Survey on the Classification of Topological Phases of Matter

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Outline

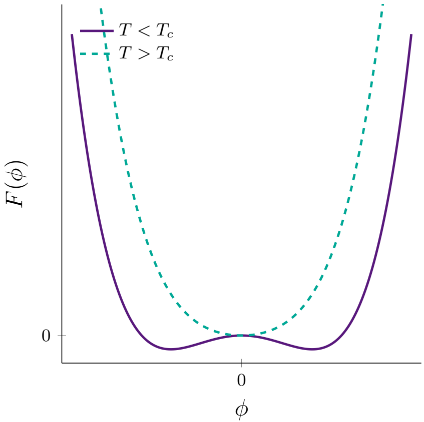
The Standard Model of Phases

The GLW Paradigm: Spontaneous Symmetry Breaking

The **Ginzburg-Landau-Wilson (GLW)** framework was the standard paradigm for classifying phases of matter.

- **Core Idea:** Phase transitions are driven by **Spontaneous Symmetry Breaking** (SSB).
- **Key Tool:** A **local order parameter** (ϕ) emerges to describe the broken-symmetry state.
  - High-T (Symmetric): $\langle \phi \rangle = 0$
  - Low-T (Broken): $\langle \phi \rangle \neq 0$

Figure 1: SSB: Free energy $F(\phi)$ vs. order parameter ϕ.



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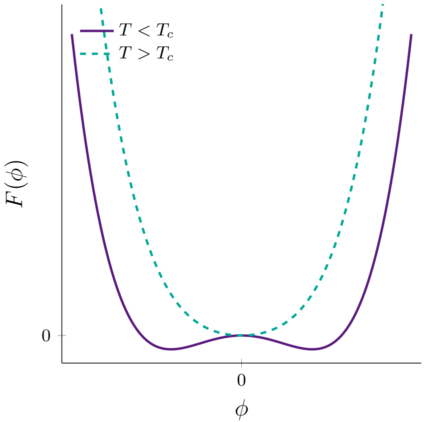

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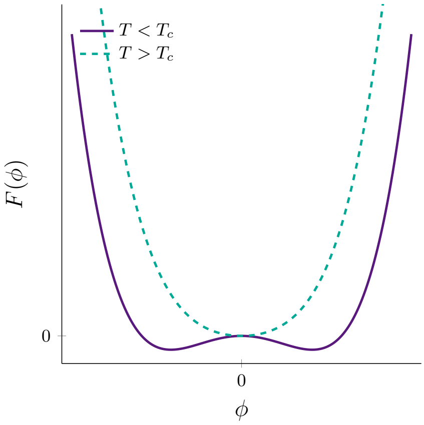

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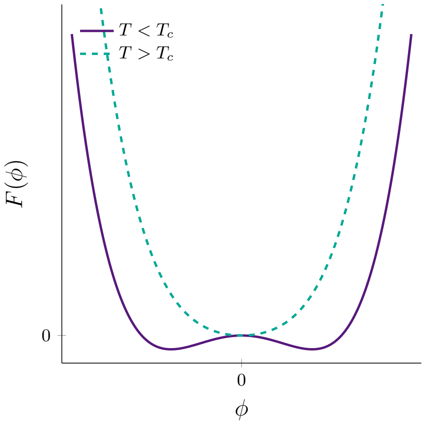

Core Contributions: The GLW Toolkit

Landau (1937)

Provided the foundational **mean-field framework**. Defined the order parameter based on **symmetry arguments**.

Ginzburg (1950)

Extended the theory to include **spatial fluctuations** ($|\nabla\phi|^2$). Introduced the **coherence length** (ξ).

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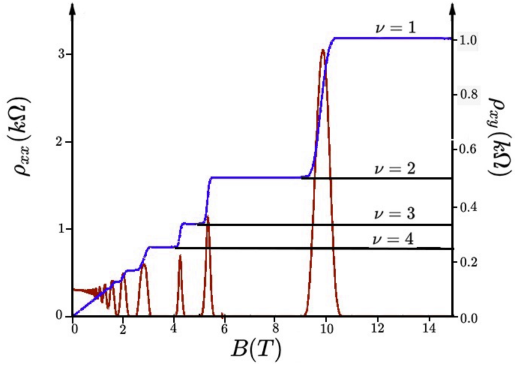
The Paradigm Fails: A Crisis in Physics

The GLW framework is fundamentally incomplete.

