscient</i>	-12.4	14.1
vanilla	15	<i>REVISE</i>	-11.3	13.1

## 142 A.1.1.3 Circles

- **Energy Penalty** (Table A3): *ECCo* consistently yields better results than *Vanilla*, though primarily for low to medium choices of the energy penalty ( $<=5$ ) during training. The same goes for *Generic*, which sometimes outperforms *ECCo* (for small energy penalty at evaluation time). *Omni* does alright for lower energy penalty at evaluation time, but loses out for higher choices. *REVISE* performs poorly across the board (except very low choices at evaluation time).
- **Cost (distance penalty)**: *ECCo* and *Generic* generally achieve the best results when no cost penalty is used during training. Both *Omni* and *REVISE* are largely unaffected.
- **Maximum Iterations**: *ECCo* consistently yields better results for higher numbers of iterations. *Generic* generally does best for a medium number (50). *Omni* is sometimes invalid (???).
- **Validity**: *ECCo* tends to outperform its *Vanilla* counterpart, though primarily for low to medium choices of the energy penalty ( $<=5$ ) during training and evaluation. *Vanilla* typically worse across the board.
- **Accuracy**: Mostly unaffected, but *REVISE* again consistently some deterioration and *ECCo* deteriorates for high choices of energy penalty during training, reflecting other outcomes above.

Table A3: Results for Circles data by energy penalty.

Objective	$\lambda_{\text{div}}(\text{train})$	Generator	Value	Std
<b>full</b>	<b>0.01</b>	<b>ECCo</b>	<b>-1.26</b>	<b>0.423</b>
full	0.01	<i>Generic</i>	-1.49	0.71
full	0.01	<i>Omniscient</i>	-5.21	5.25
full	0.01	<i>REVISE</i>	$-2.71 \cdot 10^{26}$	$6.37 \cdot 10^{26}$
vanilla	0.01	<i>ECCo</i>	-9.33	7.34
vanilla	0.01	<i>Generic</i>	-8.89	6.88
vanilla	0.01	<i>Omniscient</i>	-8.67	6.87
vanilla	0.01	<i>REVISE</i>	-8.65	6.8
full	0.05	<i>ECCo</i>	-1.29	0.397
<b>full</b>	<b>0.05</b>	<b>Generic</b>	<b>-1.21</b>	<b>0.356</b>
full	0.05	<i>Omniscient</i>	-5.08	5.09
full	0.05	<i>REVISE</i>	$-5.91 \cdot 10^{27}$	$1.36 \cdot 10^{28}$
vanilla	0.05	<i>ECCo</i>	-9.35	7.32
vanilla	0.05	<i>Generic</i>	-8.85	6.87
vanilla	0.05	<i>Omniscient</i>	-8.7	6.96
vanilla	0.05	<i>REVISE</i>	-8.52	6.76
<b>full</b>	<b>0.1</b>	<b>ECCo</b>	<b>-1.2</b>	<b>0.383</b>

Continuing table below.

<b>Objective</b>	$\lambda_{\text{div}}(\text{train})$	<b>Generator</b>	<b>Value</b>	<b>Std</b>
full	0.1	<i>Generic</i>	-1.5	0.735
full	0.1	<i>Omniscient</i>	-5.17	5.23
full	0.1	<i>REVISE</i>	$-3.06 \cdot 10^{26}$	$7.7 \cdot 10^{26}$
vanilla	0.1	<i>ECCo</i>	-9.33	7.32
vanilla	0.1	<i>Generic</i>	-8.88	6.86
vanilla	0.1	<i>Omniscient</i>	-8.69	6.9
vanilla	0.1	<i>REVISE</i>	-8.68	6.81
<b>full</b>	<b>0.5</b>	<b>ECCo</b>	<b>-1.12</b>	<b>0.217</b>
full	0.5	<i>Generic</i>	-1.21	0.352
full	0.5	<i>Omniscient</i>	-5.09	5.12
full	0.5	<i>REVISE</i>	$-5.97 \cdot 10^{27}$	$1.37 \cdot 10^{28}$
vanilla	0.5	<i>ECCo</i>	-9.35	7.3
vanilla	0.5	<i>Generic</i>	-8.89	6.92
vanilla	0.5	<i>Omniscient</i>	-8.68	6.93
vanilla	0.5	<i>REVISE</i>	-8.53	6.75
<b>full</b>	<b>1</b>	<b>ECCo</b>	<b>-1.1</b>	<b>0.163</b>
full	1	<i>Generic</i>	-1.49	0.726
full	1	<i>Omniscient</i>	-5.16	5.2
full	1	<i>REVISE</i>	$-3.09 \cdot 10^{26}$	$7.22 \cdot 10^{26}$
vanilla	1	<i>ECCo</i>	-9.34	7.36
vanilla	1	<i>Generic</i>	-8.86	6.85
vanilla	1	<i>Omniscient</i>	-8.7	6.9
vanilla	1	<i>REVISE</i>	-8.69	6.85
full	5	<i>ECCo</i>	-1.75	0.154
<b>full</b>	<b>5</b>	<b>Generic</b>	<b>-1.21</b>	<b>0.363</b>
full	5	<i>Omniscient</i>	-5.14	5.16
full	5	<i>REVISE</i>	$-1.1 \cdot 10^{28}$	$2.5 \cdot 10^{28}$
vanilla	5	<i>ECCo</i>	-9.36	7.32
vanilla	5	<i>Generic</i>	-8.88	6.91
vanilla	5	<i>Omniscient</i>	-8.7	6.93
vanilla	5	<i>REVISE</i>	-8.52	6.73
full	10	<i>ECCo</i>	$-1.02 \cdot 10^6$	$2.32 \cdot 10^6$
<b>full</b>	<b>10</b>	<b>Generic</b>	<b>-1.49</b>	<b>0.702</b>
full	10	<i>Omniscient</i>	-5.13	5.16
full	10	<i>REVISE</i>	$-3.74 \cdot 10^{26}$	$9.09 \cdot 10^{26}$
vanilla	10	<i>ECCo</i>	-9.31	7.33
vanilla	10	<i>Generic</i>	-8.87	6.86
vanilla	10	<i>Omniscient</i>	-8.7	6.89
vanilla	10	<i>REVISE</i>	-8.69	6.83
full	15	<i>ECCo</i>	$-3.31 \cdot 10^{13}$	$7.54 \cdot 10^{13}$
<b>full</b>	<b>15</b>	<b>Generic</b>	<b>-1.22</b>	<b>0.37</b>
full	15	<i>Omniscient</i>	-5.2	5.23
full	15	<i>REVISE</i>	$-9.01 \cdot 10^{27}$	$2.06 \cdot 10^{28}$
vanilla	15	<i>ECCo</i>	-9.38	7.34
vanilla	15	<i>Generic</i>	-8.86	6.87
vanilla	15	<i>Omniscient</i>	-8.69	6.96
vanilla	15	<i>REVISE</i>	-8.51	6.73