CIV102F Assignment #6: October 13th, 2025 (Version 2!)
Due: October 20th, 2025 at 11:59 pm (all sections)

Hope you had a Happy Thanksgiving! 🦃

General Instructions

[Update] The requirements of this Assignment #6 and onward have changed based upon a request from the Division of Engineering Science (same as all previous assignments, except for Assignment #1).

- There are **five (5) questions** in this assignment. **Only one (1) question will be graded.**
- **For this assignment, the question to be graded is question #2.**
- Students are **not** required to complete and submit the entire question set. You are, however, strongly encouraged to proactively complete the set every 1-1.5 weeks to obtain consistent practice and develop a routine. **This forms great preparation for your final exam and weekly quizzes!**
- Submissions which do not contain a serious attempt to solve the question to be graded will receive a grade of 0.
- Intermediate steps must be provided to explain how you arrived at your final answer. Receiving full marks requires **both** the correct process and answer. Final answers must be reported using **slide-rule precision** and **engineering (or scientific) notation** for very large or very small quantities, unless otherwise specified.
- Submissions must be prepared **neatly** and be formatted using the requirements discussed in the updated course syllabus. Marks will be deducted for poor presentation of work.

Assignment-Specific Instructions

- For your shear force diagrams and bending moment diagrams, the sign convention must be shown, and key values (i.e., local minima/maxima or axis intercepts) must be labelled with their magnitude. Locations may be indicated whenever it is convenient and/or enhances the clarity of your response.

Slide-Rule Precision Reminders

The following information is taken from the CIV102 Fall 2025 Quercus site.

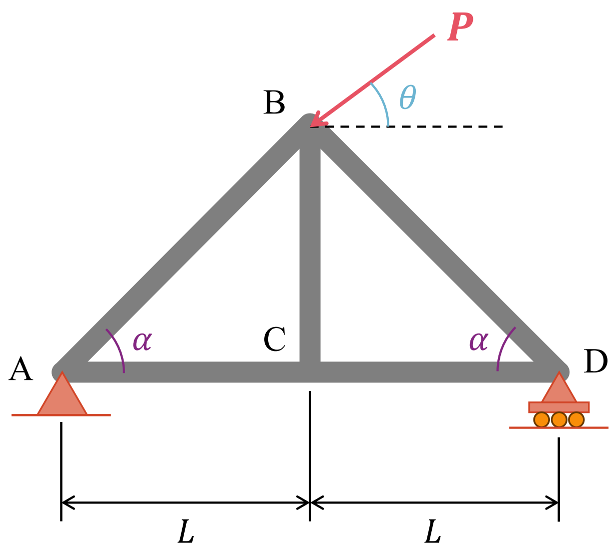
Example Scenario	Final Answer	Justification
Area of an Euclidean circle with a radius of 1 m?	3.14 m^2.	The number π has a decimal representation of 3.14159265... that is infinitely long. **Since it starts with a "3" and not a "1", we express it using three significant digits: 3.14.**
Hypotenuse length of a right-angled Euclidean triangle, with legs of 1 m each?	1.414 m.	The exact length comes from the Pythagorean theorem: $\sqrt{1^2+1^2}=\sqrt{2}$. The square root of 2 is irrational and has a decimal representation of 1.41421356... that is also infinitely long. **Since it starts with a "1", we express it using four significant digits: 1.414.**
Speed of light in a vacuum, expressed in m/s?	3.00 x 10^8 m/s (scientific notation; exponent is 8). 300. x 10^6 m/s (engineering notation; exponent is a multiple of 3).	A commonly quoted value is 299,792,458 m/s. This is a large number! To express it with 3 significant digits, let's use scientific notation. You can also use engineering notation, if you want! Ensure the **coefficient of the 10^n has 3 significant digits. The exponent, n, is an integer.**
Hydrogen atom radius (approximate), expressed in m?	5.30 x 10^{-11} m (scientific notation; exponent is -11). 53.0 x 10^{-12} m (engineering notation; exponent is a multiple of 3).	From various online sources, the approximate size is 53 picometres (53 trillionths of a metre), which is also called the "Bohr Radius". This is a very small number! Ensure the **coefficient in front of the 10^n has 3 significant digits. The exponent, n, is an integer.**
Density of water, expressed in kg/m^3?	1000. kg/m^3 (common). 1.000 x 10^3 kg/m^3 (also acceptable, but takes longer to write).	In some cases, "1000" is interpreted as having 1 significant digit, or 4 significant digits. To be certain, you can place a decimal after the last zero, like "1000." to indicate that you intend for all the digits to be considered significant. **On quizzes and/or assignments, the safest option is to use the "1000." format.**
What if the final answer is an exact, whole number?	10 people, 314 apples, 3 oranges, 420 bricks, 117634 strands of string, etc.	**Whole numbers can be reported as-is, following the context of the question.** There is no need to format answers as "10.00 people", "314. apples", "3.00 oranges", "420. bricks", "1.176 x 10^5 strands", etc.
Mathematical expressions?	Examples include $F/6$, $3x+2$, $\sqrt{2GM/r}$, etc. Using slide rule precision, we would have $0.1667F$, $3.00x+2.00$, and $1.414\sqrt{GM/r}$.	**If not explicitly stated, then slide-rule precision should be used for all coefficients.** Depending upon the context of the question, it will be **explicitly stated** whether the answer should be reported using slide-rule precision or can be kept in an exact mathematical form.
Complex-valued answers?	One example is the principal square root of the imaginary unit: $\sqrt{i}=0.707+0.707i$.	Complex numbers, $z=a+bi$, consist of a real part, a, and an imaginary part, b. The imaginary unit is defined as the principal square root of negative one: $i=\sqrt{-1}$ and $i^2=-1$. **Both the real part and the imaginary part** should be expressed using slide-rule precision.

