CIV102F Assignment #4: September 28th, 2025

Due: October 5th, 2025 at 11:59 pm (all sections)

General Instructions

[Update] The requirements of Assignment #4 and onward have changed based upon a request from the Division of Engineering Science (same as Assignment #3).

- There are **six (6) questions** in this assignment. **Only one (1) question will be graded.**
- **<u>For this assignment, the question to be graded is question #5.</u>**
- Students are **not** required to complete and submit the entire question set. You are, however, strongly encouraged to proactively complete the set every 1-1.5 weeks to obtain consistent practice and develop a routine. **This forms great preparation for your final exam and weekly quizzes!**
- Submissions which do not contain a serious attempt to solve question #5 will receive a grade of 0.
- Intermediate steps must be provided to explain how you arrived at your final answer. Receiving full marks requires **both** the correct process and answer. Final answers must be reported using **slide-rule precision** and **engineering (or scientific) notation** for very large or very small quantities, unless otherwise specified.
- Submissions should be prepared **neatly** and be formatted using the requirements discussed in the updated course syllabus. Marks will be deducted for poor presentation of work, in some cases.

Assignment-Specific Instructions

- Appendices from the course notes can be used if some information is missing.
- Tension members can be indicated as positive (+) while compression members can be indicated with (-).
- If you have any questions, you can come to TA office hours, send us quick emails, or ask a friend!

Slide-Rule Precision Reminders

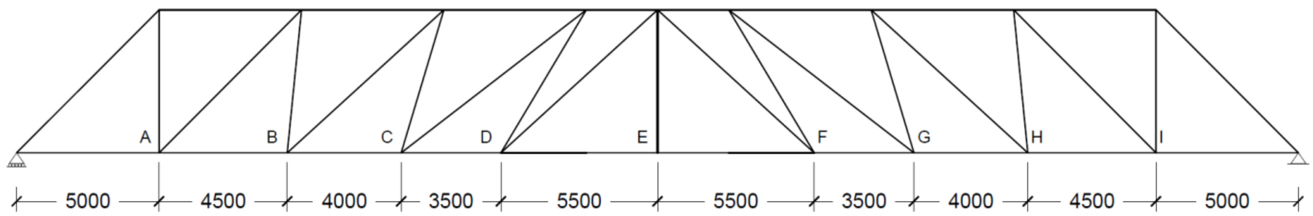
The following information is taken from the CIV102 Fall 2025 Quercus site.

