CIV102F Assignment #7: October 20th, 2025

Due: November 3rd, 2025 at 11:59 pm (all sections)

Have a fun Fall Study Break (Reading Week)!

General Instructions

[Update] The requirements of Assignment #7 and onward have changed based upon a request from the Division of Engineering Science (same as all previous assignments, except for Assignment #1).

- There are **six (6) questions** in this assignment. **Only one (1) question will be graded.**
- **For this assignment, the question to be graded is question #4.**
- Students are **not** required to complete and submit the entire question set. You are, however, strongly encouraged to proactively complete the set every 1-1.5 weeks to obtain consistent practice and develop a routine. **This forms great preparation for your final exam and weekly quizzes!**
- Submissions which do not contain a serious attempt to solve the question to be graded will receive a grade of 0.
- Intermediate steps must be provided to explain how you arrived at your final answer. Receiving full marks requires **both** the correct process and answer. Final answers must be reported using **slide-rule precision** and **engineering (or scientific) notation** for very large or very small quantities, unless otherwise specified.
- Submissions must be prepared **neatly** and be formatted using the requirements discussed in the updated course syllabus. Marks will be deducted for poor presentation of work.

Assignment-Specific Instructions

- For all shear force diagrams and bending moment diagrams, the sign convention must be shown, and key values (i.e., local minima/maxima or axis intercepts) must be labelled with their magnitude. Locations may be indicated whenever it is convenient and/or enhances the clarity of your response.

Slide-Rule Precision Reminders

The following information is taken from the CIV102 Fall 2025 Quercus site.

Example Scenario	Final Answer	Justification
Area of an Euclidean circle with a radius of 1 m?	3.14 m^2.	The number π has a decimal representation of 3.14159265... that is infinitely long. **Since it starts with a "3" and not a "1", we express it using three significant digits: 3.14.**
Hypotenuse length of a right-angled Euclidean triangle, with legs of 1 m each?	1.414 m.	The exact length comes from the Pythagorean theorem: $\sqrt{1^2+1^2}=\sqrt{2}$. The square root of 2 is irrational and has a decimal representation of 1.41421356... that is also infinitely long. **Since it starts with a "1", we express it using four significant digits: 1.414.**
Speed of light in a vacuum, expressed in m/s?	3.00×10^8 m/s (scientific notation; exponent is 8). $300. \times 10^6$ m/s (engineering notation; exponent is a multiple of 3).	A commonly quoted value is 299,792,458 m/s. This is a large number! To express it with 3 significant digits, let's use scientific notation. You can also use engineering notation, if you want! Ensure the **coefficient of the 10^n has 3 significant digits. The exponent, n, is an integer.**
Hydrogen atom radius (approximate), expressed in m?	5.30×10^{-11} m (scientific notation; exponent is -11). 53.0×10^{-12} m (engineering notation; exponent is a multiple of 3).	From various online sources, the approximate size is 53 picometres (53 trillionths of a metre), which is also called the "Bohr Radius". This is a very small number! Ensure the **coefficient in front of the 10^n has 3 significant digits. The exponent, n, is an integer.**
Density of water, expressed in kg/m^3?	1000. kg/m^3 (common). 1.000×10^3 kg/m^3 (also acceptable, but takes longer to write).	In some cases, "1000" is interpreted as having 1 significant digit, or 4 significant digits. To be certain, you can place a decimal after the last zero, like "1000." to indicate that you intend for all the digits to be considered significant. **On quizzes and/or assignments, the safest option is to use the "1000." format.**
What if the final answer is an exact, whole number?	10 people, 314 apples, 3 oranges, 420 bricks, 117634 strands of string, etc.	**Whole numbers can be reported as-is, following the context of the question.** There is no need to format answers as "10.00 people", "314. apples", "3.00 oranges", "420. bricks", "1.176×10^5 strands", etc.
Mathematical expressions?	Examples include $F/6$, $3x+2$, $\sqrt{2GM/r}$, etc. Using slide rule precision, we would have $0.1667F$, $3.00x+2.00$, and $1.414\sqrt{GM/r}$.	**If not explicitly stated, then slide-rule precision should be used for all coefficients.** Depending upon the context of the question, it will be **explicitly stated** whether the answer should be reported using slide-rule precision or can be kept in an exact mathematical form.
Complex-valued answers?	One example is the principal square root of the imaginary unit: $\sqrt{i}=0.707+0.707i$.	Complex numbers, $z=a+bi$, consist of a real part, a, and an imaginary part, b. The imaginary unit is defined as the principal square root of negative one: $i=\sqrt{-1}$ and $i^2=-1$. **Both the real part and the imaginary part** should be expressed using slide-rule precision.

