# A Chiral Symmetric Dirac Equation
# Watson & Musielak (Int. J. Mod. Phys. A, 2020)

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## What is Chirality?

A Lorentz-invariant property distinguishing how spinor components transform under boosts $\Lambda$:

$$\psi = \psi_L + \psi_R \tag{1}$$
$$\psi_L \to \Lambda\psi_L \quad , \quad \psi_R \to \Lambda^{-1}\psi_R \tag{2}$$

The components transform under inequivalent Lorentz group representations:

- **Left-handed ($\psi_L$):** $(\frac{1}{2}, 0)$
- **Right-handed ($\psi_R$):** $(0, \frac{1}{2})$

## Chiral Symmetric Dirac Equation

$$\left(i\gamma^\mu\partial_\mu - me^{-i2\alpha\gamma^5}\right)\psi = 0 \tag{3}$$

**The properties that the DECS must satisfy**

In the theory of irreducible representations of the Poincaré group, each irreducible representation is defined by how the objects it describes transform under $\mathcal{P}$.

- **Poincaré Principle:** Physics is unchanged under Lorentz transformations and translations.
- **Gauge Invariance:** Interactions emerge from requiring local symmetry of the Lagrangian.
- **Locality:** Locality: Fields interact only at the same space–time point.

## DECS derivation

**1. Start with the translation operator eigenvalue equation:**

Given that $P^\mu = i\partial^\mu$:

$$i\,\partial_\mu\phi = k_\mu\phi \quad \longmapsto \quad X^\mu\partial_\mu\psi = -Y\psi \tag{4}$$

**2. Impose Lorentz invariance:**

This yields the most general first-order equation with two degrees of freedom ($y_L/x_R$, $y_R/x_L$):

$$\left[i\gamma^\mu\partial_\mu + i\left(\frac{y_L}{x_R}P_L + \frac{y_R}{x_L}P_R\right)\right]\psi = 0 \tag{5}$$

**3. Squaring the Hamiltonian reveals the mass term:**

The squared Hamiltonian derived from eq. (5) reads:

$$H^2\psi = \left(\partial_k\partial^k - \frac{y_Ly_R}{x_Lx_R}\right)\psi \tag{6}$$

**4. Identify the mass $m$ and chiral angle $\alpha$:**

The remaining degrees of freedom are identified as:

$$m \equiv \pm i\sqrt{\frac{y_Ly_R}{x_Lx_R}} \quad \alpha \equiv -\frac{i}{2}\ln\left(\mp\sqrt{\frac{x_Ly_L}{x_Ry_R}}\right) \tag{7}$$

Substituting these into eq. (5) gives the DECS eq. (3).

## Chiral Angle and Mass

**What is the Chiral Angle?**

Rewriting the DECS eq. (3) reveals scalar and pseudoscalar mass terms:

$$\left(i\gamma^\mu\partial_\mu - M - \tilde{M}\gamma^5\right)\psi = 0 \tag{8}$$

Where $\alpha$ mixes the **scalar mass** $M$ and **pseudoscalar mass** $\tilde{M}$:

- $M = m\cos 2\alpha$ (Scalar)
- $\tilde{M} = -im\sin 2\alpha$ (Pseudoscalar)

## Chiral Angle and Mass

**Generating Mass: A Two-Field Higgs Model**

These $M$ and $\tilde{M}$ terms can arise from Yukawa couplings to two Higgs-like fields: a scalar $\phi_1$ and a pseudoscalar $\phi_2$.

$$\mathcal{L}_Y \approx -\frac{\lambda_1 v_1}{\sqrt{2}}\bar{\psi}\psi - \frac{\lambda_2 v_2}{\sqrt{2}}\bar{\psi}\gamma^5\psi \tag{9}$$

This *fixes* the parameters $m$ and $\alpha$:

$$m = \sqrt{\frac{\lambda_1^2 v_1^2 - \lambda_2^2 v_2^2}{2}} \tag{10}$$
$$\alpha = \frac{i}{4}\ln\left(\frac{\lambda_1 v_1 + \lambda_2 v_2}{\lambda_1 v_1 - \lambda_2 v_2}\right) \tag{11}$$

## Neutrinos and Dark Matter

**Neutrino Mass Proposal (and its flaw)**

- **Paper's Goal:** Explain small $\nu$ mass by cancellation between $M$ and $\tilde{M}$:

$$m_\nu = \frac{1}{2\sqrt{2}}(\lambda_1 v_1 - \lambda_2 v_2) \tag{12}$$

- **Fundamental Contradiction:**
- The paper **states** $\nu$ has no right-chiral component ($\frac{1}{2}(1+\gamma^5)\nu = 0$).
- *But...* Dirac mass terms (both $M$ and $\tilde{M}$) **require** both left and right chiral components to be non-zero.

**Dark Matter Candidate**

Despite the neutrino flaw, the model offers a DM candidate:

- The **pseudoscalar Higgs** $\phi_2$ is proposed as Dark Matter.
- It couples to the SM only via the pseudoscalar Yukawa (primarily to neutrinos).
- This makes it massive, long-lived, and "dark"—a viable WIMP-like particle.

## Knowledge Gap

- Hermiticity of the Chiral Symmetric Hamiltonian
- A Chiral Equation for higher spins (Proca, Rarita-Schwinger-like equations)
- Detection and measurement of the free parameters that specify the model
- Validity of the proposed neutrino mass as a Dirac mass

## Conclusions

- The DECS is a valid generalization of the Dirac equation, formally introducing scalar ($M$) and pseudoscalar ($\tilde{M}$) mass terms mixed by a chiral angle $\alpha$.
- A two-field Higgs model ($\phi_1, \phi_2$) can generate these mass terms.
- **Key Flaw:** The paper's application of this model to neutrinos is contradictory, as Dirac mass terms cannot apply to a purely left-chiral field.
- The pseudoscalar Higgs ($\phi_2$) remains a viable WIMP-like Dark Matter candidate.

## References

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