- **The Problem:** It cannot describe phases **without** a broken symmetry.
- **Key Example:** The **Quantum Hall Effect (QHE)**.
- **The Issue:**
  - QHE states (plateaus) are distinct phases of matter.
  - Yet, they all share the same symmetries.
  - No SSB $\implies$ no local ϕ.

Figure 2: QHE: ρ_{xy} is quantized; ρ_{xx} vanishes.



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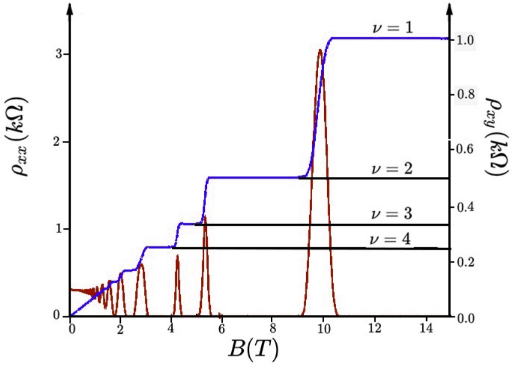

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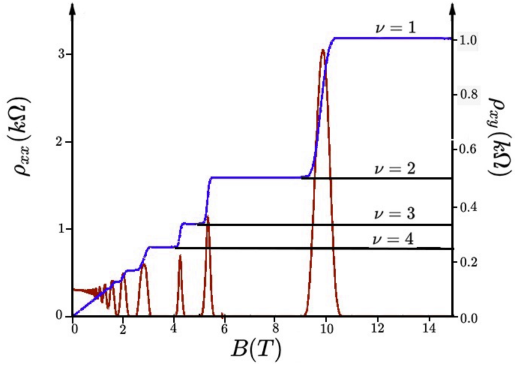

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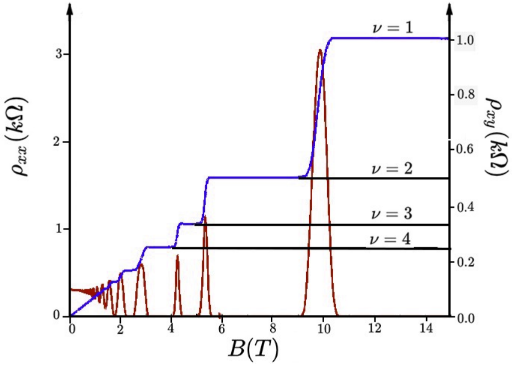

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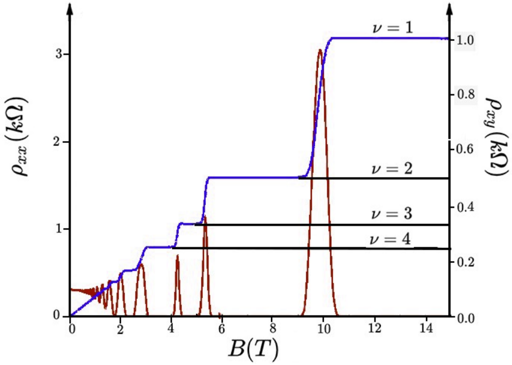

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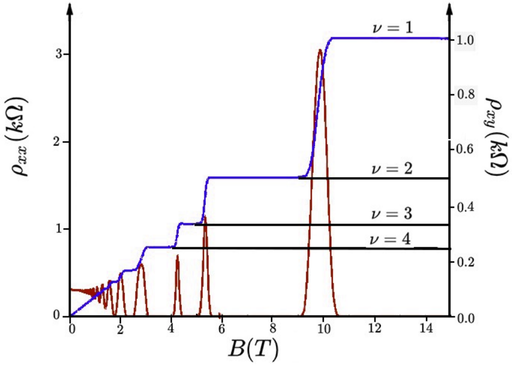

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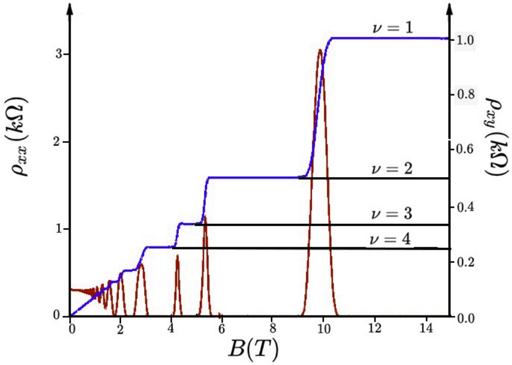

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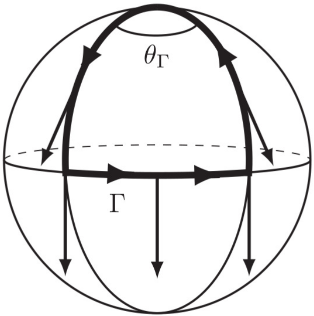
The New Paradigm: Topology



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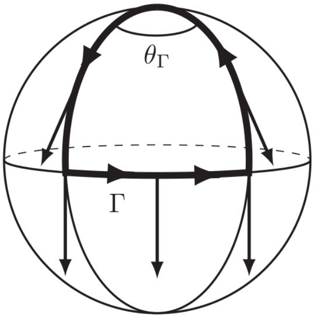
- The parameters of its Hamiltonian return to their start.
- ...but the wavefunction acquires an unexpected phase.

