Quantum Entanglement

Spooky and Hidden

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Universidad Distrital Francisco José de Caldas

The Clash of Titans: Einstein's Realism

- **Deterministic View:** The universe is knowable and orderly.
- **Objective Reality:** Physical properties exist independently of measurement.
- **No Fundamental Randomness:** Probability is not intrinsic to nature.

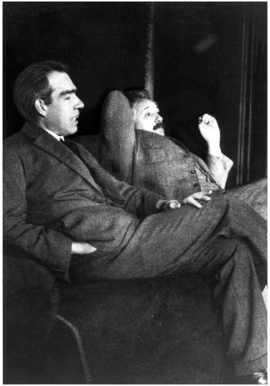

Figure 1: Niels Bohr with Albert Einstein at Paul Ehrenfest's home in Leiden [1].

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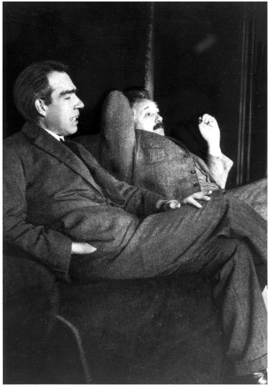

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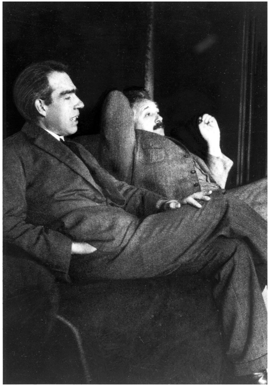

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The EPR Paradox: A Challenge in 1935

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- In 1935, Einstein, Podolsky, and Rosen published a key paper.
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MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

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Any serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

In attempting to judge the success of a physical theory, we may ask ourselves two questions: (1) "Is the theory correct?" and (2) "Is the description given by the theory complete?" It is only in the case in which positive answers may be given to both of these questions, that the concepts of the theory may be said to be satisfactory. The correctness of the theory is judged by the degree of agreement between the conclusions of the theory and human experience. This experience, which alone enables us to make inferences about reality, in physics takes the form of experiment and measurement. It is the second question that we wish to consider here, as applied to quantum mechanics.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory*. We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

The elements of the physical reality cannot be determined by *a priori* philosophical considerations, but must be found by an appeal to results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We shall be satisfied with the following criterion, which we regard as reasonable. *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.* It seems to us that this criterion, while far from exhausting all possible ways of recognizing a physical reality, at least provides us with one

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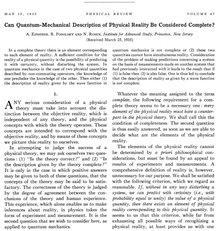

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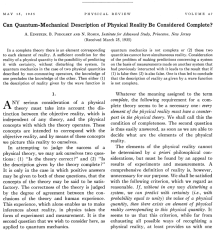

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The EPR Criterion of Reality

The “rule” to define an element of reality

To formalize their attack, EPR introduced an explicit and, seemingly, irrefutable criterion:

> *“If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.”*

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Violation of the Criterion: Position and Momentum

Example with a state $\psi = e^{(2\pi i/h)p_0 x}$

- When applying the momentum operator (p), we get an exact value:

$$p\psi = \left(\frac{h}{2\pi i}\frac{\partial}{\partial x}\right)\psi = p_0\psi$$

  It is concluded that the momentum, p_0, is an **element of reality**.

- However, when applying the position operator (x), an exact value is not obtained:

$$x\psi \neq \text{constant} \cdot \psi$$

  We can only calculate the **probability** of finding the particle in an interval $[a, b]$, which turns out to be $P(a, b) = b - a$.

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The Fundamental Dichotomy

From the previous example, it follows that there are only two possibilities:

1. Either the quantum-mechanical description of reality given by the wave function **is not complete**.
2. Or when the operators for two physical quantities do not commute, the two quantities **cannot have simultaneous reality**.

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The Principle of Locality

A fundamental idea in physics

- An object can only be directly influenced by its **immediate surroundings**.
- Any influence at a distance cannot be instantaneous; it must propagate at a finite speed, not exceeding the speed of light (c).
- For Einstein, locality was a sacred axiom. Its violation would imply the disintegration of the universe's **causal structure** (cause-and-effect).