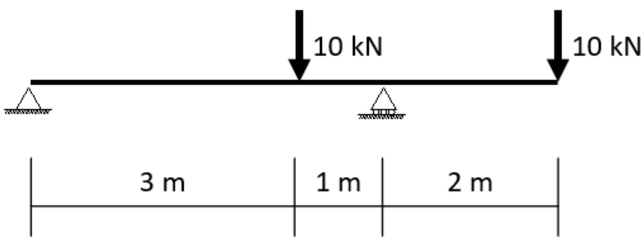
New addition(s):

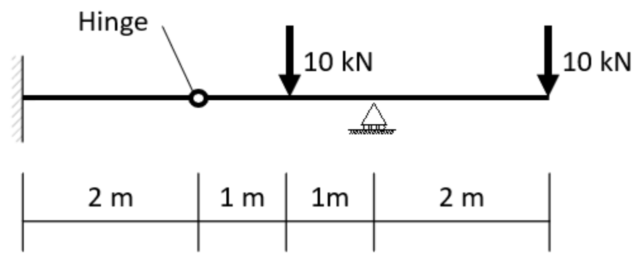
Currency?	Examples include \$2.00, 5 cents, \$169.99, \$0.31, etc.	Currency can be formatted using a whole number, or with two decimal places after the number. **Slide-rule precision does not apply here.**

Question 1.

For the two beam structures shown below, please determine the reaction forces, the shear force diagram (SFD), and the bending moment diagram (BMD). Remember to label important values for each diagram!

Hint: At hinge locations, the internal bending moment is guaranteed to be 0. Therefore, a cut taken at hinge locations will only result in 2 unknown internal forces (N, V, M = 0) whereas a cut taken at other locations will result in 3 unknown internal forces (N, V, M).

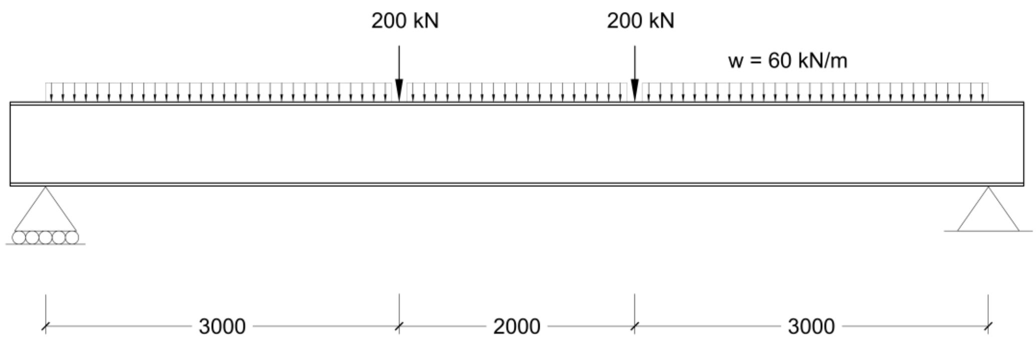


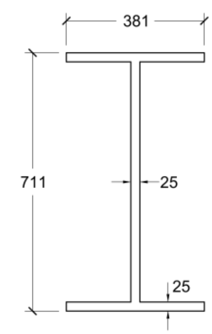


Question 2.

A steel beam shown below is subjected to two concentrated loads of 200 kN and a uniformly distributed load of 60 kN/m. Neglect the self-weight of the beam. All dimensions are in mm.