The Geometric Phase: A Conceptual View



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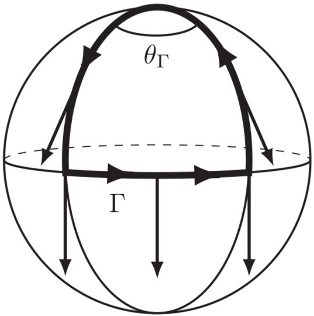

- A system evolves along a cyclic path $\mathcal{C}$.
- The parameters of its Hamiltonian return to their start.
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The Two Phases

Dynamical Phase

- **Source:** The system's own energy over time.
- **Depends on:** How long the evolution takes (the duration T).
- **Analogy:** A clock hand sweeping.

Geometric Phase (Berry Phase)

- **Source:** The *geometry* of the parameter space.
- **Depends on:** The path C taken (its shape, its enclosed area).
- **Analogy:** The change in longitude after travelling a loop on a globe.

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A Conceptual Timeline

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A trail of clues in physics:

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The Topological Era

Unifying geometry with matter:

- **Integer QHE** (1982)
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Analogy: Geometry (Gauss-Bonnet)

Integrating *local* curvature over a *global* (closed) space yields a *quantized* invariant.

$$\frac{1}{2\pi}\int_{\mathcal{M}} K\,dA = \chi(\mathcal{M}) = 2 - 2g$$

Here, $g = 2$, so $\chi = -2$.

Application: Physics (1D SSH Model)

The exact same principle defines the topological phase.

$$\nu = \frac{1}{\pi}\oint_{\mathrm{BZ}} A_\nu(k)\,dk = \begin{cases} 1 & \text{(Topological)} \\ 0 & \text{(Trivial)} \end{cases}$$

Here, $\nu \in \mathbb{Z}$ is the invariant.

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The Analogy at a Glance

Concept	Geometry (Gauss-Bonnet)	Physics (Topological Phase)
Local Property	Gaussian Curvature (K)	Berry Connection/Curvature ($A_k, \mathcal{F}$)
Global Space	Closed Surface ($\mathcal{M}$)	Brillouin Zone (BZ $\cong T^n$)
Quantized Invariant	Euler Characteristic ($\chi \in \mathbb{Z}$)	Winding/Chern Number ($\nu, C \in \mathbb{Z}$)

The Takeaway: Robustness

- The invariant (ν or χ) is robust to smooth, local deformations.
- (e.g., small bumps on the manifold, or small changes to the Hamiltonian $H(k)$).
- It can *only* change if the topology is broken (i.e., the energy gap closes).

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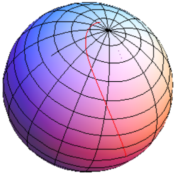
Topological Manifolds in Physics

Spinor Evolution

System: Adiabatic Spinor in Rotating Magnetic Field

Phase Type: Berry Phase

Manifold: Unit Sphere ($\mathbf{S}^2$) / Bloch Sphere



Crystalline Electrons

System: Electrons in Crystalline Solids

Phase Type: Zak Phase / Berry Phase

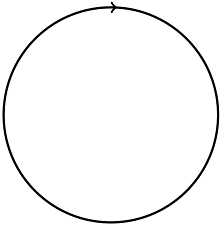
Manifold: Brillouin Zone ($\mathbf{T}^d$) / Torus

1D Crystal

System: One-Dimensional (1D) Crystal

Phase Type: Zak Phase

Manifold: Closed Loop ($\mathbf{S}^1$)



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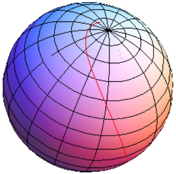

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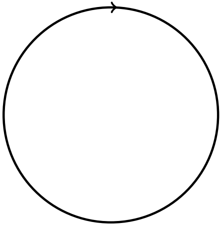
System: One-Dimensional (1D) Crystal