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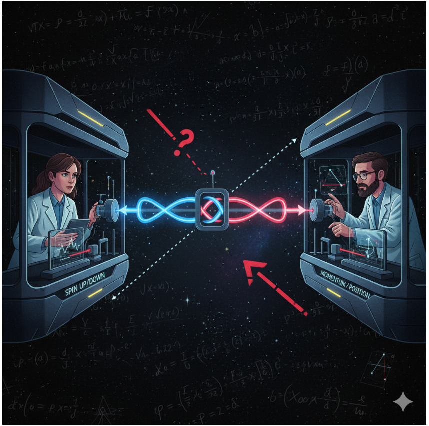
The EPR Thought Experiment

The two-particle system

- A system of two particles is prepared in an entangled state and then separated by a large distance.
- **Option 1:** If Alice measures the **momentum** of her particle, she can predict with certainty the momentum of Bob's particle.
- **Option 2:** If Alice measures the **position**, she can predict with certainty Bob's position.
- **Partial Conclusion:** Both the position and momentum of Bob's particle must be "elements of reality".



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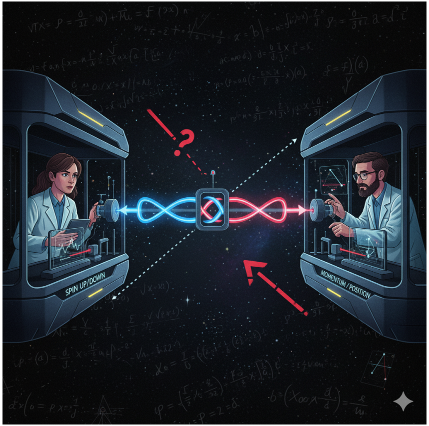

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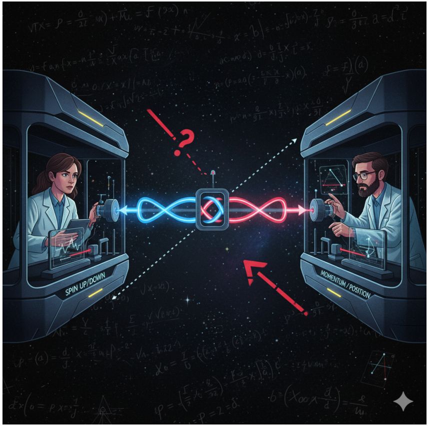

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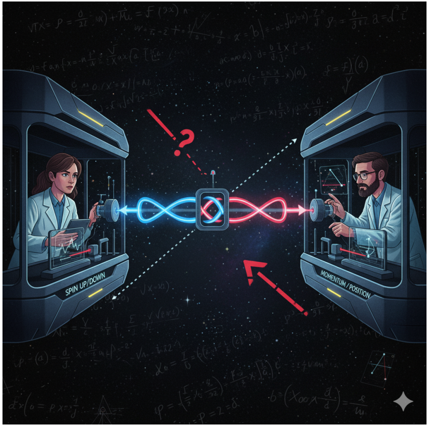

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The Contradiction and EPR's Conclusion

The Conflict with Quantum Mechanics

- The conclusion that Bob's particle has a definite position and momentum **simultaneously**...
- ...is in direct conflict with the **Heisenberg Uncertainty Principle**, which forbids it.

The EPR Conclusion

Faced with this dilemma, if local realism is correct, the only possible conclusion is that:

- The description of reality provided by the wave function must be **incomplete**.
- There must be an underlying theory ("hidden variables") that provides a complete description.

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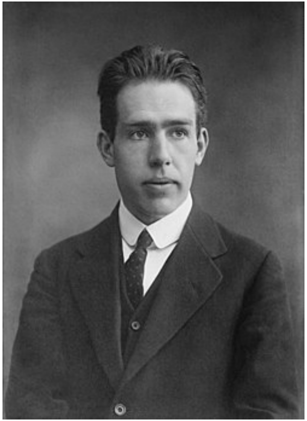
Mutually Exclusive, Yet Necessary Concepts

- Complementarity posits that pairs of classical concepts are necessary for a complete description.
- But they are **mutually exclusive** in any single experiment.
- They are not contradictory properties of the object, but complementary aspects of a **phenomenon** revealed by incompatible experimental arrangements.