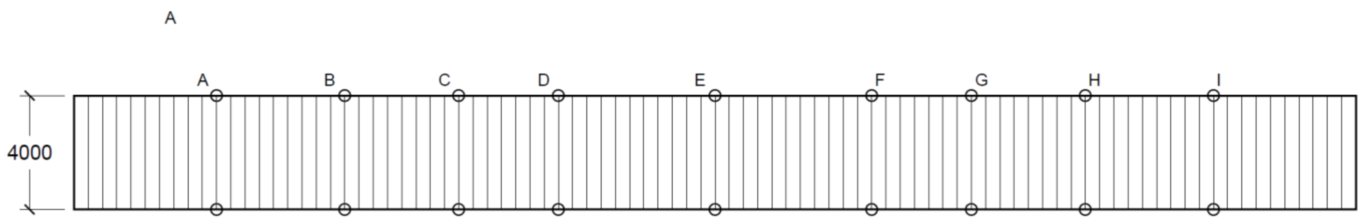
Example Scenario	**Final Answer**	**Justification**
Area of an Euclidean circle with a radius of 1 m?	3.14 m^2.	The number π has a decimal representation of 3.14159265... that is infinitely long. **Since it starts with a "3" and not a "1", we express it using three significant digits: 3.14.**
Hypotenuse length of a right-angled Euclidean triangle, with legs of 1 m each?	1.414 m.	The exact length comes from the Pythagorean theorem: $\sqrt{1^2+1^2}=\sqrt{2}$. The square root of 2 is irrational and has a decimal representation of 1.41421356... that is also infinitely long. **Since it starts with a "1", we express it using four significant digits: 1.414.**
Speed of light in a vacuum, expressed in m/s?	3.00 x 10^8 m/s (scientific notation; exponent is 8). 300. x 10^6 m/s (engineering notation; exponent is a multiple of 3).	A commonly quoted value is 299,792,458 m/s. This is a large number! To express it with 3 significant digits, let's use scientific notation. You can also use engineering notation, if you want! Ensure the **coefficient of the 10^n has 3 significant digits. The exponent, n, is an integer.**
Hydrogen atom radius (approximate), expressed in m?	5.30 x 10^{-11} m (scientific notation; exponent is -11). 53.0 x 10^{-12} m (engineering notation; exponent is a multiple of 3).	From various online sources, the approximate size is 53 picometres (53 trillionths of a metre), which is also called the "Bohr Radius". This is a very small number! Ensure the **coefficient in front of the 10^n has 3 significant digits. The exponent, n, is an integer.**
Density of water, expressed in kg/m^3?	1000. kg/m^3 (common). 1.000 x 10^3 kg/m^3 (also acceptable, but takes longer to write).	In some cases, "1000" is interpreted as having 1 significant digit, or 4 significant digits. To be certain, you can place a decimal after the last zero, like "1000." to indicate that you intend for all the digits to be considered significant. **On quizzes and/or assignments, the safest option is to use the "1000." format.**
What if the final answer is an exact, whole number?	10 people, 314 apples, 3 oranges, 420 bricks, 117634 strands of string, etc.	**Whole numbers can be reported as-is, following the context of the question.** There is no need to format answers as "10.00 people", "314. apples", "3.00 oranges", "420. bricks", "1.176 x 10^5 strands", etc.
Mathematical expressions?	Examples include $F/6$, $3x+2$, $\sqrt{2GM/r}$, etc. Using slide rule precision, we would have $0.1667F$, $3.00x+2.00$, and $1.414\sqrt{GM/r}$.	**If not explicitly stated, then slide-rule precision should be used for all coefficients.** Depending upon the context of the question, it will be **explicitly stated** whether the answer should be reported using slide-rule precision or can be kept in an exact mathematical form.
Complex-valued answers?	One example is the principal square root of the imaginary unit: $\sqrt{i}=0.707+0.707i$.	Complex numbers, $z=a+bi$, consist of a real part, a, and an imaginary part, b. The imaginary unit is defined as the principal square root of negative one: $i=\sqrt{-1}$ and $i^2=-1$. **Both the real part and the imaginary part** should be expressed using slide-rule precision.

New addition(s): Currency?

Currency?	Examples include $2.00, 5 cents, $169.99, $0.31, etc.	Currency can be formatted using a whole number, or with two decimal places after the number. **Slide-rule precision does not apply here.**

1. The figure shown below is a truss bridge. The first image shows an elevation view (from the side), and the second image show a plan view (from the top, also known as "bird's-eye view"). Calculate the joint loads at joints A to I if the bridge was carrying a uniformly distributed pressure load of w = 10 kPa, applied to the full deck area. All dimensions are in mm.

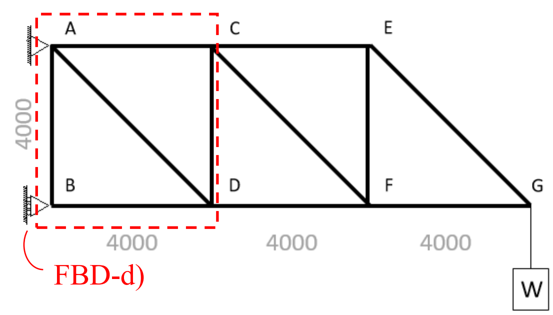




2. The truss structure shown below is holding a block with a mass of m = 28,830 kg. All given dimensions are in millimetres. Perform the following consecutive tasks:

a) Draw a free body diagram (FBD) of the entire structure where the only forces acting on the free body are:

- the force of the weight W,
- the reactions forces of the pin support at joint A,
- the reaction force of the roller support at joint B.