Phase Type: Zak Phase

Manifold: Closed Loop ($\mathbf{S}^1$)



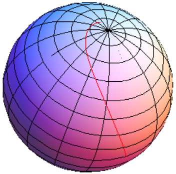
Topological Manifolds in Physics

Spinor Evolution

System: Adiabatic Spinor in Rotating Magnetic Field

Phase Type: Berry Phase

Manifold: Unit Sphere ($\mathbf{S}^2$) / Bloch Sphere



Crystalline Electrons

System: Electrons in Crystalline Solids

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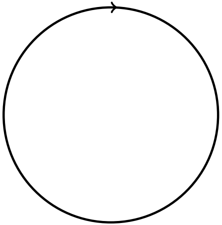

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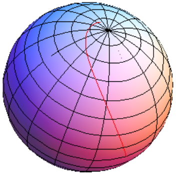

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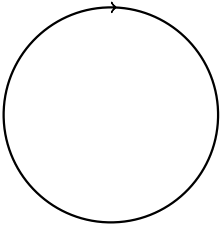

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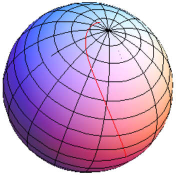

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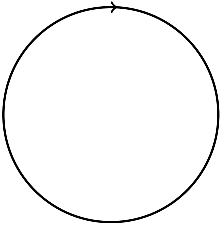

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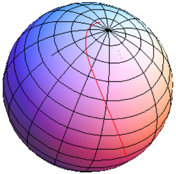

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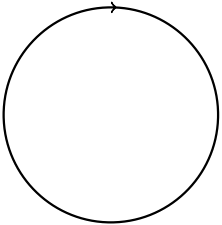

The Classification

The 10-Fold Way: Symmetries

The Three Fundamental Symmetries

- Time-Reversal (T) $T^2 = 0, +1, -1$, Particle-Hole (P) $P^2 = 0, +1, -1$
- Chiral (S) $S = TP, \quad S^2 = 0, 1$

		10 Symmetry Classes					
		TRS	**PHS**	**CS**	**d=1**	**d=2**	**d=3**
Standard (Wigner-Dyson)	A (unitary)	0	0	0	$--$	$\mathbb{Z}$	$--$
	AI (orthogonal)	$+1$	0	0	$--$	$--$	$--$
	AII (symplectic)	-1	0	0	$--$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
Chiral	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$	$--$	$\mathbb{Z}$
	BDI (chiral orthogonal)	$+1$	$+1$	1	$\mathbb{Z}$	$--$	$--$
	CII (chiral symplectic)	-1	-1	1	$\mathbb{Z}$	$--$	$\mathbb{Z}_2$
BdG	D (p-wave SC)	0	$+1$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	$--$
	C (d-wave SC)	0	-1	0	$--$	$\mathbb{Z}$	$--$
	DIII (p-wave TRS SC)	-1	$+1$	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI (d-wave TRS SC)	$+1$	-1	1	$--$	$--$	$\mathbb{Z}$

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The Math-Physics Link: The Periodic Table

The “Engine”: Clifford Algebras

The 10 AZ symmetry classes have a 1-to-1 correspondence with the 10 real and complex Clifford Algebras.

The “Pattern”: Bott Periodicity

This mathematical structure dictates that the classification is periodic:

- Period 2 for Complex classes.
- Period 8 for Real classes.

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Conclusions & Future Work

Conclusions: The Topological Paradigm

Problem: The Old Paradigm (GLW)

- Based on Spontaneous Symmetry Breaking (SSB).
- Relies on a **local order parameter**.
- FAILS for phases with no local order (e.g., QHE).

Solution: The New Paradigm (Topology)

- Replaces local order → global topological invariant ($\nu, C \in \mathbb{Z}$).
- Invariant is computed from the **Berry Phase**.
- This phase encodes the **geometry** of the parameter space (BZ).

Result: A New "Periodic Table"

- Symmetries (T, P, S) + Topology $\implies$ **10-fold Way**.
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Future Work: Deeper Connections

The Foundation: K-Theory

- Bott Periodicity & K-Theory
- Classification of Vector Bundles
- Cohomology & Characteristic Classes

The Dynamics: TQFT

- Topological Quantum Field Theory (TQFT)
- Effective Field Theories (e.g., Chern-Simons)
- Fractional Statistics & Anyonic Braiding
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From **static classification** → **dynamic computation**.

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Thank you!

Questions or Comments?

Survey on the Classification of Topological Phases of Matter

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November 13, 2025

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INTERNATIONAL YEAR OF Quantum Science and Technology