Lab 2

Exponential Random Graph Models

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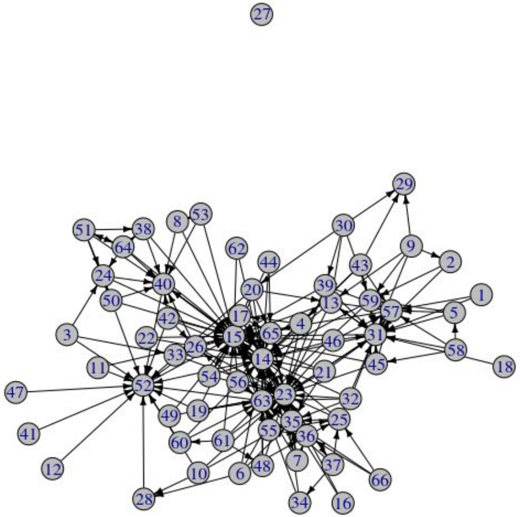
Part I Tips

- Plot the base (buy-in) network and include it in your report. Explain whether this plot supports or rejects Hypothesis 1 (Indegree popularity effects)?



A. This question is asking what you think about H1 based on the plot. That said, there is no right and wrong to support or reject H1. Your reasoning is more important.

B. Focus on whether there are few employees receiving more ties than others (that’s what indegree popularity effects are).

Hypothesis Testing

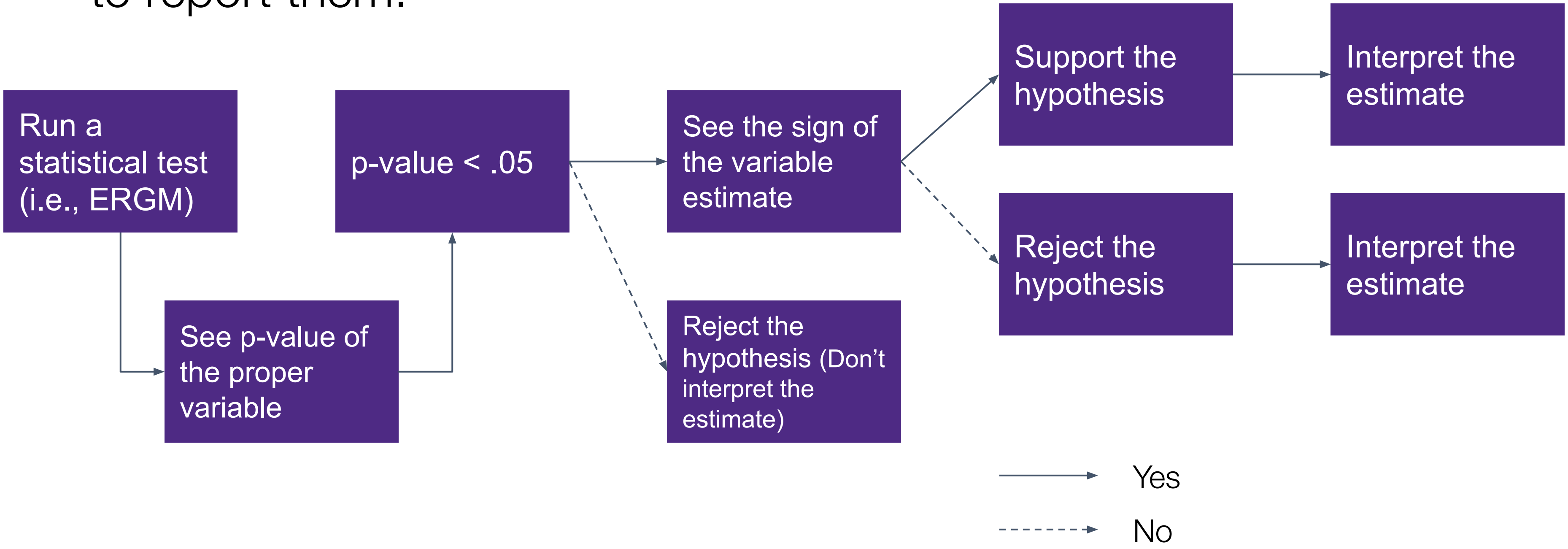
- A **hypothesis**: a conjecture statement about how two or more variables are related based on “educated guess”

- H_A: Individuals are *less* likely to report buy-in ties from others than not to report them.
- → The probability of having a buy-in tie in the network is *lower* than 50% (random chance).
- → There is a *negative* relationship between the number of buy-in ties and the network.

$$Network \sim -\theta * Edges$$

Hypothesis Testing

- H_A: Individuals are *less* likely to report buy-in ties from others than not to report them.



Run a statistical test (i.e., ERGM)

See p-value of the proper variable

p-value < .05

See the sign of the variable estimate

Reject the hypothesis (Don't interpret the estimate)