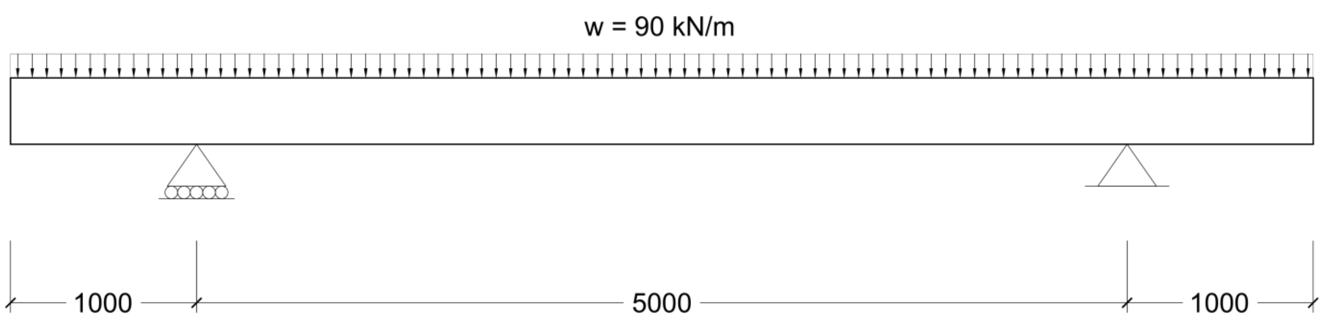
a. Calculate the second moment of area, I, for the given wide-flange cross-section, expressed in mm^4.
b. Construct the SFD and BMD. Label important values.
c. Determine the maximum tensile and compressive stress magnitudes present in the beam.
d. Indicate the location(s) of where these maximum stress magnitudes occur within the beam.





Question 3.

The spruce beam shown below is subjected to a high uniform load of 90 kN/m. It has a solid square cross-section with a side length of 356 mm. Calculate the factor of safety against **flexural failure** for this beam. Use Appendix A to source the material properties of spruce. Neglect the self-weight of this beam. All dimensions are given in mm.



Question 4.

The setup shown below in Figure 1 (Hibbeler, 2007) is used to straighten the bent beam DAE via a stiff hydraulic jack (AB). For this analysis, suppose we take each beam to have the *simplified* cross-section specified in Figure 2. In Figure 2, the top flange of the beam has the same thickness as the bottom flange (24 mm).

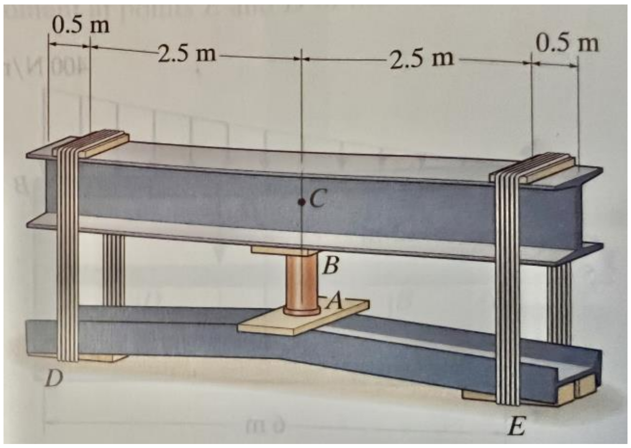


Figure 1. Diagram of the beam-straightening setup.

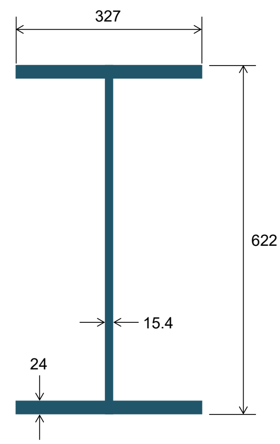


Figure 2. Cross-section dimensions of each steel beam.

Each beam is made of hot-rolled steel with a yield stress of 400 MPa and has a mass of approximately 1,168 kg. This mass is uniformly distributed throughout each beam's length. At the ends of each beam, there is an arrangement of wooden bearing plates and five closed loops of high-alloy steel cables. Each cable has a diameter of d. Suppose we neglect the masses and thicknesses of all the wooden bearing plates, in addition to the self-weight of the steel cables and the jack. Support reactions can be resolved into point forces. For the bottom wooden supports, let us also neglect the angle of inclination. When hydraulic oil is pumped into the jack, the internal piston will push upon each beam and experience an axial compressive force of P_j. From external instrumentation, a measured jack force of P_j = 22 kN is sufficient to straighten beam DAE.