Then, solve for the reaction forces using $\Sigma F_x=0$, $\Sigma F_y=0$, $\Sigma M=0$.

b) Draw a FBD of joint B where the only forces acting are:

- The reaction force of the roller at joint B,
- The unknown member forces in members AB and BD.

Then, solve for the forces in AB and BD using $\Sigma F_x=0$, $\Sigma F_y=0$.

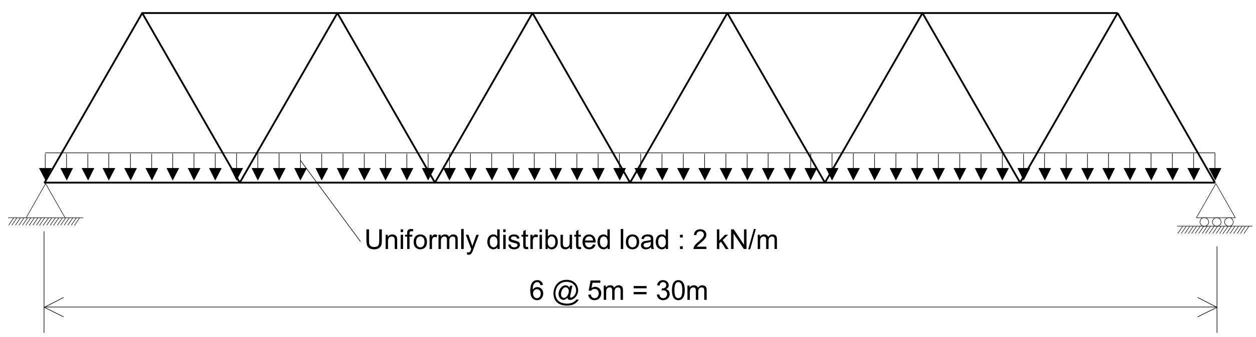
c) Draw a FBD of joint A and solve for the forces in member AD and AC similarly to part b).

d) Draw the FBD (FBD-d shown in diagram) where the only forces acting on the free body are:

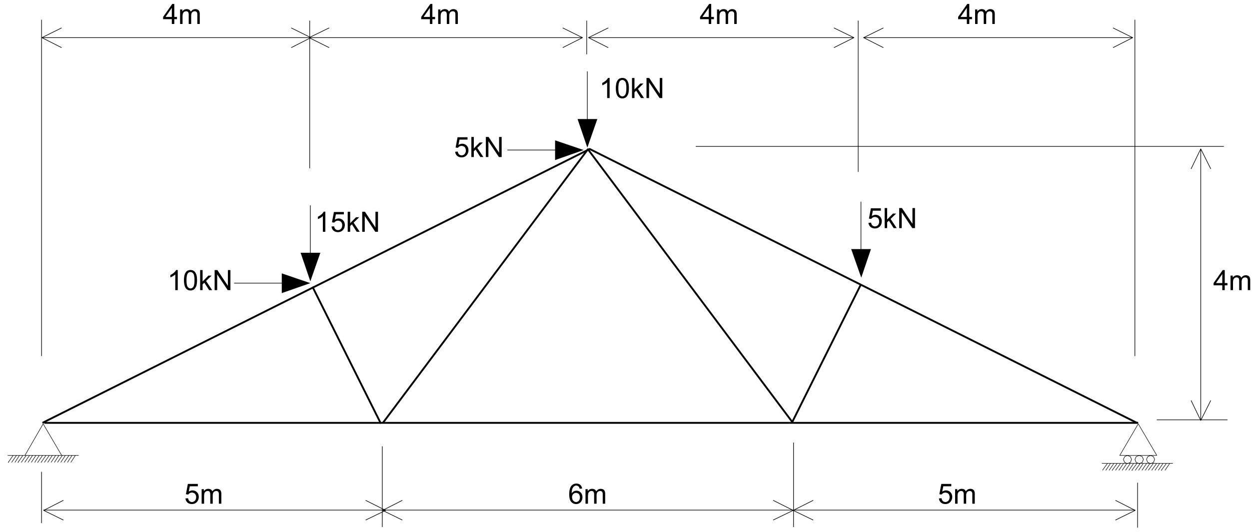
- The reaction forces at joints A and B,
- The unknown member forces in member CE, CF, and DF.