Support the hypothesis

Reject the hypothesis

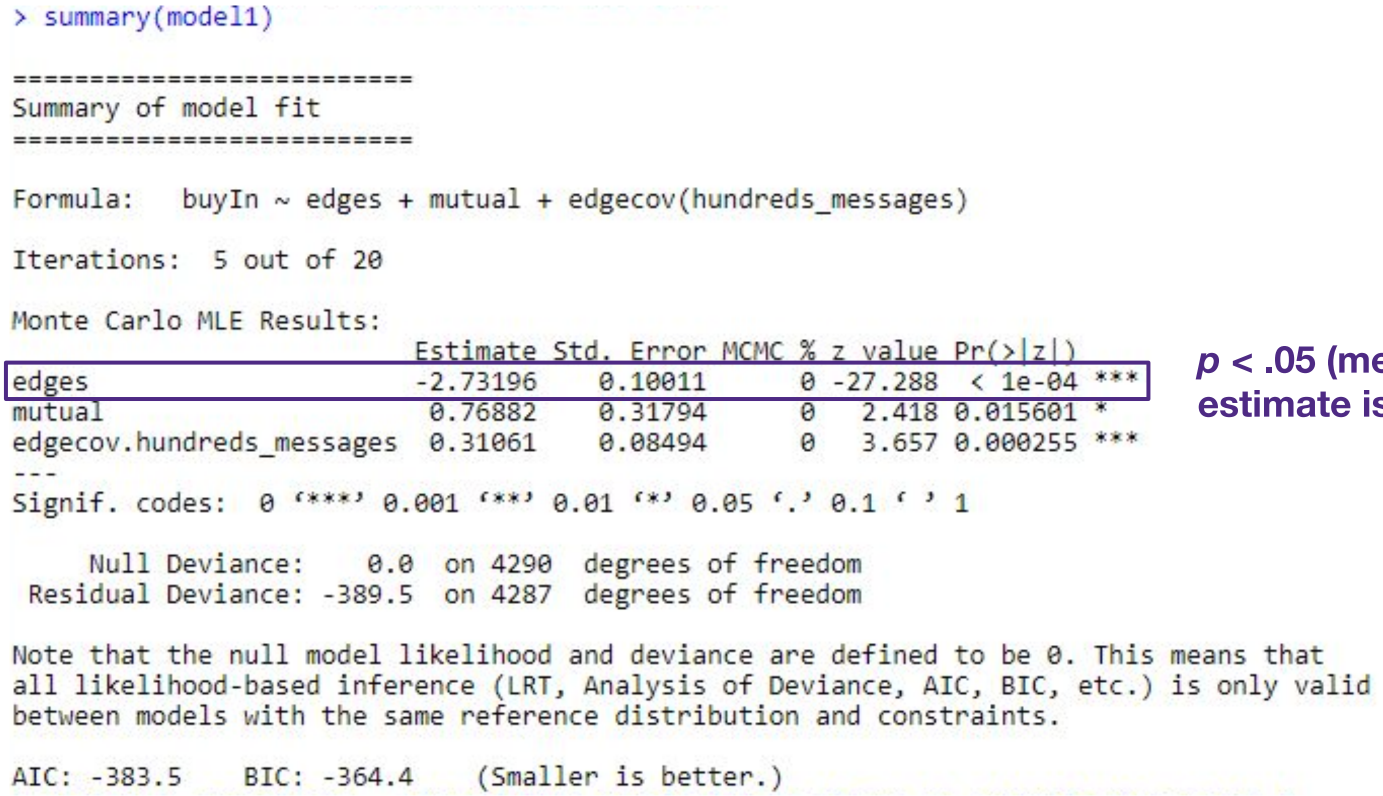
Interpret the estimate

Interpret the estimate

Yes

No

Results of Model 1

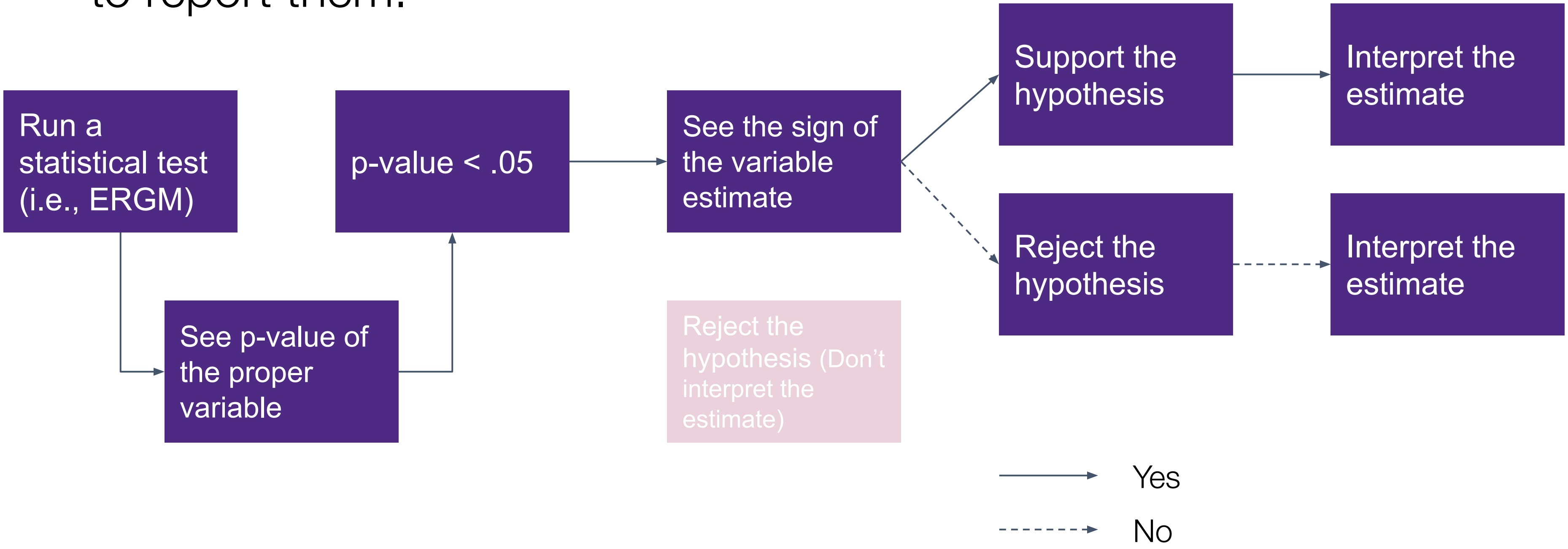


```
> summary(model1)

==========================
Summary of model fit
==========================

Formula:   buyIn ~ edges + mutual + edgecov(hundreds_messages)

Iterations:  5 out of 20

Monte Carlo MLE Results:
                          Estimate Std. Error MCMC % z value Pr(>|z|)
edges                     -2.73196    0.10011      0 -27.288  < 1e-04 ***
mutual                     0.76882    0.31794      0   2.418 0.015601 *
edgecov.hundreds_messages  0.31061    0.08494      0   3.657 0.000255 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

     Null Deviance:    0.0  on 4290  degrees of freedom
 Residual Deviance: -389.5  on 4287  degrees of freedom

Note that the null model likelihood and deviance are defined to be 0. This means that
all likelihood-based inference (LRT, Analysis of Deviance, AIC, BIC, etc.) is only valid
between models with the same reference distribution and constraints.

AIC: -383.5    BIC: -364.4    (Smaller is better.)
```

p < .05 (meaning the estimate is not 0 ($\theta \neq 0$))

Hypothesis Testing

- H_A: Individuals are ***less*** likely to report buy-in ties from others than not to report them.



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See p-value of the proper variable

p-value < .05

See the sign of the variable estimate

Reject the hypothesis (Don't interpret the estimate)