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Bohr's Response: The Principle of Complementarity

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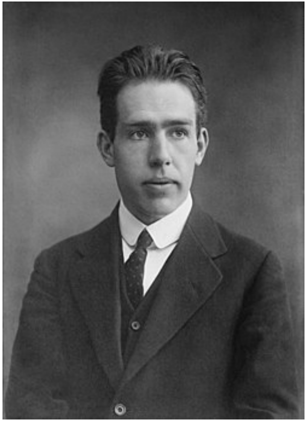

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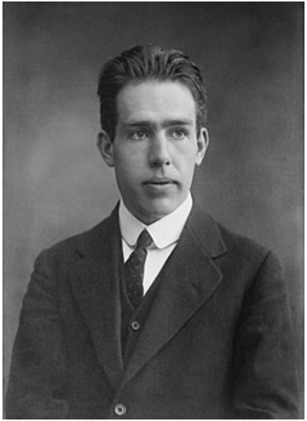

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Is Quantum Mechanics Complete? Bohr’s View

Radical Revision of Physical Reality

- The experimenter’s “freedom of choice” does not reveal pre-existing realities, but rather **creates different experimental conditions** and phenomena.
- Quantum mechanics is **complete** because it correctly and exhaustively describes the results of *every possible, well-defined experiment*.
- Bohr compares this revision of physical reality to the modification of ideas about the absolute character of phenomena introduced by the **Theory of Relativity**.
- This philosophical debate was crucial and led to **Bell’s Theorem** and subsequent experiments, turning EPR into a testable question.

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Schrödinger's Response: The Birth of "Entanglement"

Context and the Essence of the New Physics

- A few months after EPR, Schrödinger published his response: "Discussion of Probability Relations between Separated Systems".
- He demonstrated a deep understanding and took the EPR argument to its most extreme conclusions.
- He understood that the phenomenon was not an anomaly, but **the characteristic trait of quantum mechanics**.
- He coined the term **"Entanglement" (Verschränkung)**.

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“Steering” and the Schmidt Decomposition

The most unsettling aspect of Entanglement

- Schrödinger identified an experimenter’s ability to **“steer” (steuern)** the state of a distant system.
- This is achieved through the local choice of measurement.
- He described it as “rather discomforting that the theory allows a system to be steered... at the whim of the experimenter, even though they have no access to it”.
- Rigorously demonstrated with a mathematical formalism: the **Schmidt decomposition**.

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- This is achieved through the local choice of measurement.
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Bohr vs. Schrödinger: Two Approaches to the Paradox

Fundamental Differences in the Response to EPR

- **Bohr:** Offered a philosophical framework (complementarity) to *dissolve the paradox*, declaring that EPR's questions were ill-posed.
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- **Named and defined the central concept:** He coined the term "entanglement".
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- Together with **Yakir Aharonov** (in 1957), they simplified the example from continuous variables to the **discrete variable of spin**.
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An example with spin-1/2 particles

- Consider a molecule with total spin zero that decays into two atoms (A and B) with spin 1/2.
- It is an **entangled** state where the total spin is zero, regardless of the measurement axis.

The entangled wave function:

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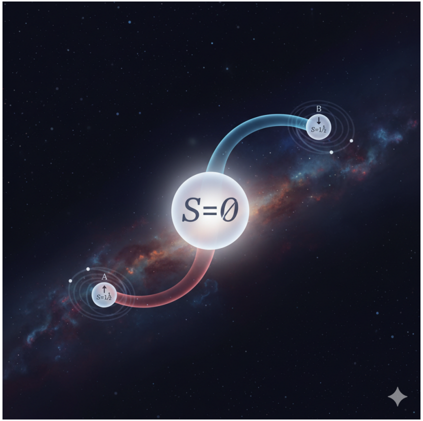

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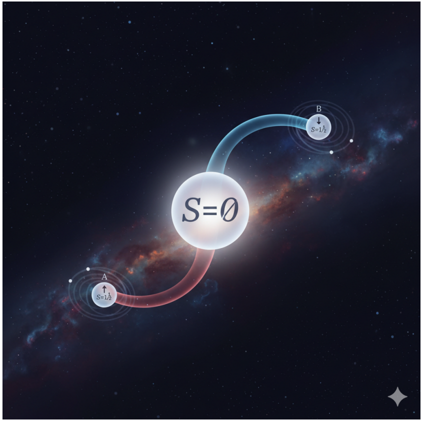

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The Core of the Paradox: Classical vs. Quantum

Classical Interpretation (Local Realism)

- The atoms have **pre-existing** and perfectly anti-correlated spin vectors.
- The measurement on A only **reveals** an already existing property.
- There is no instantaneous influence at a distance; the correlation was established locally at the source.

