



A Chiral Symmetric Dirac Equation

Watson & Musielak (Int. J. Mod. Phys. A, 2020)

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What is Chirality?

Chiral Symmetric Dirac Equation

$$(i\gamma^\mu \partial_\mu - me^{-i2\alpha\gamma^5}) \psi = 0 \quad (1)$$

Chiral Angle and Mass

What is the Chiral Angle?

The DECS can be re-written (eq. (1)):

$$(i\gamma^\mu \partial_\mu - M - \tilde{M}\gamma^5) \psi = 0 \quad (2)$$

Where:

- $M = m \cos 2\alpha$ is the standard scalar mass.
- $\tilde{M} = -im \sin 2\alpha$ is a pseudoscalar mass. The chiral angle α mixes the scalar and pseudoscalar mass terms.

Generating Mass: A Two-Field Higgs Model

These M and \tilde{M} terms can arise from Yukawa couplings to two Higgs-like fields: a scalar ϕ_1 and a pseudoscalar ϕ_2 .

$$\mathcal{L}_Y \approx -\frac{\lambda_1 v_1}{\sqrt{2}} \bar{\psi} \psi - \frac{\lambda_2 v_2}{\sqrt{2}} \bar{\psi} \gamma^5 \psi \quad (3)$$

This fixes the parameters m and α :

$$m = \sqrt{\frac{\lambda_1^2 v_1^2 - \lambda_2^2 v_2^2}{2}} \quad (4)$$

$$\alpha = \frac{i}{4} \ln \left(\frac{\lambda_1 v_1 + \lambda_2 v_2}{\lambda_1 v_1 - \lambda_2 v_2} \right) \quad (5)$$

Neutrinos and Dark Matter

Neutrino Failure

The paper proposes its main application in explaining "anomalously small" neutrino masses. It suggests the scalar (M) and the pseudoscalar (\tilde{M}) mass contributions, which arise from ϕ_1 and ϕ_2 respectively, nearly cancel each other out.

$$m_\nu = \frac{1}{2\sqrt{2}} (\lambda_1 v_1 - \lambda_2 v_2) \quad (6)$$

However, the paper's derivation contains a fundamental contradiction. It states that the neutrino field has no right-chiral component (Eq. 26: $\frac{1}{2}(1 + \gamma^5)\nu = 0$). As is well-known, a Dirac mass term (both scalar and pseudoscalar) requires both chiral components to be non-zero.

Dark Matter Candidate

- The pseudoscalar Higgs field, ϕ_2 , is proposed as a Dark Matter candidate.
- It couples to Standard Model particles only via this pseudoscalar Yukawa coupling, primarily interacting with neutrinos.
- This makes it massive, long-lived, and "dark," satisfying the requirements for a WIMP-like particle.

References

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