Support the hypothesis

Interpret the estimate

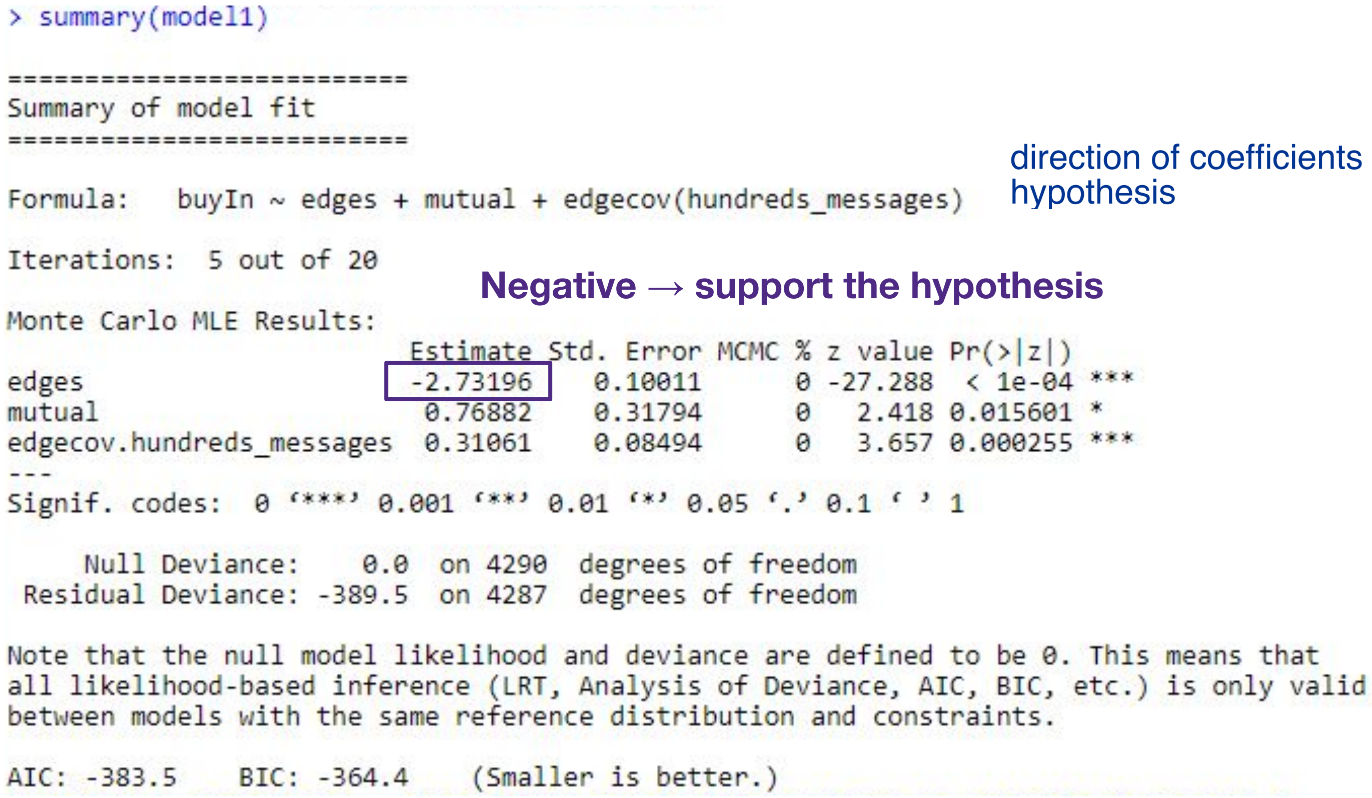
Reject the hypothesis

Interpret the estimate

Yes

No

Testing Hypothesis A



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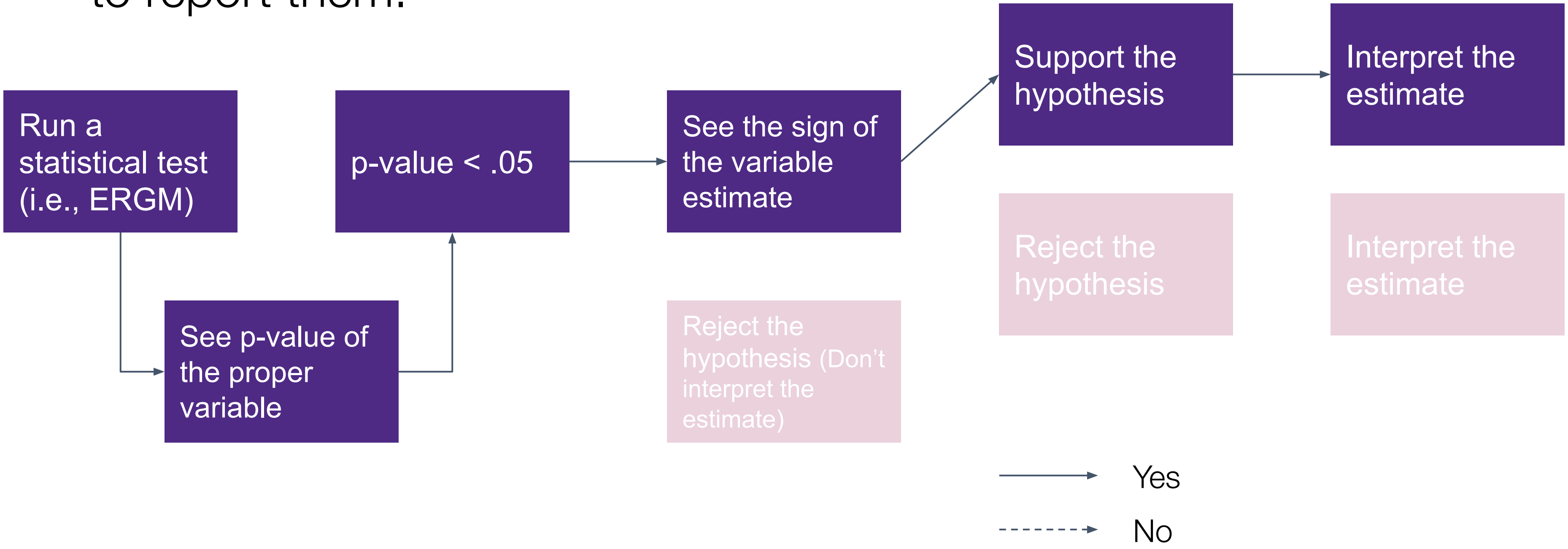
AIC: -383.5    BIC: -364.4    (Smaller is better.)
```

Negative → support the hypothesis

direction of coefficients matches with our hypothesis

Testing Hypothesis A

- H_A: Individuals are ***less*** likely to report buy-in ties from others than not to report them.



Run a statistical test (i.e., ERGM)
See p-value of the proper variable
p-value < .05
See the sign of the variable estimate
Support the hypothesis
Interpret the estimate
Reject the hypothesis
Interpret the estimate
Reject the hypothesis (Don't interpret the estimate)
Yes
No

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Interpret the Estimate of Edges

Run `exp(-2.73196)` this is the coefficient

→ 0.06509159

- The estimate (**θ**): Conditional log-odds ratio

$$Network \sim -2.73196 * Edges$$

exp of the log odds ratio

- Exponential: **odds ratio (if an odds ratio is 1, odds are even)**
  - The odds of individuals reporting buy-in ties are <u>0.07 times *lower* than</u> the odds of individuals *not* reporting them

messaging others / sending messaging ties are

Interpret the Estimate of Edges

Run `exp(-2.73196)/(1+exp(-2.73196))`

→ 0.0611136

- The estimate (**θ**): Conditional log-odds ratio

$$Network \sim -2.73196 * Edges$$

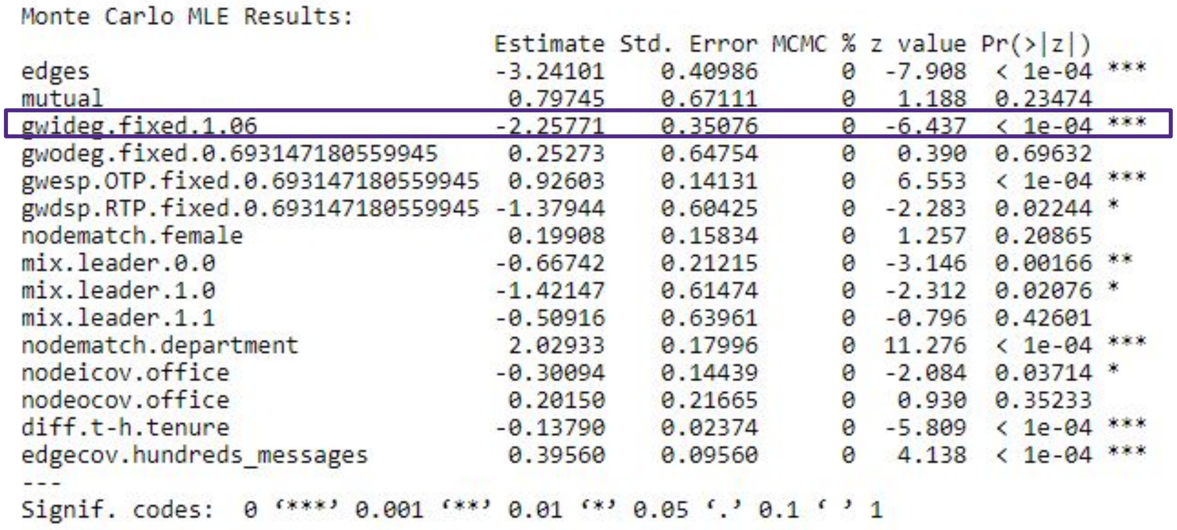
- Inverse logit (`plogis`): **probability (random chance is 50%)**
  - The probability of having a buy-in tie in the network is 6%.

just means the tie is very unlikely

One More Example in Model 2

- H1: There will be *indegree popularity effects* (tendency of a small number of nodes to receive many ties) in who people report is capable of getting buy in from them.

Geometrically weighted indegree measures a tendency *<u>against</u>* indegree preferential attachment



```
Monte Carlo MLE Results:
                                    Estimate Std. Error MCMC % z value Pr(>|z|)
edges                               -3.24101    0.40986      0  -7.908  < 1e-04 ***
mutual                               0.79745    0.67111      0   1.188  0.23474
gwideg.fixed.1.06                   -2.25771    0.35076      0  -6.437  < 1e-04 ***
gwodeg.fixed.0.693147180559945       0.25273    0.64754      0   0.390  0.69632
gwesp.OTP.fixed.0.693147180559945    0.92603    0.14131      0   6.553  < 1e-04 ***
gwdsp.RTP.fixed.0.693147180559945   -1.37944    0.60425      0  -2.283  0.02244 *
nodematch.female                     0.19908    0.15834      0   1.257  0.20865
mix.leader.0.0                      -0.66742    0.21215      0  -3.146  0.00166 **
mix.leader.1.0                      -1.42147    0.61474      0  -2.312  0.02076 *
mix.leader.1.1                      -0.50916    0.63961      0  -0.796  0.42601
nodematch.department                 2.02933    0.17996      0  11.276  < 1e-04 ***
nodeicov.office                     -0.30094    0.14439      0  -2.084  0.03714 *
nodeocov.office                      0.20150    0.21665      0   0.930  0.35233
diff.t-h.tenure                     -0.13790    0.02374      0  -5.809  < 1e-04 ***
edgecov.hundreds_messages            0.39560    0.09560      0   4.138  < 1e-04 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Interpret the Estimate of H1

Run `exp(-2.25771)/(1+exp(-2.25771))`

→ 0.0946865

Or run `exp(2.25771)/(1+exp(2.25771))`

→ 0.9053135

- Inverse logit (`plogis`): **probability**
  - The probability of those who are *not* popular receiving a tie in the network is 9%.
  - The probability of those who are popular receiving a tie in the network is 91%.