Please answer the following questions.

a. For part (a.), beam DAE is now straight and has elastically recovered. Sketch the bending moment diagram (kN·m) for the top beam. Label important values!

b. Suppose we forget to turn off the jack and beam DAE deforms into a curve again, now concave up. Determine the maximum allowable jack force, $P_{j,\text{max.}}$, and the minimum allowable cable diameter, $d_{\text{min.}}$, that will not cause yielding in neither beam nor any cables. *Hint: draw two free-body diagrams!*

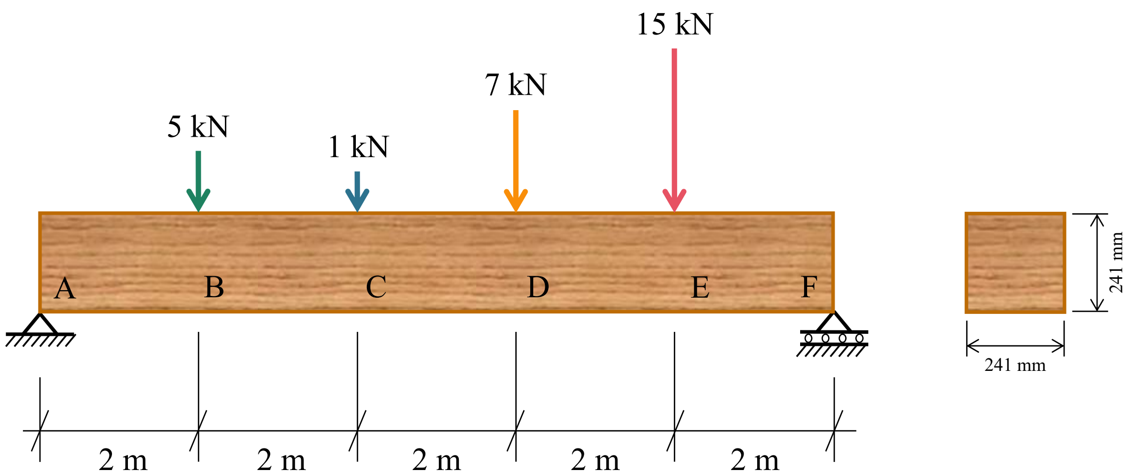
c. Calculate the maximum flexural stress (in MPa) each beam experiences based upon the maximum allowable jack force you obtained in part (b.).

Question 5.

A 10-m long timber beam, with a solid square cross-section, is subjected to four concentrated loads shown below. Ignore the self-weight of the beam.

a. Draw the bending moment diagram. Please show all the important values.

b. Find the maximum tensile flexural stress and maximum compressive flexural stress in the beam. Indicate where these occur.



Question 6. (Optional.)

During lecture, you learned about the curvature of beams. The professor may have commented about how applying a pure bending moment to a uniform and prismatic beam will bend it into an arc of some curve. In this optional question, you will go through a guided derivation and learn of one method to solve a *nonlinear* ordinary differential equation (NODE).

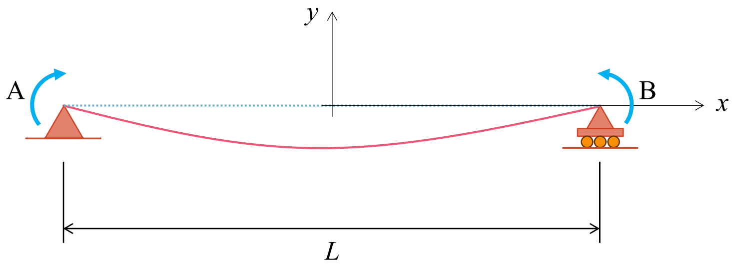


Figure 3. Pure bending scenario of a uniform beam. Positive bending moment effect is shown.

