A Chiral Symmetric Dirac Equation
Watson & Musielak (Int. J. Mod. Phys. A, 2020)

Julian L. Avila-Martinez Laura Y. Herrera-Martinez Sebastian Rodriguez-Garcia

Física, Universidad Distrital Francisco José de Caldas

What is Chirality?

Chiral Symmetric Dirac Equation

$$\left(i\gamma^\mu\partial_\mu - me^{-i2\alpha\gamma^5}\right)\psi = 0 \tag{1}$$

Chiral Angle and Mass

What is the Chiral Angle?

The DECS can be re-written (eq. (1)):

$$\left(i\gamma^\mu\partial_\mu - M - \tilde{M}\gamma^5\right)\psi = 0 \tag{2}$$

Where:

- $M = m\cos 2\alpha$ is the standard scalar mass.
- $\tilde{M} = -im\sin 2\alpha$ is a pseudoscalar mass. The chiral angle α mixes the scalar and pseudoscalar mass terms.

Generating Mass: A Two-Field Higgs Model

These M and $\tilde{M}$ terms can arise from Yukawa couplings to two Higgs-like fields: a scalar ϕ_1 and a pseudoscalar ϕ_2.

$$\mathcal{L}_Y \approx -\frac{\lambda_1 v_1}{\sqrt{2}}\bar{\psi}\psi - \frac{\lambda_2 v_2}{\sqrt{2}}\bar{\psi}\gamma^5\psi \tag{3}$$

This *fixes* the parameters m and α:

$$m = \sqrt{\frac{\lambda_1^2 v_1^2 - \lambda_2^2 v_2^2}{2}} \tag{4}$$

$$\alpha = \frac{i}{4}\ln\left(\frac{\lambda_1 v_1 + \lambda_2 v_2}{\lambda_1 v_1 - \lambda_2 v_2}\right) \tag{5}$$

Neutrinos and Dark Matter

Neutrino Failure

The paper proposes its main application in explaining "anomalously small" neutrino masses. It suggests the scalar (M and the pseudoscalar $\tilde{M}$ mass contributions, which arise form ϕ_1 and ϕ_2 respectively, nearly cancel each other out.

$$m_\nu = \frac{1}{2\sqrt{2}}(\lambda_1 v_1 - \lambda_2 v_2) \tag{6}$$

However, the paper's derivation contains a fundamental contradiction. It states that the neutrino field has no right-chiral component (Eq. 26: $\frac{1}{2}(1+\gamma^5)\nu = 0$). As is well-known, a Dirac mass term (both scalar and pseudoscalar) requires both chiral components to be non-zero.

Dark Matter Candidate

- The pseudoscalar Higgs field, ϕ_2, is proposed as a Dark Matter candidate.
- It couples to Standard Model particles only via this pseudoscalar Yukawa coupling, primarily interacting with neutrinos.
- This makes it massive, long-lived, and "dark," satisfying the requirements for a WIMP-like particle.

References

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