New addition(s):

Currency?	Examples include \$2.00, 5 cents, \$169.99, \$0.31, etc.	Currency can be formatted using a whole number, or with two decimal places after the number. **Slide-rule precision does not apply here.**

Question 1.

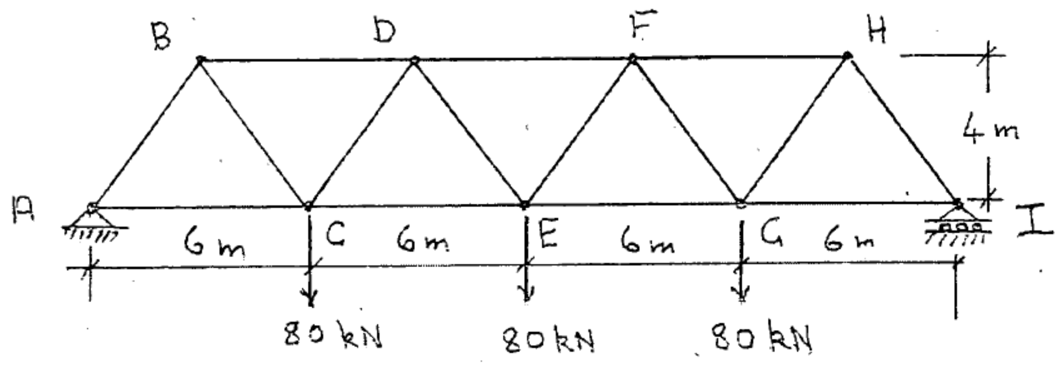
For the small truss shown below, an applied load of magnitude P is applied at an angle, θ, at joint B. This truss has a pin support at A and a roller support at D. All truss members possess the same elastic modulus (E) and the same cross-sectional area (A). Suppose we set $\alpha = 45°$, impose $0° \leq \theta \leq 180°$, and neglect the self-weight of each truss member. What value(s) of θ will cause the *largest* vertical deflection at joint C, and what is/are the deflection value(s)? Exact expressions, where appropriate, are allowed. Otherwise, report your final answer using slide-rule precision. Also do your best to simplify your intermediate steps and final answer(s) as much as possible.



The creation of your own table, similar to this table shown below, may be helpful (but is not mandatory) when you are working through the calculations:

Member	**Real Force (F)**	**Virtual Force (f)**	**Length (ℓ)**	**Elastic Modulus (E)**	**Area (A)**	**Displacement Contribution**
AB						
AC						
BC						
BD						
CD						
Sum						

Question 2.
Consider the truss bridge shown below. The three members comprising the **top chord** of this truss are **HSS 178×178×4.8**. The four members comprising the **bottom chord** are **HSS 127×51×4.8**. All remaining **diagonal members are HSS 127×127×4.8.** All members have a yield stress of 350 MPa and an elastic modulus of 200 GPa. Note the truss geometry and loading are *symmetric*. Lines that appear horizontal/vertical are horizontal/vertical.