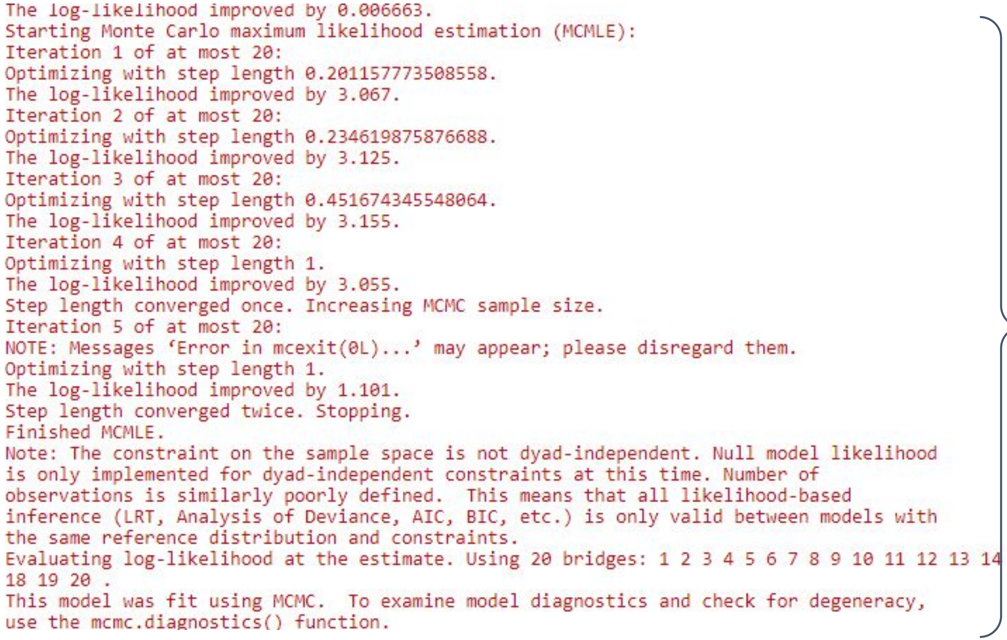
Part III: Model Convergence

- **Model convergence:** examining whether the estimate process is converged or not

- **Goodness-of-fit test:** examining whether your model estimate represents your data

Model Convergence

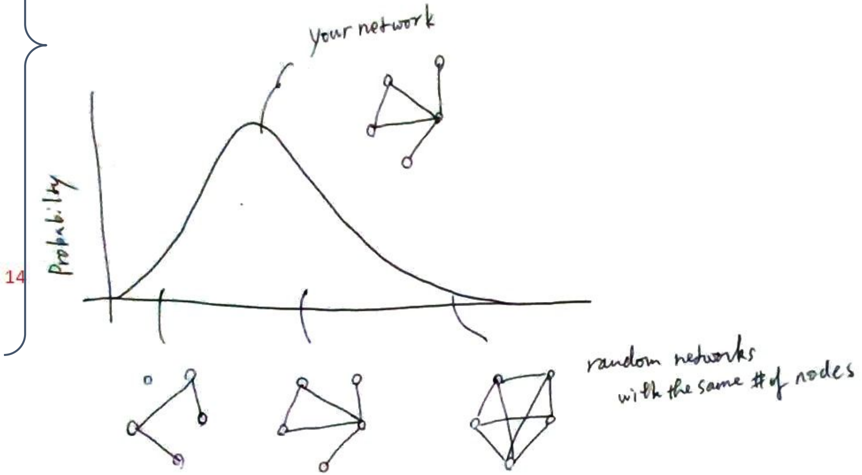
- When you run ergm(buyIn ~ edges +...) in R



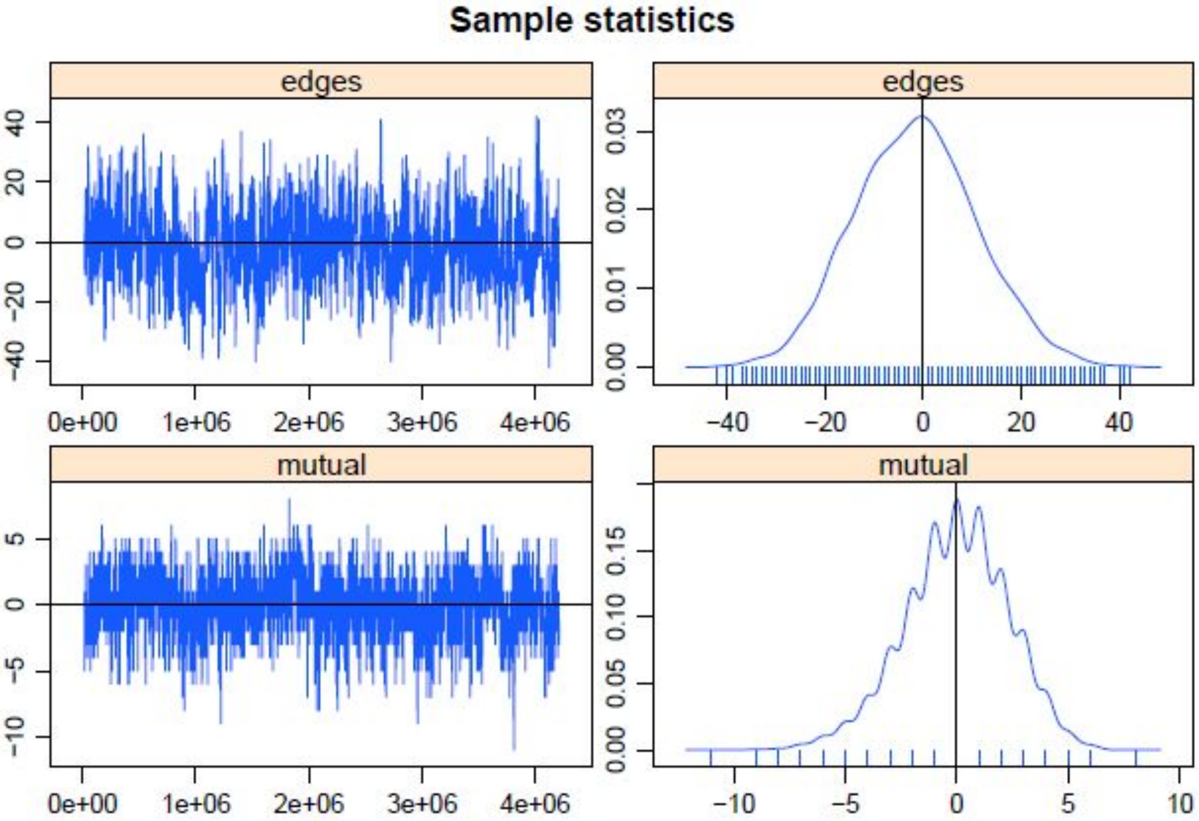
```
The log-likelihood improved by 0.006663.
Starting Monte Carlo maximum likelihood estimation (MCMLE):
Iteration 1 of at most 20:
Optimizing with step length 0.201157773508558.
The log-likelihood improved by 3.067.
Iteration 2 of at most 20:
Optimizing with step length 0.234619875876688.
The log-likelihood improved by 3.125.
Iteration 3 of at most 20:
Optimizing with step length 0.451674345548064.
The log-likelihood improved by 3.155.
Iteration 4 of at most 20:
Optimizing with step length 1.
The log-likelihood improved by 3.055.
Step length converged once. Increasing MCMC sample size.
Iteration 5 of at most 20:
NOTE: Messages ‘Error in mcexit(0L)...’ may appear; please disregard them.
Optimizing with step length 1.
The log-likelihood improved by 1.101.
Step length converged twice. Stopping.
Finished MCMLE.
Note: The constraint on the sample space is not dyad-independent. Null model likelihood
is only implemented for dyad-independent constraints at this time. Number of
observations is similarly poorly defined.  This means that all likelihood-based
inference (LRT, Analysis of Deviance, AIC, BIC, etc.) is only valid between models with
the same reference distribution and constraints.
Evaluating log-likelihood at the estimate. Using 20 bridges: 1 2 3 4 5 6 7 8 9 10 11 12 13 14
18 19 20 .
This model was fit using MCMC.  To examine model diagnostics and check for degeneracy,
use the mcmc.diagnostics() function.
```