Quantum Interpretation

- A particle's spin state is **not defined until it is measured**.
- The choice to measure A's spin **realizes** its value, and due to entanglement, simultaneously realizes the opposite value for B.
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The Alternative Hypothesis: Entanglement Breaking

Entanglement that “decays” with distance

- The idea is raised that the quantum many-body formulation **might not be valid** for widely separated particles.
- The proposal suggests that entanglement is a phenomenon that **decreases with distance**.
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Alternative state (statistical mixture):

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- **Avoidance of the EPR Paradox:** It restores local realism; the measurement on A only reveals a pre-existing spin direction.
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Wave function of the system:

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Measurement Process and Setups

- Polarization is measured indirectly via **Compton Scattering**.
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The Crucial Ratio and the Predictions

The ratio R is defined as the quotient of the coincidence rates between the two geometries:

$$R = \frac{\text{Coincidence Rate(parallel)}}{\text{Coincidence Rate(perpendicular)}}$$

- **Quantum Mechanics Prediction (Entangled State):**
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- Local Realism Predictions (Statistical Mixture):
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The Bridge to Reality: The CHSH Inequality

From Theory to Practice

- The CHSH inequality was the key theoretical tool that allowed for the design of the Freedman and Clauser experiment.

The Decisive Inequality (δ)

For the angles of predicted maximum violation ($\phi = 22.5^\circ$ and $3\phi = 67.5^\circ$), the inequality was simplified to a single expression:

$$\delta = \frac{R(22.5^\circ)}{R_0} - \frac{R(67.5^\circ)}{R_0} - \frac{1}{4}$$

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Generating Entangled Pairs

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- The detectors were high-sensitivity **photomultiplier tubes (PMTs)**.
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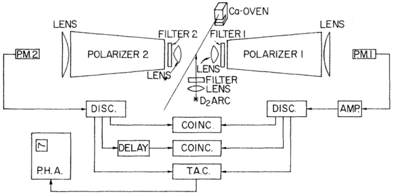

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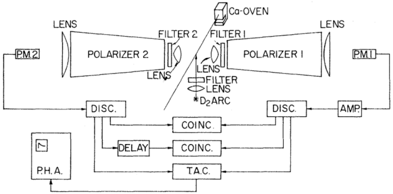

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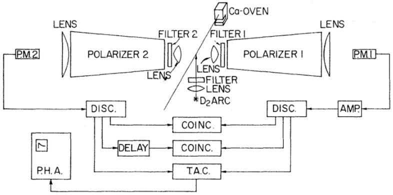

Figure 9: Scheme of the experimental setup [9].

Protocol and Result

- The experiment lasted approximately **200 hours** due to the low coincidence rate.
- A robust real-time normalization system was used to cancel out equipment drift.

The Decisive Experimental Result

The measured value was: $\delta = 0.050 \pm 0.008$

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Mathematical Framework for Entanglement

Motivation: From Feature to Formalism

The Origin of Entanglement

- Emerged as a mathematical feature of early quantum mechanics.
- Now rigorously described via experiment and theory.
- Core Idea: How to represent states of *composite quantum systems*.

A Note on Interpretation

While the math is well-established, its physical interpretation remains a subject of active debate—a common theme in quantum theory.

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Departure from Classical State Spaces

Classical Composite Systems

For a two-particle system, the total state space is the Cartesian product of individual phase spaces:

$$\Gamma_{AB} = \Gamma_A \times \Gamma_B$$

The state (x_A, x_B) of the parts completely defines the whole.

A Naive Quantum Extrapolation

A simple guess might be $\mathcal{H}_{AB} = \mathcal{H}_A \times \mathcal{H}_B$. **This is incorrect.**

- It fails to incorporate the *superposition principle* for combined states.
- An ordered pair $(|\psi_A\rangle, |\phi_B\rangle)$ cannot represent linear combinations.

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The Tensor Product Structure

The Correct Postulate

The state space for a composite quantum system is the *tensor product* of the individual Hilbert spaces:

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B \tag{1}$$

General Pure State

A general pure state $|\Psi\rangle \in \mathcal{H}_{AB}$ is a superposition:

$$|\Psi\rangle = \sum_{i,j} c_{ij}(|a_i\rangle \otimes |b_j\rangle) \tag{2}$$

where $\{|a_i\rangle\}$ and $\{|b_j\rangle\}$ are orthonormal bases for $\mathcal{H}_A$ and $\mathcal{H}_B$.

