342 Assignment 5: Rex $<->$ eNFA minimization

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Total points: 30

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Due Date: March 8 2021

1. (10 points) Give the e-NFA for (a*.b*)* Follow the algorithm given instead of creating your own eNFA

   eNFA drawing

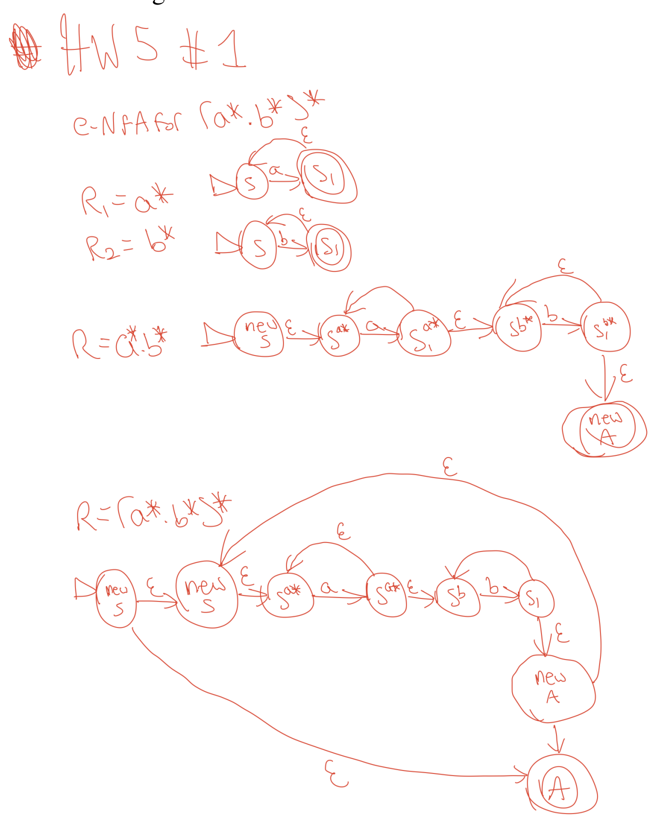


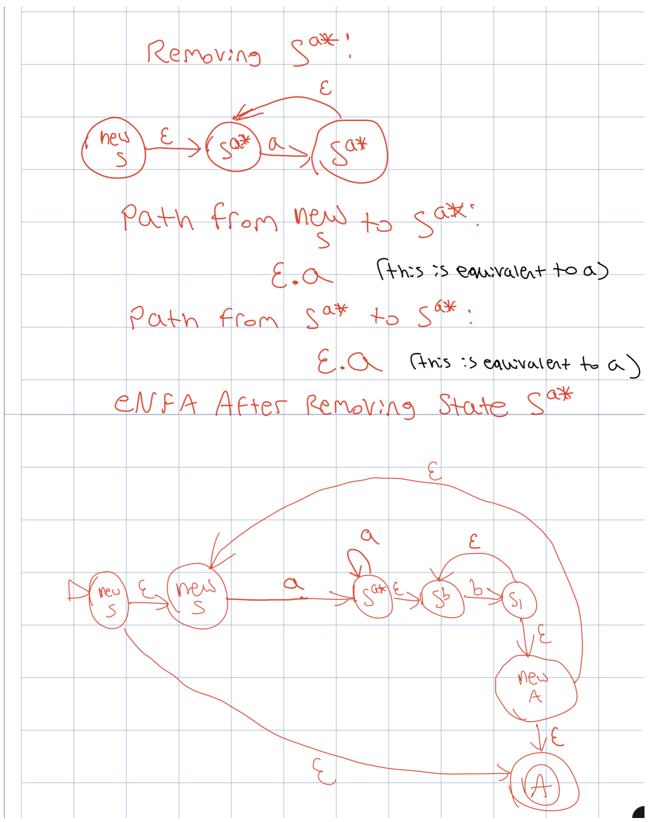
   Table Form:
   If $R = R1$
   L(R) = {x | x = y.z where y ∈ L(R1) and Z ∈ L(R2)}
   M1 = REX2eNFA(Σ, R1) with components M1.Σ, M1.Q, M1.S, M1.A, M1.T
   Return the following M:

   - Alphabet = Σ
   - Set of States = rename(M1,Q)∪(newS, newA)
   - Start State = newS
   - Accepting State = newA
   - Transition Table:
     - <newS, epsilon> = {M1.S}

- <M1.A, epsilon> = {newA}
- <M1.A, epsilon> = {M1.S}
- <State, Θ,or epsilon> = Copy from M1.T

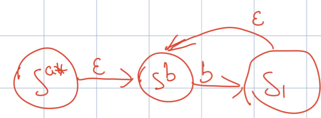
2. (10 points) the state removal algorithm was given in class for DFA; it can be used without any change whatsoever for eNFA, NFA etc.

   (a) Remove one state form your eNFA in Q1 showing all the steps (draw the auxiliary figure showing states with transitions in and out of the state you want to remove; then for (in,out) pairs of such states, give the path through the state being removed; then remove the state, and draw the e-NFA with new transitions; Then combine transitions with the same source and target using —)



   (b) Remove another state form your e-NFA you for after A showing all the steps (draw the auxiliary figure showing states with transitions in and out of the state you want to remove; then for (in,out) pairs of such states, give the path through the state being removed; then remove the state, and draw the e-NFA with new transitions; Then combine transitions with the same source and target using —)

Removing S^b :



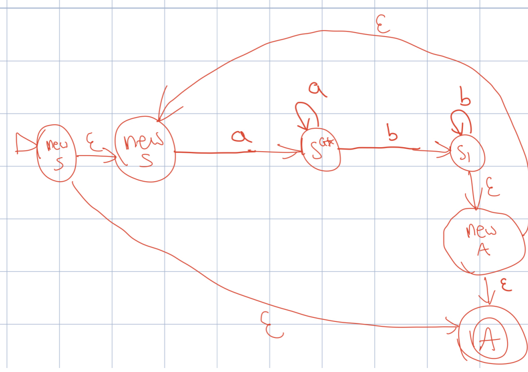
Path from S^{a*} to S_1:

$\varepsilon . b$ (this is equivalent to b)

Path from S_1 to S_1 :

$\varepsilon . b$ (this is equivalent to b)

eNFA After Removing State Sb



3. Extra credit (10 points) finish Q2 by removing all states and getting a regular expression for this - does this agree with the one given in Q1?

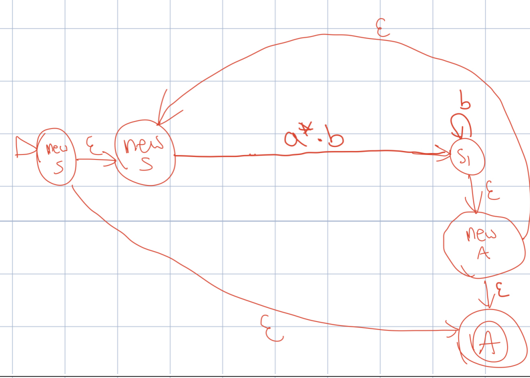
3. Removing all States:

Remove S^{a*}

Path from new s to S_1:

$a^* . b$

eNFA After Removing State S^{a*}

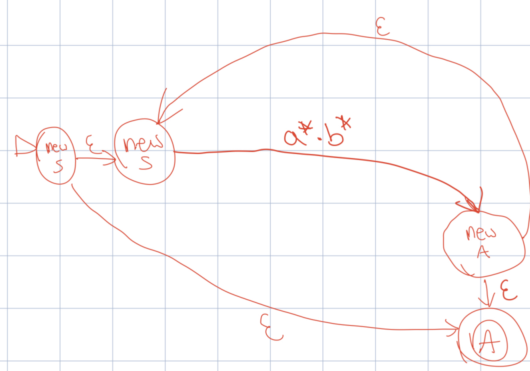


Remove S_1

Path from new s to new A:

$a^* . b^* . \varepsilon$ (equivalent to $a^* . b^*$)

eNFA After Removing State S1



Remove new A

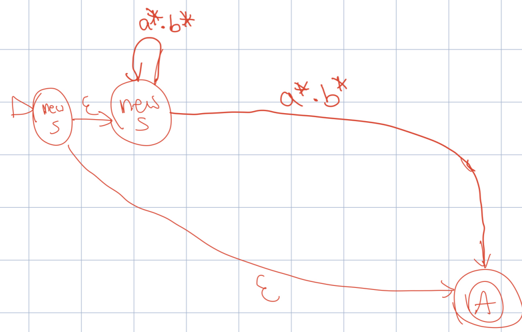
Path from new S to new S :