From high school, you may recall the second derivative of a function, $y''(x)$, can be interpreted as a measure of the curvature of the function $y(x)$ around a specified point. Later in your calculus studies, you will observe that the ***actual definition of curvature***, $\phi(x)$, is more complicated[1]:

$$\phi(x) = \frac{y''(x)}{(1 + [y'(x)]^2)^{3/2}}$$

When we apply our moments at each end of our beam, the curvature is equal to M_0/EI and is constant along the entire beam. Let us denote M_0/EI as ϕ_0, which is a constant. We may then substitute ϕ_0 into the above equation and obtain…

$$\phi_0 = \frac{y''(x)}{(1 + [y'(x)]^2)^{3/2}}$$

To solve for the deflected shape of our beam, $y(x)$, we must solve a *nonlinear* ODE (scary! 😱). For convenience, we will make the substitution $g(x) = y'(x)$ so that $g'(x) = y''(x)$. Performing this substitution has *reduced the overall order* of our ODE:

$$\phi_0 = \frac{g'(x)}{(1 + [g(x)]^2)^{3/2}}$$

We can replace instances of $g'(x)$ with the Leibniz notation, $\mathrm{d}g/\mathrm{d}x$. We may also re-notate $g(x)$ as g for brevity. Neglecting mathematical rigour, we may also treat $\mathrm{d}g/\mathrm{d}x$ as a fraction and rearrange the above equation to obtain the following:

$$\phi_0 \,\mathrm{d}x = \frac{1}{(1 + g^2)^{3/2}} \,\mathrm{d}g$$

[Let's flip to the next page! >>>]

[1] In CIV102, we analyze beams based on the assumption of *small slopes*, meaning the magnitude of the first derivative, $|y'(x)|$, is very small compared to 1: $|y'(x)| \ll 1$. Thus, this simplifies our nonlinear ODE into a linear ODE: $\phi(x) \approx y''(x)$, which is what you observed in lecture.

Question 6. (Let's have a *positive moment*, together!)

Now, we may *integrate* both sides of the equation shown below and solve for g.

$$\phi_0 \, \mathrm{d}x = \frac{1}{(1+g^2)^{3/2}} \, \mathrm{d}g$$

a. Let us first integrate the left-hand side with respect to x. Doing so will give us $\phi_0 x + C_1$, where C_1 is any arbitrary, real-valued constant. This constant is also known as the **constant of integration (don't forget to include this when writing your calculus exams!)**. For the right-hand side, however, the integral is substantially more difficult to determine.

   Basically, we would like to find a function, $F(g)$, such that when we differentiate it, we will obtain $F'(g) = (1+g^2)^{-3/2}$. This is difficult to accomplish without knowing the proper technique or having prior experience. Luckily, you have a benevolent teaching assistant who can guide you! We will consider the following function:

$$H(g) = 2g(g^2+1)^{-1/2}$$

   Please compute the first derivative, $H'(g)$. Simplify the result as much as possible. Hence, use this result to determine $F(g)$.

b. After you have performed the integration, rearrange the resulting equation to isolate for $g(x)$ and re-express the equation in terms of $y'(x)$. While solving, you may **merge** arbitrary constants into one arbitrary constant called C_1.

c. To solve for $y(x)$, perform one more integration on $y'(x)$. Use a (clever) substitution to assist with the integration and find an implicit expression for $y(x)$. Your result should now consist of two arbitrary constants, C_1 and C_2.

d. Leverage your algebraic manipulation skills to obtain the following solution form. y is a shorthand for $y(x)$. Recall the unknown constants are arbitrary:

$$(y - C_2)^2 + (x - C_1)^2 = \frac{1}{{\phi_0}^2}$$

   Does this implicit curve resemble a familiar shape? If so, then what is that shape? You can visualize it using a graphing calculator, such as Desmos. What does the quantity $1/\phi_0$ physically represent?

e. From Figure 3, the beam does not displace at either end. We also note the displaced shape is *symmetric*. Use these constraints to solve for C_1 and C_2. Then, proceed to determine an exact expression for the maximum vertical deflection, $\Delta_{\text{max.}}$, this beam experiences. When determining C_2, recall we are applying a *positive* bending moment to the beam. Express all of your final answers in terms of the variables ϕ_0 and L.

f. From part (e.), what happens to the magnitude of $\Delta_{\text{max.}}$ as $\phi_0 \to 0$? Does this appear to make some sense?