Dimension: $\dim(\mathcal{H}_{AB}) = \dim(\mathcal{H}_A) \cdot \dim(\mathcal{H}_B)$

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Two Fundamental Classes of States

Separable (Product) States

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$$|\Psi\rangle_{\text{sep}} = |\psi_A\rangle \otimes |\psi_B\rangle \tag{3}$$

Subsystems have definite, independent properties (classical analogue).

Entangled States

A state is *entangled* if it *cannot* be written in the separable form.

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The Symmetrization Postulate

The formalism $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ implicitly assumes particles are distinguishable.

For identical particles, quantum mechanics imposes the symmetrization postulate:

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Consequence for Fermions

A Restricted State Space

- Physically allowed states live in the *antisymmetric subspace* $\mathcal{H} \wedge \mathcal{H}$, not the full tensor product space $\mathcal{H} \otimes \mathcal{H}$.
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Entanglement for Identical Particles

A New Paradigm: Mode Entanglement

Entanglement is reformulated based on correlations between modes (e.g., spatial vs. spin), not between labeled particles.

- The role of a separable state is now played by a single Slater determinant.

For two fermions in single-particle states $|\phi_1\rangle, |\phi_2\rangle$:

$$|\Psi\rangle_{\text{Slater}} = \frac{1}{\sqrt{2}}\left(|\phi_1\rangle_1 \otimes |\phi_2\rangle_2 - |\phi_2\rangle_1 \otimes |\phi_1\rangle_2\right)$$

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Defining Fermionic Entanglement: Slater Rank

Condition for Entanglement
A pure fermionic state is considered **entangled if and only if its Slater rank is greater than one**.

- This means it requires a superposition of multiple Slater determinants to be described.
- Slater Rank 1 $\iff$ "Separable" (unentangled)
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Example: Pauli Exclusion *Forces* Entanglement

Two Electrons in the Same Spatial Orbital

The total wave function $|\Psi\rangle_{\text{total}} = |\psi\rangle_{\text{spatial}} \otimes |\chi\rangle_{\text{spin}}$ must be antisymmetric.

1. **Spatial State:** $|\psi\rangle_{\text{spatial}}$ is *symmetric* (same orbital).
2. **Spin State:** To ensure total antisymmetry, $|\chi\rangle_{\text{spin}}$ *must be antisymmetric*.
3. **Result:** The unique antisymmetric spin state for two spin-1/2 particles is the maximally entangled spin-singlet:

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2)$$

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An operator O_A on subsystem A is represented on $\mathcal{H}_{AB}$ as a local operator:

$$O_A \to O_A \otimes I_B$$

Inner Product

The inner product is defined by linear extension:

$$(\langle\phi_A| \otimes \langle\phi_B|)(|\psi_A\rangle \otimes |\psi_B\rangle) = \langle\phi_A\rangle\, \psi_A \,\langle\phi_B\rangle\, \psi_B$$

Formalism: Operators on Composite Systems

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Why We Need the Density Matrix (ρ)

Handling Incomplete Information

- When we look at only one part of an entangled pair, we have incomplete information about it.
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The Density Matrix: Pure vs. Mixed States

Pure States

For a pure state $|\Psi\rangle$, the density matrix is a projection operator:

$$\rho_{\text{pure}} = |\Psi\rangle\langle\Psi| \tag{4}$$

It is a projector ($\rho^2 = \rho$), so purity can be tested:

$$\text{Tr}(\rho^2) = 1$$

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For a statistical ensemble of pure states $\{p_i, |\psi_i\rangle\}$:

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Quantifying Uncertainty: Von Neumann Entropy

Definition

The degree of uncertainty or “mixedness” is quantified by the Von Neumann entropy:

$$S(\rho) = -\mathrm{Tr}(\rho \ln \rho) = -\sum_i \lambda_i \ln \lambda_i \tag{6}$$

where λ_i are the eigenvalues of ρ.

