New Mexico Tech (Spring 2020)

CSE 342:
Formal Languages Automata HW 1

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1. (1 point) Define a set
Answer:
A Collection of unique un-ordered objects.

2. (1 point) Define a function
Answer:
A mapping from elements of one set to elements of another set, such that every element in the first set is mapped to some element of the second set, and such that no element of the first set is mapped to multiple elements of the second set.

3. (1 point) Define a bijection
Answer:
A function where every element of the second set is mapped, and every element in the second set has exactly one element in the first set mapping to it

4. (1 point) Define 'alphabet'
Answer:
A non-empty, finite set

5. (5 points extra credit) Which of the following are alphabets

(a) { x||x is an even prime number > 2 }
Answer: This is NOT an alphabet because the set is infinite

(b) {"11", "00"}
Answer: This is an alphabet

(c) {"000", "001", "101"}
Answer: This is an alphabet

(d) {x||x is the set of strings over {0,1}}
Answer:
This is NOT an alphabet because the set is infinite

(e) {""}
Answer: This is an alphabet because ϵ is a finite symbol

6. (1 point) Define a string over an alphabet $\sum$
Answer:
A finite sequence of elements over the alphabet ($\sum$)

7. (1 point) How many strings of length n are there over an alphabet $\sum$
(use $|S|$ to denote the cardinality of the set S).
Answer:
There are $\sum^n$ many strings over an alphabet $\sum$, so $|S| = \sum^n$

8. (1 point) How many strings are there over $\sum$? (0, finite, countably infinite, uncountable infinite
Answer:
Countably Infinite

9. (1 point) (1 point) What is a language over an alphabet $\sum$?
Answer:
A set of strings over $\sum$ which can be empty, finite, or infinite.

10. (1 point each) How many strings do the following languages over 0,1 have? if the answer is finite, give the number if it is infinite, state whether it is countable or uncountable. If the question is not a language over 0,1, state this.

(a) L = {}
Answer: Finite, 0

(b) L = {ϵ}
Answer: Finite, 1

(c) L = {x $\|x = \epsilon$}
Answer: Finite, 1

(d) {x|| x is the set of strings over 0,1 of length 3}
Answer: Finite, 8 strings

(e) L = {x || number of a's in x = number of b's in x}
Answer: Not a language over 0,1

(f) L is the set of all strings over {0,1}
Answer: Uncountably Infinite

11. (1 point each) What is the cardinality of the following sets? (answer is one of 0, finite, countably infinite, uncountably infinite; give the largest one that fits.)

(a) A given language L (note that L is a set of strings). you are asked without specifying which language.
Answer: Finite

(b) the set of all strings over an alphabet.
Answer: Countably Infinite

(c) the set of all languages.
Answer: Uncountably infinite

(d) the set of all compilable C-programs
Answer: Countably infinite

(e) the set of all C-programs with 25 global variables
Answer: Uncountably infinite

(f) the set of all C-programs with 25 lines of code
Answer: Countably infinite

12. (2 points) Define (What is) the decision problem associated with a language L over $\sum$?
Answer: Given a Language L over an alphabet $\sum$ and a string s over $\sum$, is $s \in L$?

13. (2 points) Define (What is) the language accepted by a machine/program M? Assume the inputs of M are strings over $\sum$ and M's return type is Boolean.
Answer: The language accepted by a machine/program M is the set of all strings over $\sum$ accepted by the machine M

14. (1 point) When is a machine M called the machine/program corresponding to the language L?
Answer: A machine M is called the machine/program corresponding to the language L when is solves the decision problem for that language.