Please answer the following questions.

a. Calculate the **vertical displacement of joint E** due to the current loading.

b. Using your result in part (a.), estimate the **natural frequency** (Hz) of this truss bridge.

c. Suppose all of the 80 kN point loads begin to oscillate at a frequency of 2.75 Hz and the truss bridge has a damping ratio of 3%. Calculate the **dynamic amplification factor, DAF**.

d. If these point loads oscillate with an amplitude of 20 kN at a driving frequency of 2.75 Hz, then what would be the **maximum** value of each point load after being amplified by dynamic effects? Under these amplified loads, are any truss members *likely* to buckle or yield?

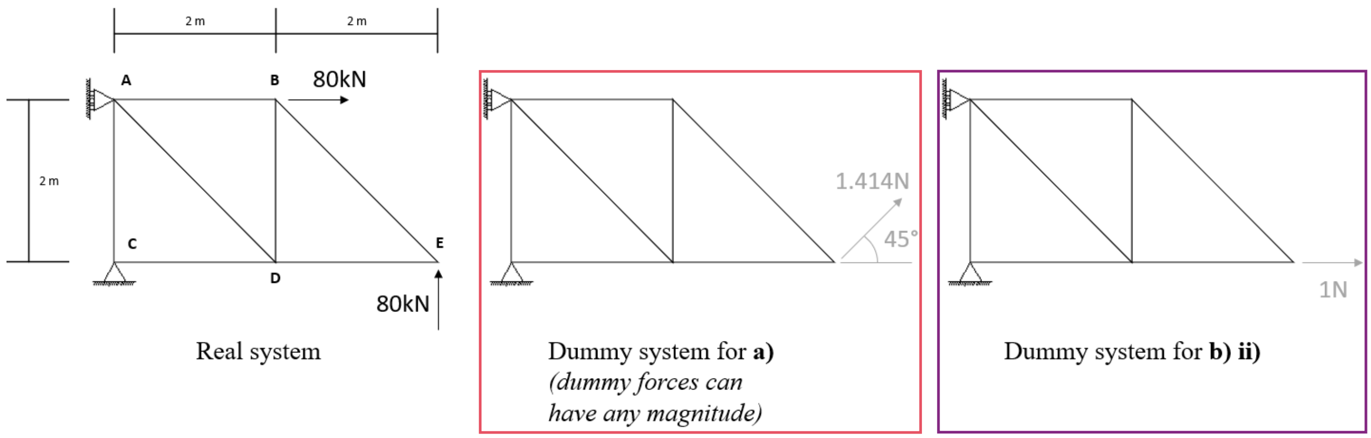
   Hint: You do not necessarily need to re-analyze the entire truss to obtain the answer (think about the characteristics of a linear system).

e. Lastly, calculate the **maximum vertical displacement of the bridge at joint E** due to dynamic effects.

   Hint: You do not necessarily need to reapply the method of virtual work across all members (again, think about the characteristics of a linear system).

Question 3.

The cantilever-like truss structure shown below consists of seven members. All members have a cross-sectional area of 1,000 mm^2 and an elastic modulus of 200 GPa.



Real system

Dummy system for **a)** *(dummy forces can have any magnitude)*

Dummy system for **b) ii)**

Please answer the following questions.

a. Calculate the distance joint E moves in the direction 45º from the horizontal ($\Delta_{E,45}$) using the **method of virtual work**. Set up a dummy system with a dummy force vector that points 45º from the horizontal as indicated by the red box.

b. Calculate the horizontal displacement of joint E ($\Delta_{E,x}$) using two different methods:

   i. Compute the x-component of $\Delta_{E,45}$ via trigonometry.
   ii. Set up a dummy system with a horizontal dummy force vector at joint E and use the method of virtual work (see the purple box).

c. Which method in part (b.) gives the **correct** answer to part (b.)? Explain the differences between the two quantities calculated with the use of a diagram. No calculations are required.