- Pure state ($\rho = |\psi\rangle \langle\psi|$): $S(\rho) = 0$ (maximal knowledge).
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The state of subsystem A is found via the *reduced density matrix*:

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Quantifying and Detecting Entanglement

The Schmidt Decomposition Theorem

Any pure bipartite state $|\Psi\rangle_{AB}$ can be written in a special orthonormal basis:

$$|\Psi\rangle_{AB} = \sum_{k=1}^{r_S} \sqrt{\lambda_k}\, |k\rangle_A \otimes |k\rangle_B \tag{7}$$

- $\{|k\rangle_A\}$ and $\{|k\rangle_B\}$ are orthonormal bases (the Schmidt bases).
- The number of non-zero terms, r_S, is the *Schmidt rank*.

A Simple Criterion

A state is *separable if and only if its Schmidt rank is 1.*

For pure states, the degree of entanglement is uniquely quantified by the *entropy of entanglement*:

$$E(|\Psi\rangle) = S(\rho_A) = -\sum_{k=1}^{r_S} \lambda_k \ln \lambda_k \tag{8}$$

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Mixed States: A Zoo of Measures

Quantifying mixed-state entanglement is complex; no single measure exists.

Entanglement of Formation (E_F)

Question: What is the minimum average pure-state entanglement needed to create ρ?

$$E_F(\rho) = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i E(|\psi_i\rangle)$$

Relative Entropy of Entanglement (E_R)

Question: How "distant" is ρ from the set of separable states (SEP)?

$$E_R(\rho) = \min_{\sigma \in \text{SEP}} S(\rho || \sigma) = \min_{\sigma \in \text{SEP}} \text{Tr}(\rho \ln \rho - \rho \ln \sigma)$$

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The Positive Partial Transpose (PPT) Test

A simple but powerful necessary condition for separability.

1. Start with a state ρ_{AB}.
2. Compute the *partial transpose* on one subsystem, e.g., B: ρ^{T_B}.
3. Check if ρ^{T_B} is positive semidefinite (all eigenvalues ≥ 0).

The Punchline

- If ρ_{AB} is separable $\implies$ ρ^{T_B} is positive.
- Therefore, if ρ^{T_B} has any **negative eigenvalues**, the state ρ_{AB} is certified as **entangled**.

(This condition is also sufficient only for 2×2 and 2×3 systems.)

Detecting Mixed-State Entanglement: The PPT Criterion

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Definition

A mixture of a Bell state and a maximally mixed state ($p \in [0,1]$):

$$\rho_W = p \left|\Psi^-\right\rangle \left\langle\Psi^-\right| + \frac{1-p}{4}\mathbb{I}_4$$

where $\left|\Psi^-\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$.

In the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$:

$$\rho_W = \frac{1}{4}\begin{pmatrix} 1-p & 0 & 0 & 0 \\ 0 & 1+p & -2p & 0 \\ 0 & -2p & 1+p & 0 \\ 0 & 0 & 0 & 1-p \end{pmatrix}$$

Definition

A mixture of a Bell state and a maximally mixed state ($p \in [0, 1]$):

$$\rho_W = p \left|\Psi^-\right\rangle \left\langle\Psi^-\right| + \frac{1-p}{4}\mathbb{I}_4$$

where $\left|\Psi^-\right\rangle = \frac{1}{\sqrt{2}}(\left|01\right\rangle - \left|10\right\rangle)$.

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The Partial Transpose

We apply the transpose operation only to the second qubit's subspace. This swaps the $|01\rangle\langle 10|$ and $|10\rangle\langle 01|$ matrix elements.

$$\rho_W^{T_B} = \frac{1}{4}\begin{pmatrix} 1-p & 0 & 0 & -2p \\ 0 & 1+p & 0 & 0 \\ 0 & 0 & 1+p & 0 \\ -2p & 0 & 0 & 1-p \end{pmatrix}$$

Worked Example: Werner State (Conclusion)

Eigenvalues of the Partial Transpose

The eigenvalues of $\rho_W^{T_B}$ are:

$$\lambda_{1,2,3} = \frac{1+p}{4} \quad \lambda_4 = \frac{1-3p}{4} \tag{9}$$

Result

The eigenvalue λ_4 becomes negative if $1 - 3p < 0$:

$$p > 1/3$$

Therefore, the Werner state is **entangled for** $p > 1/3$.