Then, solve for the forces in member CE, CF, and DF using $\Sigma F_x=0$, $\Sigma F_y=0$, $\Sigma M=0$.

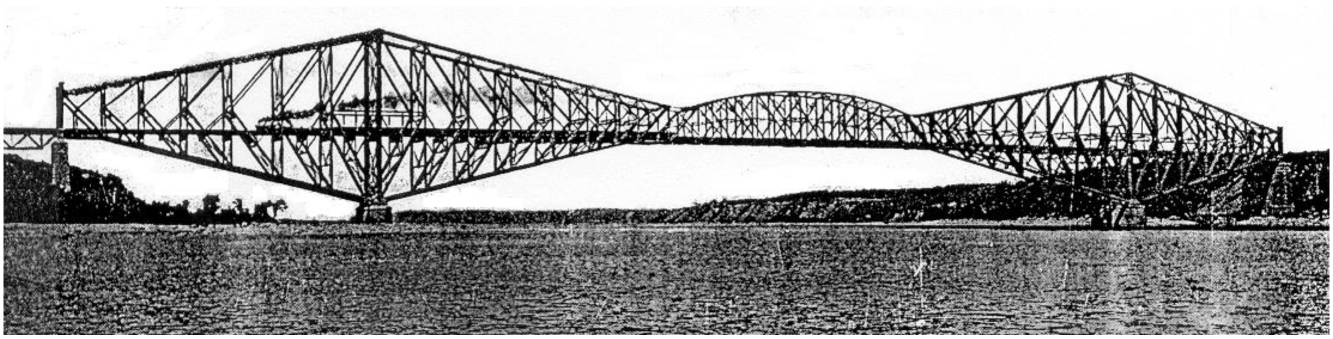
3. Calculate the applied joint loads acting on all bottom joints from the uniformly distributed load and solve for the forces in all members of the Warren truss shown below. All members have the same length. This truss must be solved using the method of joints or the method of sections. Sample calculations are only required for the first two joints or sections. Please present your final member forces on a truss diagram.



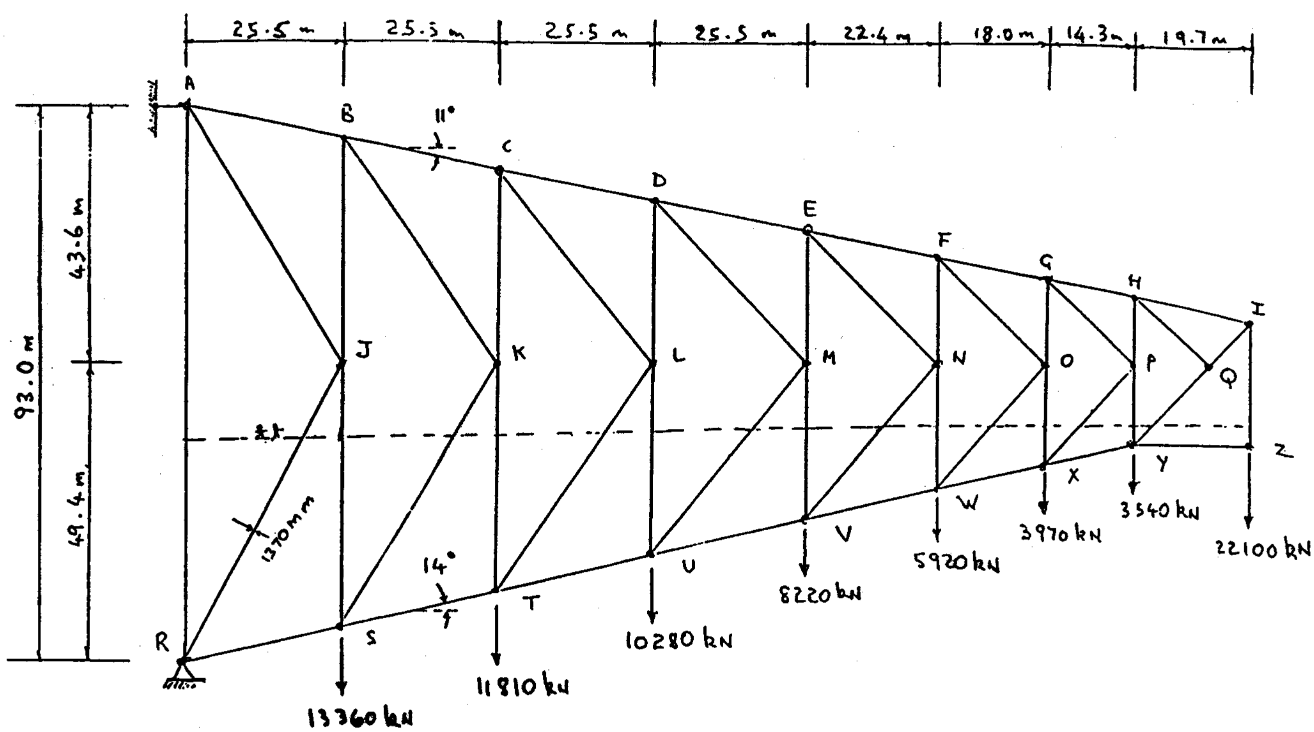
4. Solve for the loads in all the members of this Fink roof truss. Please write the calculated member loads on the diagram.