   Hints (some may be helpful; others may not be as helpful):
   - From ESC103, recall ***vector projections***… the vector $\vec{a} = \langle 1,1 \rangle$ has an angle of 45°.
   - $\Delta_{E,y}$ can be found to be 10.12 mm (upward), via the method of virtual work.
   - The two-dimensional (2-D) displacement vector for joint E, is $\vec{\Delta}_E = \langle \Delta_{E,x}, \Delta_{E,y} \rangle$.

Question 4.
For the 30-m long beam shown below,

a. Calculate the reaction forces at each support.

b. Draw a partial FBD of the beam from $x = 3$ m to the right end of the beam (shown below).

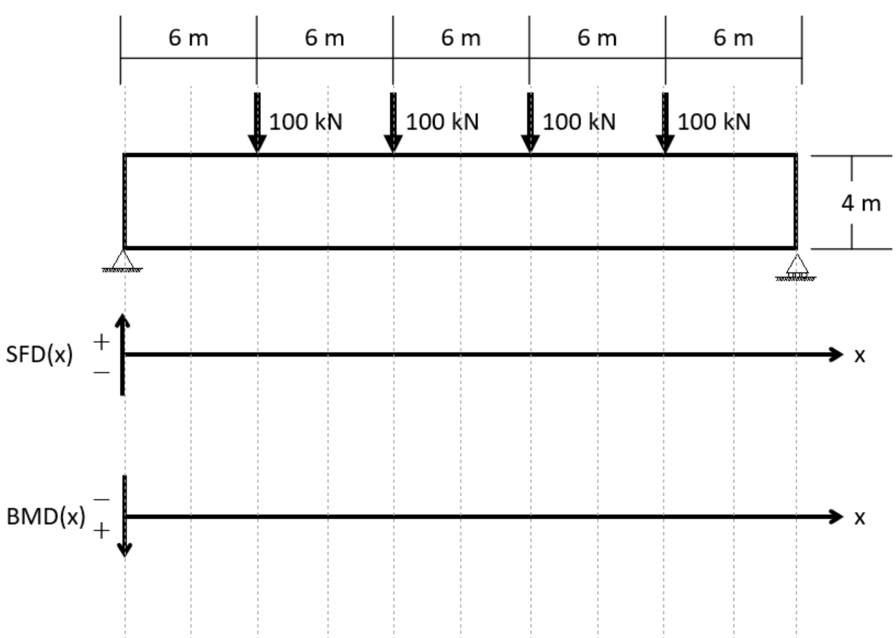
At the cut surface at $x = 3$ m, three unknown **internal forces** are exposed:
- Axial Force N (*perpendicular* to the cut surface.)
- Shear Force V (*parallel* to the cut surface.)
- Bending Moment M (curly arrow.)

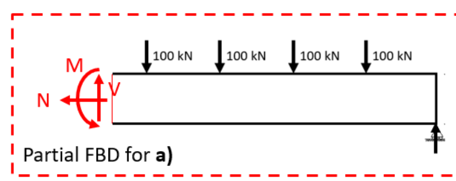
Use equilibrium to find N, V, and M, at $x = 3$ m. Plot the value V(at $x = 3$ m) on the shear force diagram (SFD) at $x = 3$ m and M(at $x = 3$ m) on the bending moment diagram (BMD).

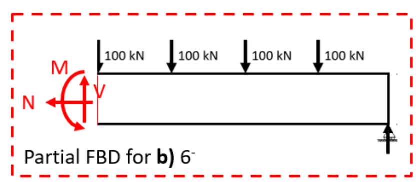
Note: The sign convention for internal bending moments is <u>not based on a single clockwise or counter-clockwise arrow</u> but whether the bending will cause the bottom of a member to be in tension (+, "smiling 😊") or the top of a member to be in tension (−, "frowning ☹").

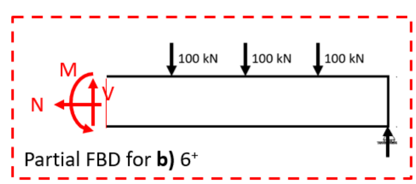
c. Consider another cut at $x = 6$ m and plot V and M. For the SFD, consider two points as $x \to 6^-$ and $x \to 6^+$.

d. Fill out the remaining points on the SFD and BMD using any method you prefer.