The magnitude of the negative eigenvalue is a measure of entanglement (a "negativity"): $\mathcal{N}(\rho_W) = \max(0, -\lambda_4)$

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Detecting Entanglement: Witnesses

Entanglement Witness

An entanglement witness is a Hermitian operator W designed such that:

- $\text{Tr}(W\rho_{\text{sep}}) \geq 0$ for *all* separable states.
- There exists at least one entangled state ρ_{ent} with $\text{Tr}(W\rho_{\text{ent}}) < 0$.

Experimental Implication

If an experiment measures an expectation value $\langle W \rangle = \text{Tr}(W\rho) < 0$, the state ρ is **certified as entangled**.

Geometrically, W is a hyperplane separating ρ_{ent} from the convex set of separable states.

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Dynamics of Entanglement

Why Study Entanglement Dynamics?

We must understand how entanglement evolves under the influence of:

- The system's Hamiltonian (internal dynamics).
- Interaction with an environment (open system dynamics).

This is crucial for quantum information, where we must **generate**, **manipulate**, and **protect** entangled states.

Propagation in Closed Systems

In many-body systems with local interactions, entanglement propagates. The **Lieb-Robinson bounds** establish a finite maximum speed for information, creating a linear "light cone" for correlations.

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Non-Equilibrium Dynamics: Quantum Quenches

Quantum Quench

A sudden change in a system's Hamiltonian $(H_0 \to H_1)$ drives the system out of equilibrium, revealing universal entanglement dynamics.

The Quasiparticle Picture

For a subsystem of length ℓ, the entropy $S_A(t)$ shows a universal pattern:

1. Initial linear growth: $S_A(t) \propto t$.
2. Saturation to a volume-law: $S_A(t) \to \text{const} \cdot \ell$.

Explanation: The quench creates entangled quasiparticle pairs. Entanglement grows as pairs are split by the subsystem boundary.

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Entanglement in Open Systems: Fragility

Decoherence

Interaction with an external environment degrades a system's coherence and entanglement over time.

Entanglement is typically far more fragile than the coherence of individual subsystems.

Entanglement Sudden Death (ESD)

A striking feature of entanglement decay.

- Unlike local coherence (which decays asymptotically), entanglement can **vanish completely at a finite time**.
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Alternative Perspectives

An Alternative View

Alternative mathematical frameworks can offer different physical insights beyond the standard Hilbert space formalism.

Entanglement in Geometric Algebra

- Entanglement is not an intrinsic property of a state, but a relationship between reference frames.
- It is described by a superposition of *relative rotations*.
- The focus shifts from abstract states to the transformation operators themselves.

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The Algebra of Physical Space (APS)

- The language is Geometric Algebra, a powerful tool for describing geometric relationships.
- In this framework, spin states are represented as *rotors* (operators that perform rotations).

Entanglement as a Transformation

Central Thesis: Entanglement is encoded within a transformation operator, the entanglor, not in the state itself.

- Entangled State = (Entanglor) acting on a (Separable State).
- The entanglor contains the geometric info of the superposition of relative rotations between particle reference frames.

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Relativistic Context and Conceptual Implications

Entangling Eigenspinors

A key advantage of APS is its natural compatibility with special relativity. A Lorentz boost (relative velocity) can be combined with an entanglor (relative rotation) into a single composite operator: the entangling eigenspinor Λ_{AB}.

Shift in Perspective

This reinterpretation has profound conceptual implications:

- The "weirdness" of QM is shifted from the state to the *transformation process*.
- Wave function collapse is not a physical change, but the **revelation of the specific geometric transformation** that was present all along.

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Applications of Quantum Entanglement (I)

1. Quantum Computing

- Entangled **qubits** are the foundation of quantum computers.
- They allow for computations impossible for classical computers.
- Advances in cryptography, drug discovery, and materials science.

2. Quantum Cryptography (QKD)

- Uses pairs of entangled photons to generate cryptographic keys.
- Offers **unconditional security** (guaranteed by the laws of physics).
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   - Foundation for a future **quantum internet**.
   - Enables hyper-secure communications and the interconnection of distributed quantum computers.
   - Development of quantum repeaters for long distances.

4. Quantum Metrology and Sensing
   - Creates **sensors much more sensitive** than classical physics allows.
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- **Quantum Simulation:** Modeling complex systems (molecules, materials) impossible for classical computers.
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Thank you!

Questions or Comments?

Quantum Entanglement

Spooky and Hidden

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October 1, 2025

Universidad Distrital Francisco José de Caldas