ADS Assignment 2018-19

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January 18, 2019

4 Asymptotic Notation

4.a $2x^4$ is $\mathcal{O}(x^3 + 3x + 2)$?

$2x^4$ is not $\mathcal{O}(x^3 + 3x + 2)$, because:

$$2x^4 \leq C \cdot (x^3 + 3x + 2)$$

$$\frac{2x^4}{x^3 + 3x + 2} \leq C$$

Therefore, as $x \to \infty$, $\frac{2x^4}{x^3+3x+2}$ also $\to \infty$, and therefore it can never be less than some constant C

4.b $4x^3 + 2x^2 \cdot logx + 1$ is $\mathcal{O}(x^3)$?

$4x^3 + 2x^2 \cdot logx + 1$ is $\mathcal{O}(x^3)$

$$4x^3 + 2x^2 \cdot logx + 1 \leq C \cdot x^3$$

$$8x^3 = 4x^3 + 2x^3 + x^3 + x^3$$

$$8x^3 \leq C \cdot x^3$$

Therefore $C = 8$ and $k = 1$ as witensses

4.c $3x^2 + 7x + 1$ is $\omega(x \cdot logx)$?

$3x^2 + 7x + 1$ is $\omega(x \cdot logx)$
We have:

$$3x^2 + 7x + 1 > C \cdot x \cdot logx$$

$$\lim_{x \to \infty} \frac{3x^2 + 7x + 1}{x \cdot logx} > C$$

This tends to infinity as it reduces to

$$\frac{6x + 7}{logx + \frac{1}{ln2}}$$

by L'Hopital's rule, and x has a quicker growth rate than $logx$, therefore the fraction $\to \infty$ as $x \to \infty$, and therefore will always be larger than some constant C. Some witnesses for this could be C as 3 ($3x^2$), where k would be 1 as $(3 + 7 + 1 > 3 \cdot 1log1 = 11 > 0)$

4.d $x^2 + 4x$ is $\Omega(x \cdot logx)$?

$x^2 + 4x$ is $\Omega(x \cdot logx)$ We have:

$$x^2 + 4x \geq C \cdot x \cdot logx$$
$$\lim_{x\to\infty} \frac{x^2 + 4x}{x \cdot logx} \geq C$$
$$\frac{2x + 4}{logx + \frac{1}{ln2}} \geq C$$

Simplified through applying L'Hopital's rule we get, the above and as x has a quicker growth rate than $logx$, we have an increasing function which $\to \infty$ and will therefore always be larger than some constant C. Some witnesses for this could be C = 1 and K = 1, as we have x^2 will definitely have a quicker growth rate than $x \cdot logx$ (which will also have a value of 0 when $x = 1$)

4.e $f(x) + g(x)$ is $\Theta(f(x) \cdot g(x))$?

I will assign $f(x) = x$ and $g(x) = x^2$
$\therefore f(x) + g(x) = x^2 + x$
However $f(x) \cdot g(x) = x^3$ and $x^2 + x \neq \Theta(x^3)$
so therefore $f(x) + g(x)$ is $\Theta(f(x) \cdot g(x))$ is false.

5 Master Theorem

5.a $T(n) = 9T(n/3) + n^2$?

$a = 9,\ b = 3,\ f(n) = n^2$
If $f(n) = \mathcal{O}(n^{\log_b a - \epsilon})$ for some $\epsilon > 0$, we get: is $f(n) = \mathcal{O}(n^{2-\epsilon})$, which, would be false.
If $f(n) = \Theta(n^{\log_b a})$ which is the same as $f(n) = \Theta(n^2)$, which is true so therefore $T(n) = \Theta(n^2 \cdot logn)$

5.b $T(n) = 4T(n/2) + 100n$?

$a = 4,\ b = 2,\ f(n) = 100n$
If $f(n) = \mathcal{O}(n^{\log_b a - \epsilon})$ for some $\epsilon > 0$, we get: is $f(n) = \mathcal{O}(n^{2-\epsilon})$,
which is true; so therefore $T(n) = \Theta(n^2)$

5.c $T(n) = 2^n T(n/2) + n^3$?

$a = 2^n,\ b = 2,\ f(n) = n^3$
Master Theorem cannot be applied as a is exponential

5.d $T(n) = 3T(n/3) + c \cdot n$?

$a = 3,\ b = 3,\ f(n) = C \cdot n$
If $f(n) = \mathcal{O}(n^{\log_b a - \epsilon})$ for some $\epsilon > 0$, we get: is $f(n) = \mathcal{O}(n^{1-\epsilon})$, which is false
If $f(n) = \Theta(n^{\log_b a})$ which is the same as $f(n)$ is $\Theta(n^1)$, which is true, so $T(n)$ is $\Theta(n \cdot logn)$

5.e $T(n) = 0.99T(n/7) + \frac{1}{n^2}$?

$a = 0.99,\ b = 7,\ f(n) = \frac{1}{n^2}$
Master Theorem cannot be applied as $a < 1$

6 Sorting

6.b Hybrid Sort Worst Case

As selection sort has no worst case due to finding the smallest value in the list of 4, and placing it at the beginning of the list and will still have the same number of comparisons if the list is already sorted, I only need to consider the worst case of merge sort. If we consider the final answer of [7,6,5,4,3,2,1,0], we can split this up into two lists of worst case comparisons: A[7,5,3,1] and B[6,4,2,0]. Which would have max comparisons as 7 compares with 6, 6 then compares with 5 (which flips between two list of which is larger) and so on, when merging the two lists. As these are length 4 or less, selection sort is applied which, as I previously explained, has no worst case, so the two lists can stay as they are. Finally for a length 8 worst case, I concatenate the two lists to get: [7,5,3,1,6,4,2,0].