Question 5. (Optional.)
The dynamic amplification factor (DAF) you learned about in lecture can be calculated using the equation shown below.

$$\mathrm{DAF} = \frac{1}{\sqrt{(1-\lambda^2)^2 + (2\beta\lambda)^2}}$$

The derivation of this factor is hinted within Chapter 19 of the CIV102 course notes. For this optional exercise, you will go through a guided derivation that shows you how the expression for DAF is obtained. The starting step of this derivation is the governing ODE, presented below.

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\beta m\omega_n\frac{\mathrm{d}x}{\mathrm{d}t} + kx = P_0\sin(\omega t)$$

As a reminder, β is the damping ratio, ω is the driving frequency, and ω_n is the structure natural frequency. λ is calculated as ω/ω_n. Using a solution technique that you will learn in Calculus I, the solution, $x(t)$, can be expressed as the sum of a **transient** component, $x_T(t)$, and a **steady-state** component, $x_S(t)$.

$$x(t) = x_T(t) + x_S(t)$$

The transient component satisfies the ODE when the right-hand side is *identically* equal to zero. This function models *damped* free vibration.

$$m\frac{\mathrm{d}^2x_T}{\mathrm{d}t^2} + 2\beta m\omega_n\frac{\mathrm{d}x_T}{\mathrm{d}t} + kx_T = 0$$

Visually, this transient component is an exponentially decaying sinusoid, which approaches zero as $t \to \infty$. In the *long term*, however, only the **steady-state** component, $x_S(t)$, **persists** and becomes the dominant structural response: $x(t) = x_S(t)$.

The steady-state component satisfies the ODE when the right-hand side is *not identically* equal to zero.

$$m\frac{\mathrm{d}^2x_S}{\mathrm{d}t^2} + 2\beta m\omega_n\frac{\mathrm{d}x_S}{\mathrm{d}t} + kx_S = P_0\sin(\omega t)$$

In practice, we will often *assume* the form of a solution by looking at what the ODE is equal to. Since the right-hand side is a sinusoid with a particular frequency, it seems appropriate to guess that $x_S(t)$ must also be a sinusoid with the *same* frequency. Specifically, we will make the following guess:

$$x_S(t) = A\cos(\omega t) + B\sin(\omega t)$$

Our guess consists of two unknown coefficients, A and B. For this particular ODE, the above guess is the correct solution form (you will see this in your calculus class later in the semester). Now, we can substitute our guess for $x_S(t)$ into the ODE and solve for the unknown coefficients A and B. For the following parts, you can retain exact expressions.

Question 5. (Continued. Let's *amplify* your learning!)

Please answer the following questions.

a. Compute the first and second derivatives of $x_S(t)$.

b. Substitute your derivative expressions from part (a.) into the ODE. Combine and rearrange like terms and see if you can obtain the following resultant equation. Find expressions for [♣] and [♠]:

$$[♣]\cos(\omega t) + [♠]\sin(\omega t) = 0$$

c. Similar to an observation made during the power series solution method (from way back in Assignment #3), it turns out the coefficients of the cosine and sine functions in part (b.) **must both be equal to zero**. More details will be revealed to you during your current/future linear algebra course. Construct and solve the resulting system of linear equations to obtain expressions for A and B. Try to condense as much as possible: you can replace instances of ω/ω_n with λ and instances of $m\omega_n{}^2$ with k.

d. At this stage, you have found expressions for A and B. Show that the maximum value of $x_S(t)$ can be expressed as the following:

$$\max[x_S(t)] = \frac{P_0}{k} \cdot q(\beta, \lambda)$$

To find the maximum value, set the first derivative, $\dot{x}_S(t)$, equal to zero and simplify. This maximum value is the steady-state amplitude. P_0/k is equivalent to the static displacement, Δ_0. Furthermore, $q(\beta, \lambda)$ is an expression involving both β and λ, which turns out to be our expression for the DAF! (See Desmos for a cool plot.)

e. Now that we have confirmed our expression for the DAF, let's ask some remaining questions.
   i. What happens to the value of the DAF when $\beta = 0$ and $\lambda = 1$? What is this phenomenon called?
   ii. For a given, non-zero value of β, what value of λ **maximizes** the DAF? What is this max. DAF?
   iii. For a given, non-zero value of β, what value of λ **minimizes** the DAF? What is this min. DAF?