5. On October 17, 1917, the first train passed over what was described by the Engineering News Record of the day as the "greatest of bridges," the Québec Bridge over the St. Lawrence. After the failure of the original bridge during construction in 1907, a three-man Royal Commission, including Professor John Galbraith of the University of Toronto, investigated the disaster. In 1911, a new design was approved, and construction began in 1913. This new bridge has a total length of 987 m with the main span being 549 m (website). This main span consists of two cantilever "arms" extending out 176.4 m from the two piers joined by a suspended span 196 m long. See the photograph below:



As well as being the longest cantilevered span bridge in the world, this bridge was notable for being the first to use the K system of web members. This system, now widely used for steel structures, was invented by Phelps Johnson and Herrick Duggan, the designers of the new bridge. The figure below shows one of the two trusses of one of the cantilever arms with its K system of web members. The loads due to the self-weight of the structure and a long, heavily loaded train passing over the bridge are also shown. The 22,100 kN load at joint Z comes from the suspended span. As can be seen, there are 26 joints in the truss, 49 members, and 3 unknown reaction forces (2 at R, and the horizontal force at A). Hence, the forces in the members can be found by solving 52 equations in 52 unknowns. As a result of this procedure, it has been found that member AR experiences a *compressive* force of 36,100 kN. Lines that look horizontal/vertical are horizontal/vertical.



(a) By examining the equilibrium of the total truss, find the 3 reaction forces.
(b) By examining the equilibrium of joint A, determine the forces in members AB and AJ.
(c) By examining the equilibrium of joint R, determine the forces in members RS and RJ.
(d) Lastly, please find the forces in members IZ, ZY, IH, IQ, QY, QH, HP and HG.

6. (Optional: try on your own time!) In class, you were presented with the following *ordinary differential equation* (ODE) which models the free vibration response of a mass-spring system.

$$m\ddot{y} = -ky$$

Here, m is the mass (kg), y is the displacement of the mass (m), $\ddot{y}$ is the acceleration of the mass (m/s^2), and k is the spring constant (stiffness: N/m). Solving an ODE, in general, is difficult as there is no single method that works for any particular ODE. The solution to the above ODE was shown to you in lecture, and in this problem, you will learn of a mathematical/graphical approach that can be used to obtain the solution to the above. To begin, let us consider a simpler ODE and see how to apply this method for the first time. ***Exact mathematical expressions are acceptable here!***

a. Right now, let us forget about the mass-spring system and instead look at the ODE given by $\dot{y} = y$, which only involves the first derivative. Suppose we would like to find solution(s) to $\dot{y} = y$. Some of you may already know of a solution to this ODE by inspection, perhaps inspired by your studies in high school mathematics (can you think of one?).