MCMC-MLE

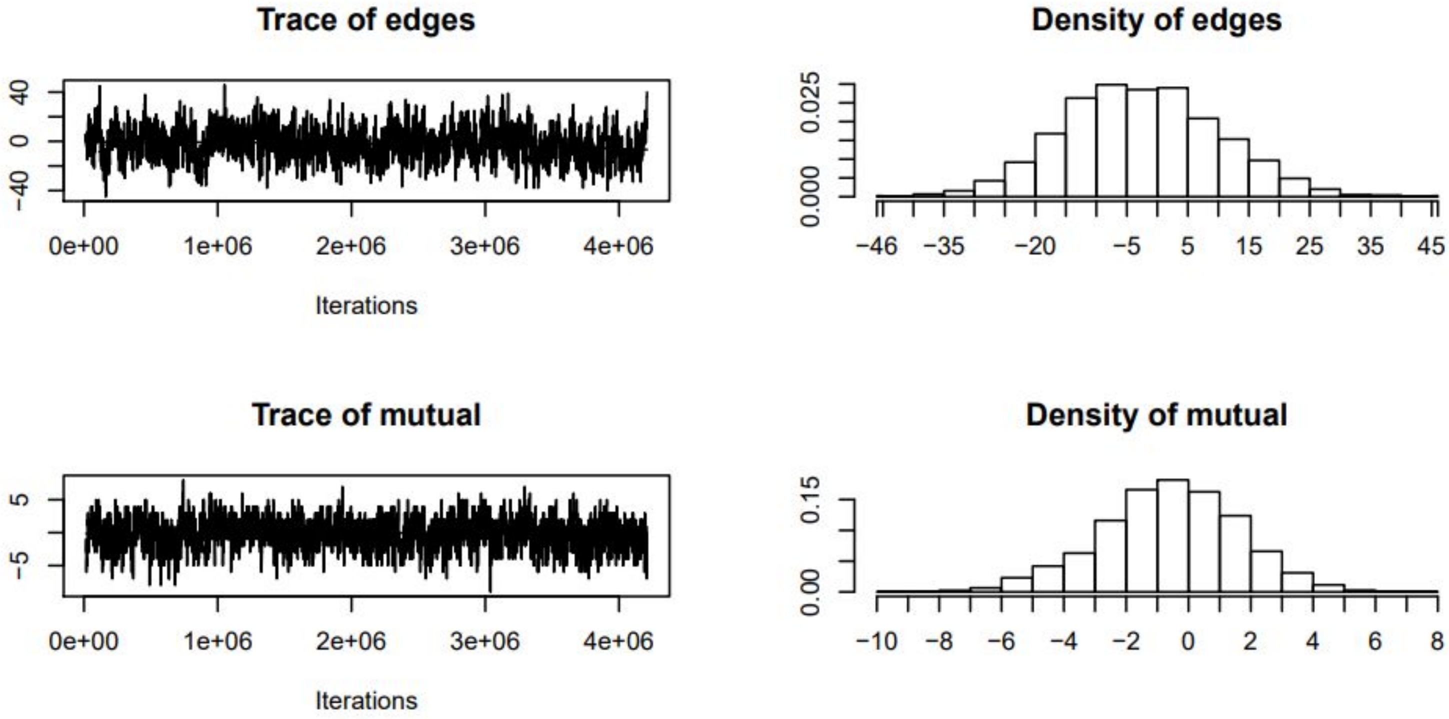
- Markov Chain: because it simulates network Yt+1 randomly based on Yt
- Monte Carlo: because of the computational implementation of the “randomly” generated part
- Maximum Likelihood Estimation



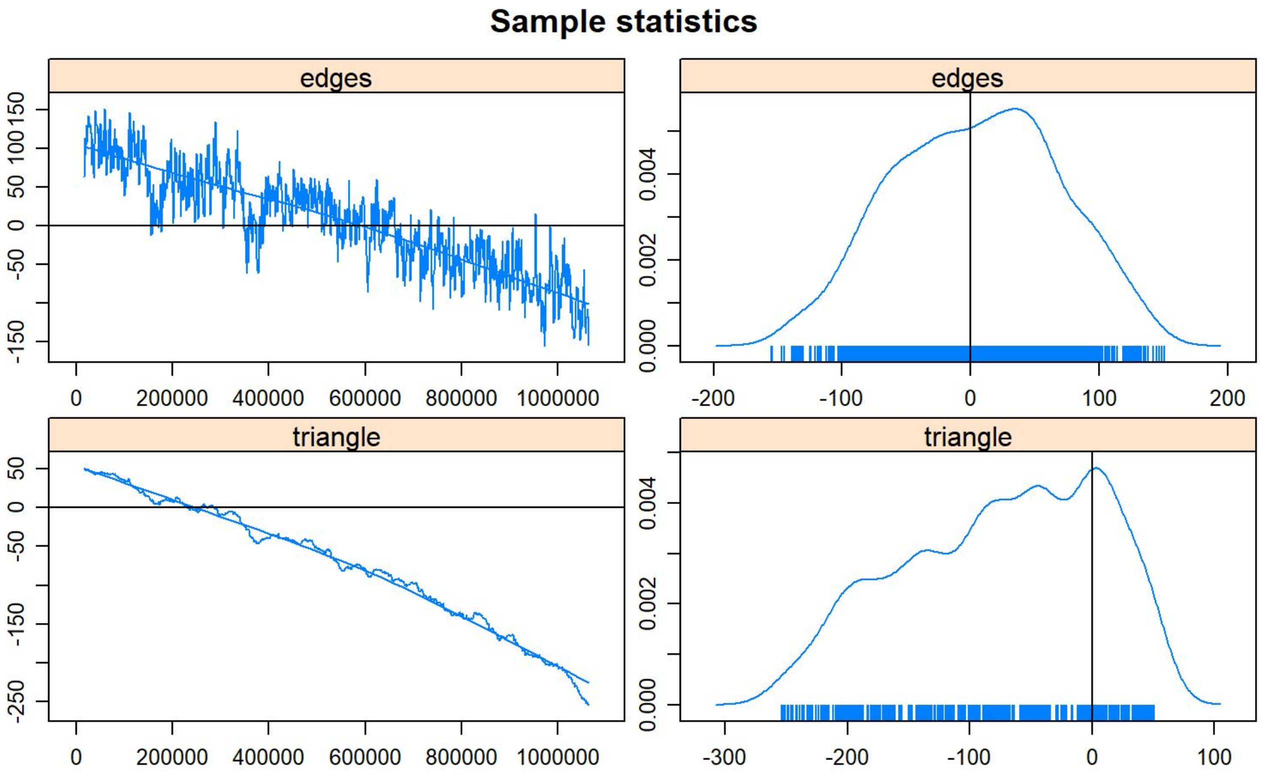
Producing a PDF file



Some People Got This

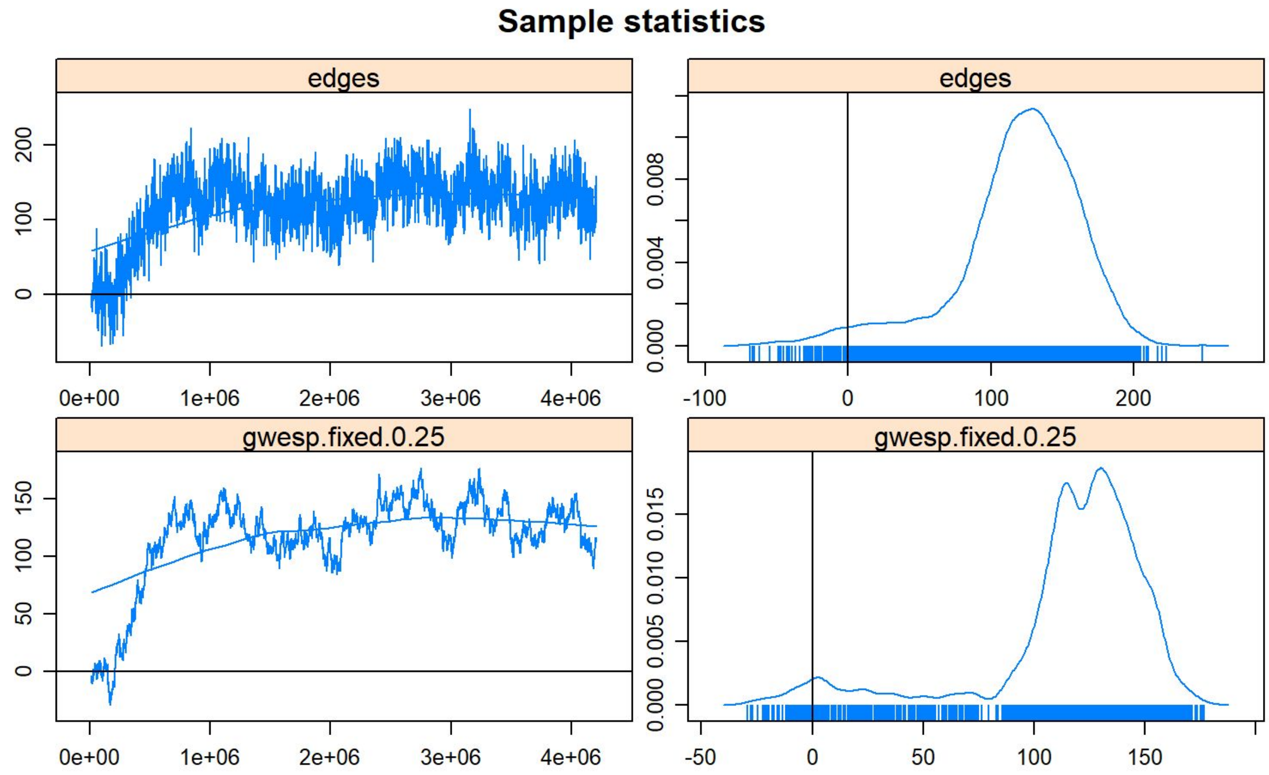


Bad Cases



http://statnet.org/Workshops/ergm_tutorial.html#7_diagnostics:_troubleshooting_and_checking_for_model_degeneracy