$a^*.b^*.\varepsilon$

Path from new S to A:

$a^*.b^*.\varepsilon$

eNFA After Removing State new A

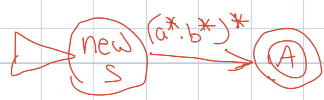


Remove new S

Path from new S to A :

$(a^*.b^*)^*$

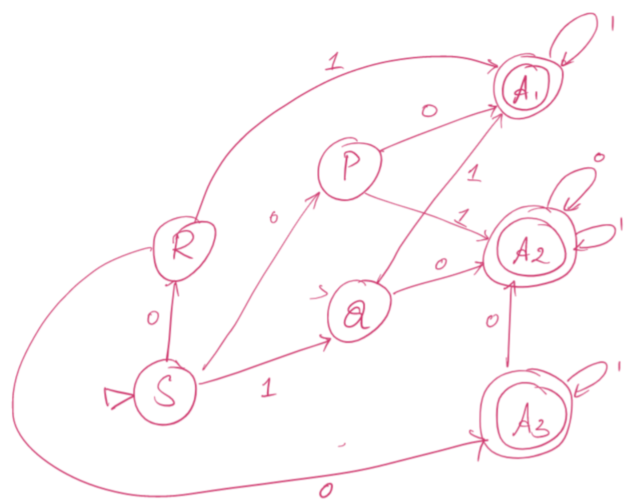
eNFA After Removing State new S



After deleting all the states, we find that the regular expression agrees with the one given in Q1.

4. (10 points) Minimize the following DFA:

   Mark which states are different, and explain how (S,T are different because one is accepting and the other is not; X,Y are different because transition on the same letter leads them to S,T, which are already known to be different)



Answer:

NFA	0	1
S	{R, P}	Q
Q	A_2	A_1
R	A_3	A_1
P	A_1	A_2
*A_1	0	A_1
*A_2	A_2	A_2
*A_3	A_2	A_3

DFA	0	1
[S]	[RP]	[Q]
[RP]	[A_1A_3]	[A_1A_2]
[Q]	[A_2]	[A_1]
*[A_1A_3]	[A_2]	[A_1]
*[A_1A_2]	[A_1]	[A_1A_2]
*[A_1]	[D]	[A_1]
*[A_2]	[A_2]	[A_2]
[D]	[D]	[D]

0 equivalent:
$\{[\mathrm{S}], [\mathrm{RP}], [\mathrm{Q}], [\mathrm{D}]\}, \{[\mathrm{A}_1 A_3], [A_1 A_2], [A_1], [A_2]\}$
$1 equivalent:$
$\{[S], [D]\}, \{[RP], [Q]\}, \{[A_1 A_3], [A_1 A_2], [A_2]\}, \{[A_1]\}$
$2 equivalent:$
$\{[S]\}, \{[D]\}, \{[RP]\}, \{[Q]\}, \{[A_1 A_3], [A_1 A_2], [A_2]\}, \{[A_1]\}$
$3 equivalent:$
$\{[S]\}, \{[D]\}, \{[RP]\}, \{[Q]\}, \{[A_1 A_3], [A_1 A_2], [A_2]\}, \{[A_1]\}$

S
$:\xrightarrow{0} [RP], \xrightarrow{1} [Q]$
Q
$:\xrightarrow{0} [A_1 A_3], [A_1 A_2], [A_2], \xrightarrow{1} [A_1]$
A_1
$:\xrightarrow{1} [A_1]$
D
$:\xrightarrow{0,1} [D]$
RP
$:\xrightarrow{0} [A_1 A_3], [A_1 A_2], [A_2], \xrightarrow{1} [A_1 A_3], [A_1 A_2], [A_2]$
$\mathrm{A}_1 A_3$
$, [A_1 A_2], [A_2] :\xrightarrow{0,1} [A_1 A_3], [A_1 A_2], [A_2]$