   Otherwise, for the purposes of this question, suppose we do not know of such a solution off the top of our head. Instead, I will introduce to you an approach that works well for solving these kinds of ODEs. One underlying assumption of this approach is that the solution, $y(t)$, may be expressed as a ***polynomial with infinitely many terms,*** written as follows:

$$y(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 + c_6 t^6 + c_7 t^7 + c_8 t^8 + c_9 t^9 + c_{10} t^{10} + \cdots$$

   Initially, this looks insane, and that was how I felt when I first encountered this method. But, if we suppress our unease and go with the flow, perhaps everything can work out in the end (*spoiler:* it does, at least in this case). Since ODEs involve functions and their derivatives, it kind of makes sense to select such a polynomial because the derivatives of polynomials are simple to compute and having infinitely many terms feels like we are covering all possible function behaviour.

   Pushing mathematical rigour aside, taking the first derivative of our polynomial involves differentiating *each* term according to the well-established *power rule*. Fill in the blanks below.

$$\dot{y}(t) = \_\_\_\_ + \_\_\_\_ t + \_\_\_\_ t^2 + \_\_\_\_ t^3 + \_\_\_\_ t^4 + \_\_\_\_ t^5 + \_\_\_\_ t^6 + \_\_\_\_ t^7 + \_\_\_\_ t^8 + \_\_\_\_ t^9 + \ldots$$

b. Once you have your derivative expression, equate it to $y(t)$ and rearrange it such that you have $\dot{y}(t) - y(t) = 0$. The left-hand side of the equation is a new polynomial while the right-hand side is equal to zero. Combine like terms together and fill in the blanks below:

$$\_\_\_\_ + \_\_\_\_ t + \_\_\_\_ t^2 + \_\_\_\_ t^3 + \_\_\_\_ t^4 + \_\_\_\_ t^5 + \_\_\_\_ t^6 + \_\_\_\_ t^7 + \_\_\_\_ t^8 + \_\_\_\_ t^9 + \ldots = 0$$

[Let’s flip to the next page to advance to the next step.]

Question 6, continued:

c. This next step involves a ***very important observation***. We are looking for a solution, $y(t)$, that satisfies our ODE for every possible real value of $t \geq 0$. Since t can be any non-negative real number, the only way for the polynomial you found in part b) to always be equal to zero is if ***all the coefficients are equal to zero***. (This is related to linear algebra.)

Upon imposing this condition, you will now have equations for each coefficient. While there are infinitely many such equations, they will all exhibit a pattern. Below is a table that has been partially filled. Let us fill in the remaining entries to see the pattern emerge*. Entries with a "…" do not need to be filled.

Term	Degree n	Coefficient Equation	Coefficient c_{n+1}	Coefficient in Terms of c_0
t^0 (constant)	0	$c_1 - c_0 = 0$	$c_1 = c_0$	$c_1 = c_0$
t^1 (linear term)	1	$2c_2 - c_1 = 0$	$c_2 = c_1/2$	$c_2 = c_0/2$
t^2 (quadratic term)	2	$3c_3 - c_2 = 0$	$c_3 = c_2/3$	$c_3 = c_0/6$
t^3 (cubic term)	3	$4c_4 - c_3 = 0$	$c_4 = c_3/4$	$c_4 = c_0/24$
t^4 (quartic term)	4	$5c_5 - c_4 = 0$	$c_5 = c_4/5$	$c_5 = c_0/120$
t^5 (quintic term)				
t^6 (sixth-degree)				
t^7 (seventh-degree)				
t^8 (eighth-degree)				
t^9 (ninth-degree)				
…	…	…	…	…
t^n (arbitrary degree)*				
…	…	…	…	…
… and so on!	…	…	…	…

*A hint: The product $n(n-1)(n-2)(n-3) \dots (3)(2)(1)$ can be rewritten as $n!$ ("n" factorial), where $n > 0$ and is an *integer*. For example, 1! = 1, 2! = 2, 3! = 6, 4! = 24, and so on. In CIV102, we define $0! = 1$ and $0^0 = 1$.