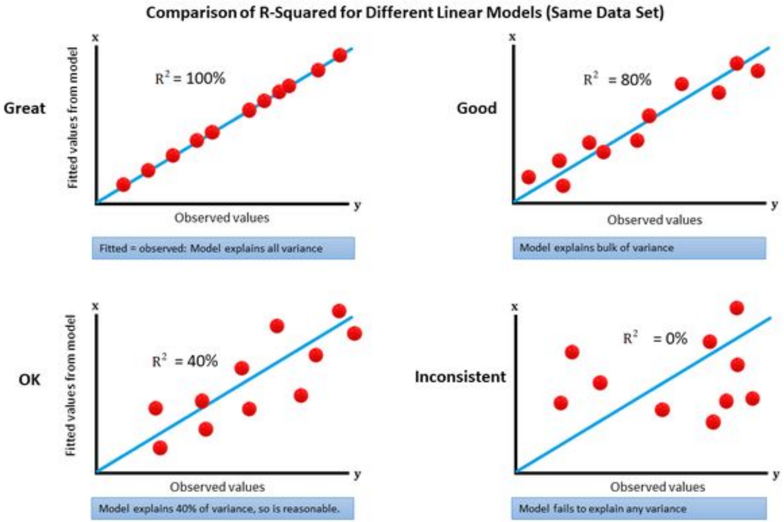
Bad Cases



http://statnet.org/Workshops/ergm_tutorial.html#7_diagnostics:_troubleshooting_and_checking_for_model_degeneracy

Goodness-of-fit Test

- **Does my model (i.e., the results of ERGM) fit well with my network data?**
- Think of a goodness-of-fit test as R-squared in regression



Comparison of R-Squared for Different Linear Models (Same Data Set)

Fitted = observed: Model explains all variance

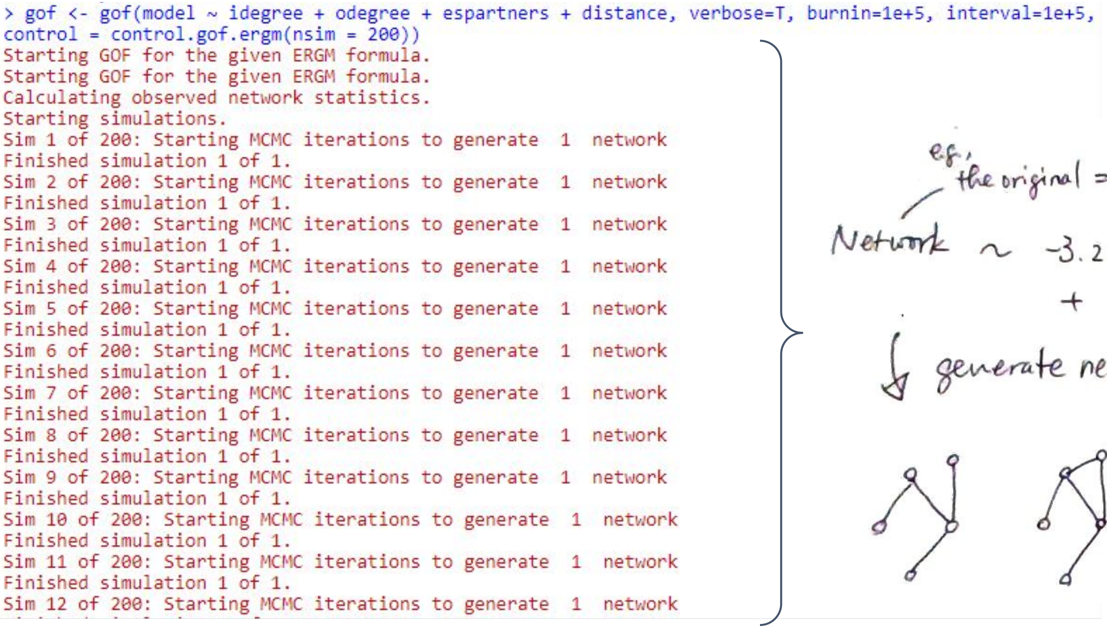
Model explains bulk of variance

Model explains 40% of variance, so is reasonable.

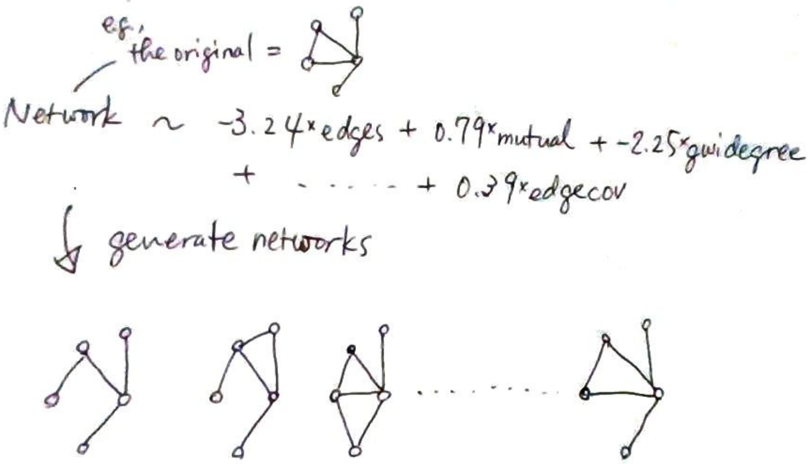
Model fails to explain any variance

Goodness-of-fit Test

If you run gof(model1 ~ …)

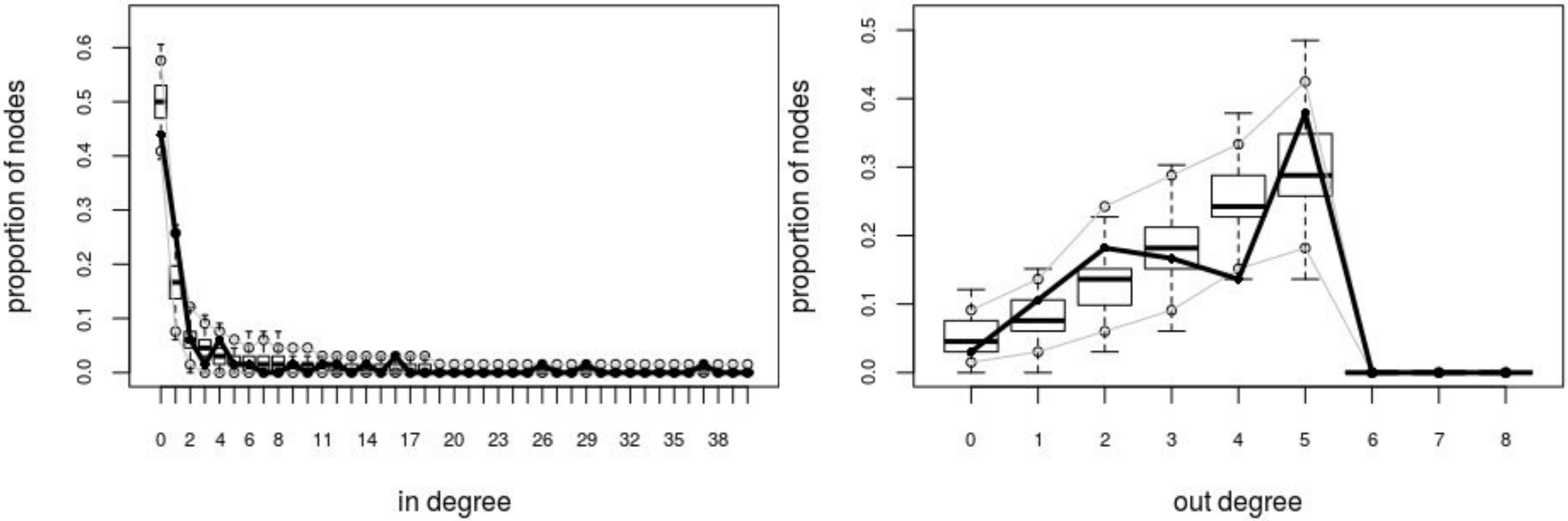


```
> gof <- gof(model ~ idegree + odegree + espartners + distance, verbose=T, burnin=1e+5, interval=1e+5,
control = control.gof.ergm(nsim = 200))
Starting GOF for the given ERGM formula.
Starting GOF for the given ERGM formula.
Calculating observed network statistics.
Starting simulations.
Sim 1 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 2 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 3 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 4 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 5 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 6 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 7 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 8 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 9 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 10 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 11 of 200: Starting MCMC iterations to generate  1  network
Finished simulation 1 of 1.
Sim 12 of 200: Starting MCMC iterations to generate  1  network
```


e.g.,
the original =
Network ~ -3.24×edges + 0.79×mutual + -2.25×gwidegree
+ . . . . . + 0.39×edgecov
generate networks
. . . .