d. Now, we have expressions for each coefficient, all in terms of c_0. We can then substitute each coefficient into our polynomial $y(t)$ and obtain the following *infinitely* long solution. During this process, we may factor out c_0 as it is common to every coefficient. Determine the values that go into the square brackets:

$$y(t) = c_0\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6} + [\quad]t^4 + [\quad]t^5 + [\quad]t^6 + [\quad]t^7 + [\quad]t^8 + [\quad]t^9 + \cdots + [\quad]t^n + \cdots\right)$$

e. At this point, we can compactly rewrite the equation in part d) via sigma (summation) notation and the use of factorials. What expression goes inside the square bracket down below?

$$y(t) = \sum_{n=0}^{\infty} [\quad]t^n$$

f. Lastly, open up a graphing calculator, such as Desmos, and play around with this infinitely long polynomial. Among these functions: $c_0 \ln(t)$, $c_0 \arctan(t)$, $c_0 \exp(t)$, and $c_0 \cosh(t)$, which one does the polynomial found in part e) approach as the degree, n, approaches infinity? Verify this function satisfies the original ODE.

Question 6, continued:

You may complete the following part on your own time.

g. Using the same method described through part a) to part f), you are now able to tackle the free vibration ODE. For starters, let us take m = 1 kg and k = 1 N/m so we have $\ddot{y} = -y$. Complete the following table.

Term	Degree	Coefficient Equation	Coefficient c_{n+2}	Coefficient in Terms of c_0 or c_1
t^0 (constant)	0	$2c_2 + c_0 = 0$	$c_2 = -c_0/2$	$c_2 = -c_0/2!$
t^1 (linear term)	1	$6c_3 + c_1 = 0$	$c_3 = -c_1/6$	$c_3 = -c_1/3!$
t^2 (quadratic term)	2	$12c_4 + c_2 = 0$	$c_4 = -c_2/12$	$c_4 = c_0/4!$
t^3 (cubic term)	3	$20c_5 + c_3 = 0$	$c_5 = -c_3/20$	$c_5 = c_1/5!$
t^4 (quartic term)	4	$30c_6 + c_4 = 0$	$c_6 = -c_4/30$	$c_6 = -c_0/6!$
t^5 (quintic term)	5	$42c_7 + c_5 = 0$	$c_7 = -c_5/42$	$c_7 = -c_1/7!$
t^6 (sixth-degree)	6			
t^7 (seventh-degree)	7			
t^8 (eighth-degree)	8			
t^9 (ninth-degree)	9			
…	…	…	…	…
…	…	…	…	…
… and so on!	…	…	…	…

h. From your results, show that the solution to $\ddot{y} = -y$ can be expressed as the following summation. Determine expressions for [♣] and [♠].

$$y(t) = c_0 \underbrace{\sum_{n=0}^{\infty} [♣] t^{2n}}_{f(t)} + c_1 \underbrace{\sum_{n=0}^{\infty} [♠] t^{2n+1}}_{g(t)}$$

i. Navigate to Desmos again and plot each polynomial separately. Suppose we set $c_0 = 1$ and $c_1 = 1$. As we add more and more higher-degree terms, what familiar function does each polynomial approach?

j. For those of you who are adventurous, use what you have learned[1] in parts a) through i) and see if you can demonstrate that the general solution to $m\ddot{y} = -ky$, for $m > 0$ and $k > 0$, can be expressed as

$$y(t) = c_0 f(\omega t) + c_1 g(\omega t)$$

where $\omega = \sqrt{k/m}$ and functions f, g were determined earlier in part i). To start, rearrange the ODE to obtain $\ddot{y} + \omega^2 y = 0$. As an additional hint, you may replace c_0 and c_1 with $c_0\omega$ and $c_1\omega$ as c_0 and c_1 are arbitrary constants.

Now you have an idea of how the solution presented in lecture can be obtained (so long as you have access to a graphing software)!

[1] This method of solving ODEs is commonly called the *power series solution approach*. In general, however, it does not always work.