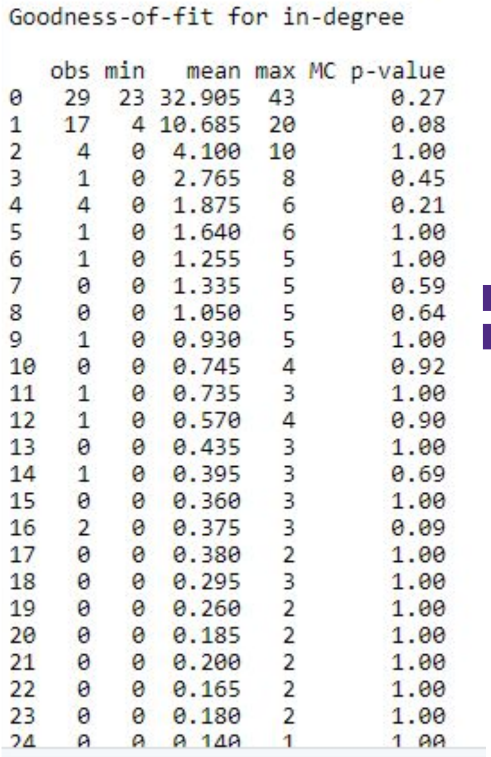
Produce Plots in Your Plots Panel

If you run plot(gof)



Interpret the Output of GOF

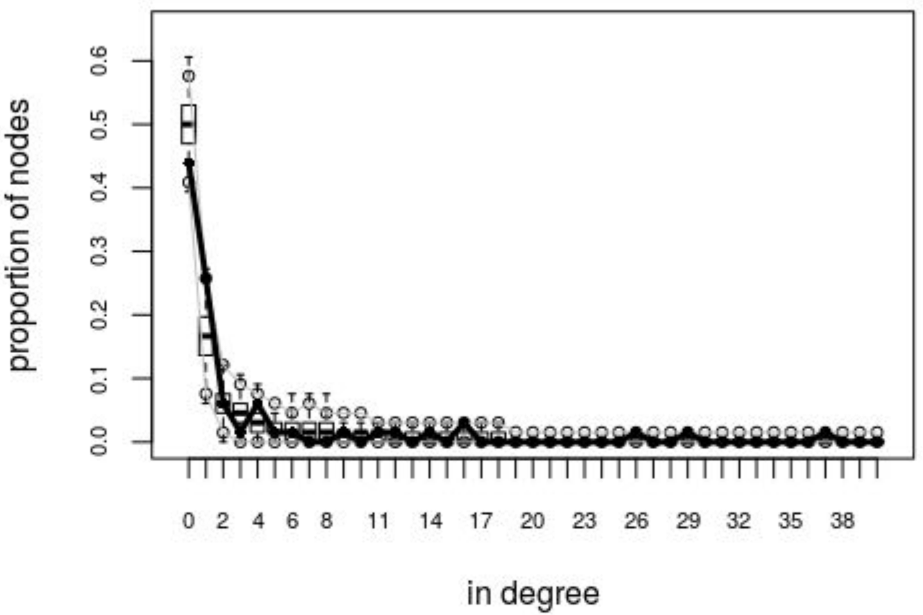
If you run gof



```
Goodness-of-fit for in-degree

   obs min   mean max MC p-value
0   29  23 32.905  43       0.27
1   17   4 10.685  20       0.08
2    4   0  4.100  10       1.00
3    1   0  2.765   8       0.45
4    4   0  1.875   6       0.21
5    1   0  1.640   6       1.00
6    1   0  1.255   5       1.00
7    0   0  1.335   5       0.59
8    0   0  1.050   5       0.64
9    1   0  0.930   5       1.00
10   0   0  0.745   4       0.92
11   1   0  0.735   3       1.00
12   1   0  0.570   4       0.90
13   0   0  0.435   3       1.00
14   1   0  0.395   3       0.69
15   0   0  0.360   3       1.00
16   2   0  0.375   3       0.09
17   0   0  0.380   2       1.00
18   0   0  0.295   3       1.00
19   0   0  0.260   2       1.00
20   0   0  0.185   2       1.00
21   0   0  0.200   2       1.00
22   0   0  0.165   2       1.00
23   0   0  0.180   2       1.00
24   0   0  0.140   1       1.00
```

=



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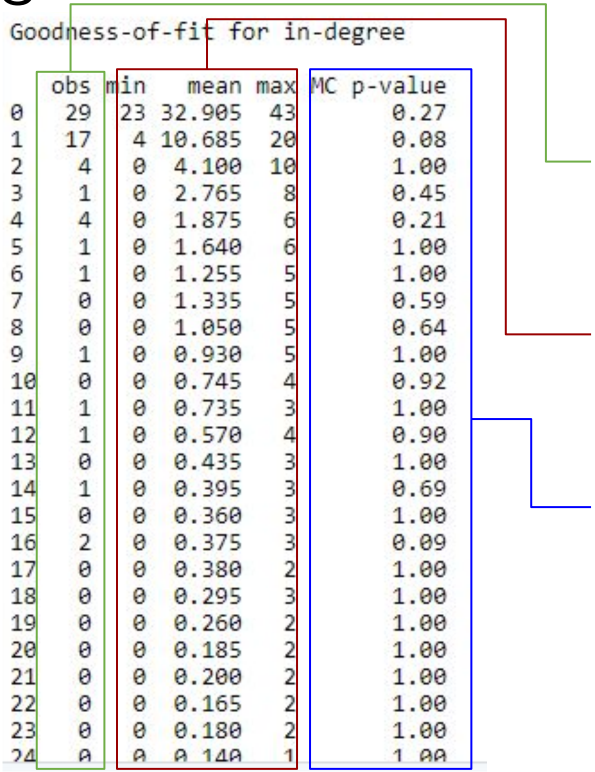

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22   0   0  0.165   2       1.00
23   0   0  0.180   2       1.00
24   0   0  0.140   1       1.00
```

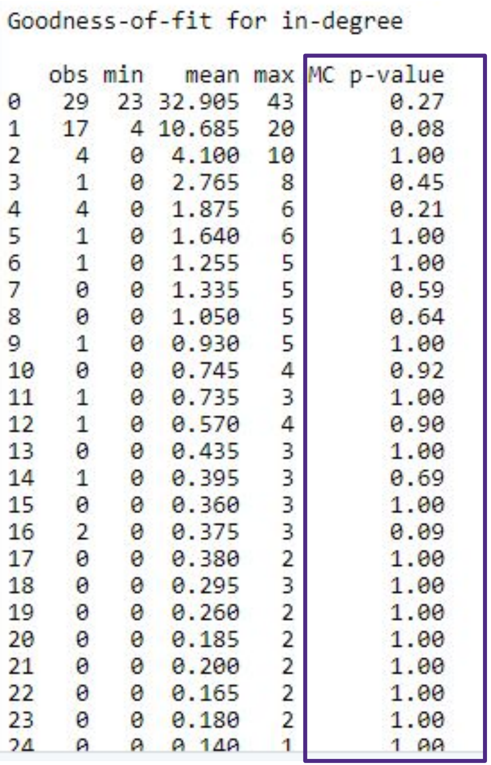
Actual count from the original network

Counts from the simulated network based on the model (i.e., the results of ERGM)

p-value

Interpret the Output of GOF

If you run gof



```
Goodness-of-fit for in-degree

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3    1   0  2.765   8       0.45
4    4   0  1.875   6       0.21
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7    0   0  1.335   5       0.59
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20   0   0  0.185   2       1.00
21   0   0  0.200   2       1.00
22   0   0  0.165   2       1.00
23   0   0  0.180   2       1.00
24   0   0  0.140   1       1.00
```
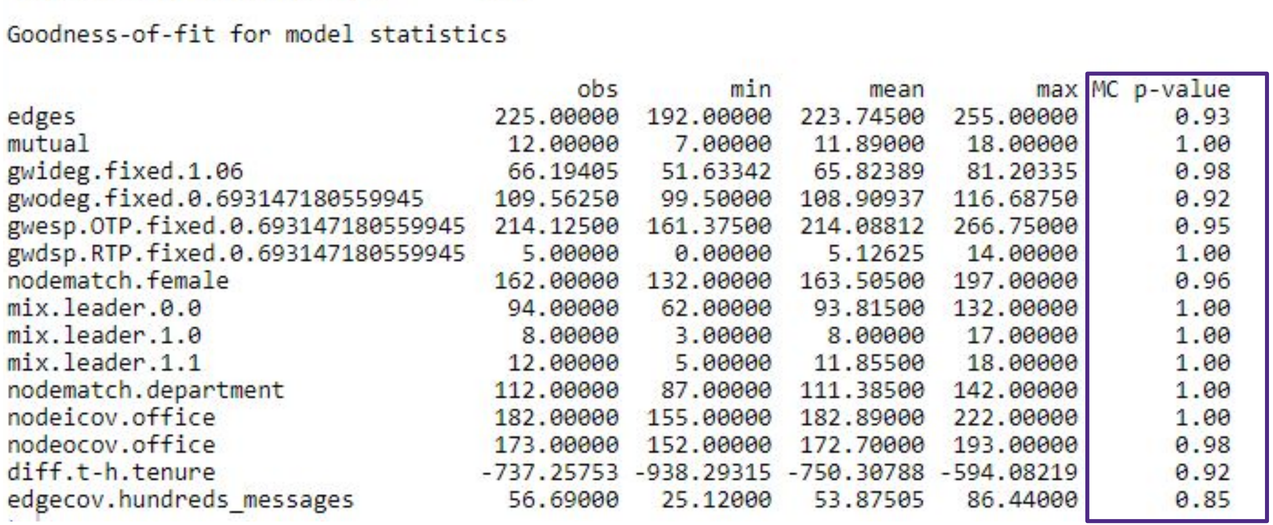
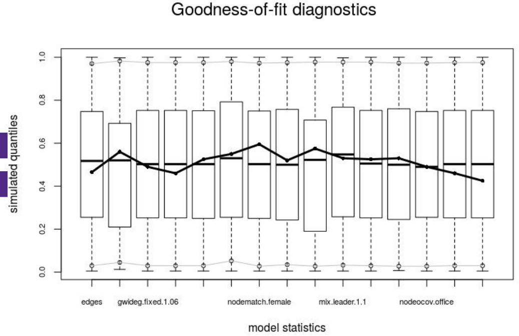

If p > .05, it indicates that the model fits well with the original network in terms of the in-degree distribution

Interpret the Output of GOF

If you run gof



```
Goodness-of-fit for model statistics

                                         obs        min       mean        max MC p-value
edges                              225.00000  192.00000  223.74500  255.00000       0.93
mutual                              12.00000    7.00000   11.89000   18.00000       1.00
gwideg.fixed.1.06                   66.19405   51.63342   65.82389   81.20335       0.98
gwodeg.fixed.0.693147180559945     109.56250   99.50000  108.90937  116.68750       0.92
gwesp.OTP.fixed.0.693147180559945  214.12500  161.37500  214.08812  266.75000       0.95
gwdsp.RTP.fixed.0.693147180559945    5.00000    0.00000    5.12625   14.00000       1.00
nodematch.female                   162.00000  132.00000  163.50500  197.00000       0.96
mix.leader.0.0                      94.00000   62.00000   93.81500  132.00000       1.00
mix.leader.1.0                       8.00000    3.00000    8.00000   17.00000       1.00
mix.leader.1.1                      12.00000    5.00000   11.85500   18.00000       1.00
nodematch.department               112.00000   87.00000  111.38500  142.00000       1.00
nodeicov.office                    182.00000  155.00000  182.89000  222.00000       1.00
nodeocov.office                    173.00000  152.00000  172.70000  193.00000       0.98
diff.t-h.tenure                   -737.25753 -938.29315 -750.30788 -594.08219       0.92
edgecov.hundreds_messages           56.69000   25.12000   53.87505   86.44000       0.85
```



If p > .05, it indicates that the model fits well with the original network in terms of the variables

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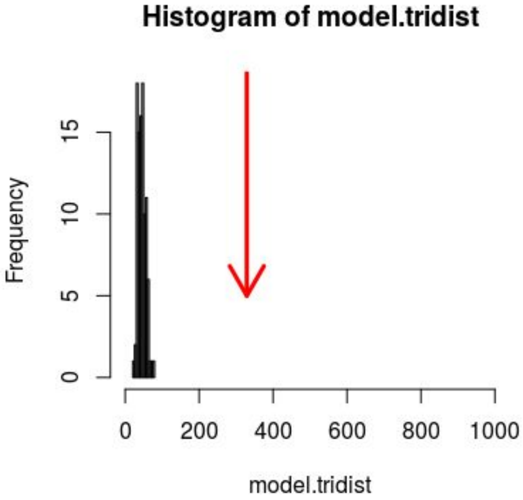
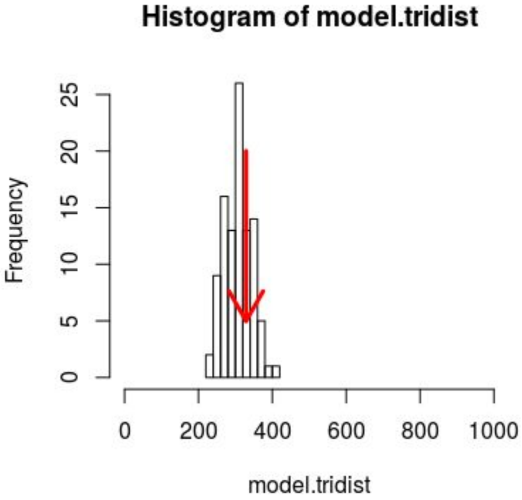
GOF for Triangles

Model 1



t-statistic = 26.3656
→ p < .001

Model 2



t-statistic = 0.6205
→ p > .50

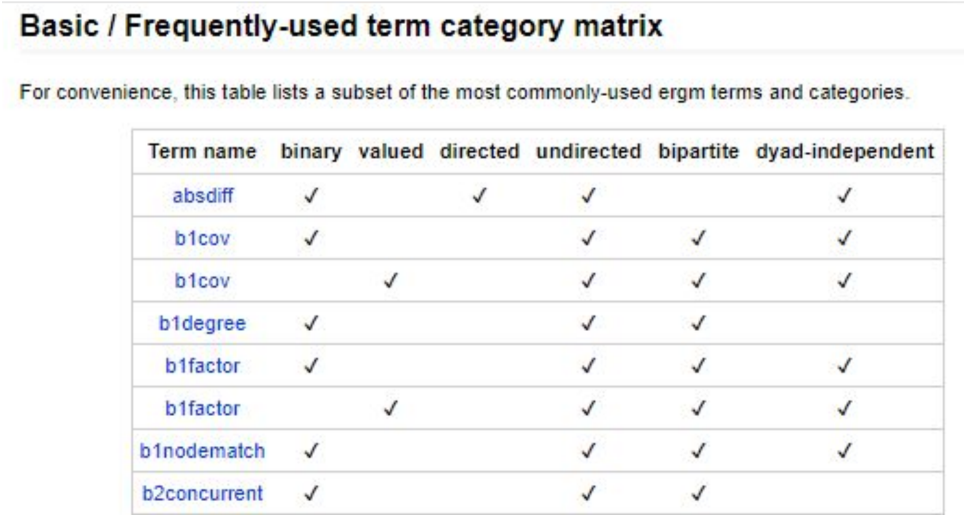
If p > .05, it indicates that the model fits well with the original network in terms of the triangles

```
2*pt(t, df=100, lower.tail = FALSE)
```

Q & A

ERGM Terms

- This is not for Lab 2, but if you use ERGMs for your final project, please take a look at this page: https://cran.r-project.org/web/packages/ergm/vignettes/ergm-term-crossRef.html



Basic / Frequently-used term category matrix

For convenience, this table lists a subset of the most commonly-used ergm terms and categories.

Term name	binary	valued	directed	undirected	bipartite	dyad-independent
absdiff	✓		✓	✓		✓
b1cov	✓			✓	✓	✓
b1cov		✓		✓	✓	✓
b1degree	✓			✓	✓	
b1factor	✓			✓	✓	✓
b1factor		✓		✓	✓	✓
b1nodematch	✓			✓	✓	✓
b2concurrent	✓			✓	✓	