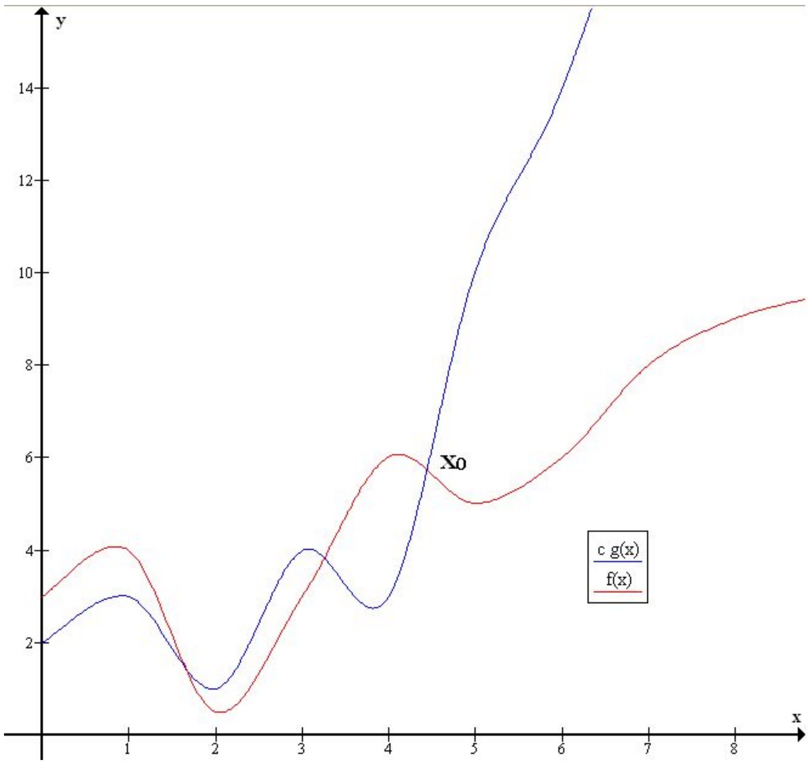
Algorithm Analysis ~ 08

CSE/IT 122 ~ Algorithms & Data Structures

BIG-OH

- Formal Definition:
  - Let *f* and *g* be functions defined on the same set of nonnegative real numbers. Then *f* is at order at most *g*, written *f(x)* is *O(g(x))*, iff there exists a positive number *c* and a non negative real number n_0 such that $|f(x)| \le c \cdot |g(x)|$, for all real numbers $x > n_0$
- Informal Definition:
  - *f(n)* is *O(g(n))* if *f* grows at most as fast as *g*. In other words *g* is an upper bound.

Big-Oh

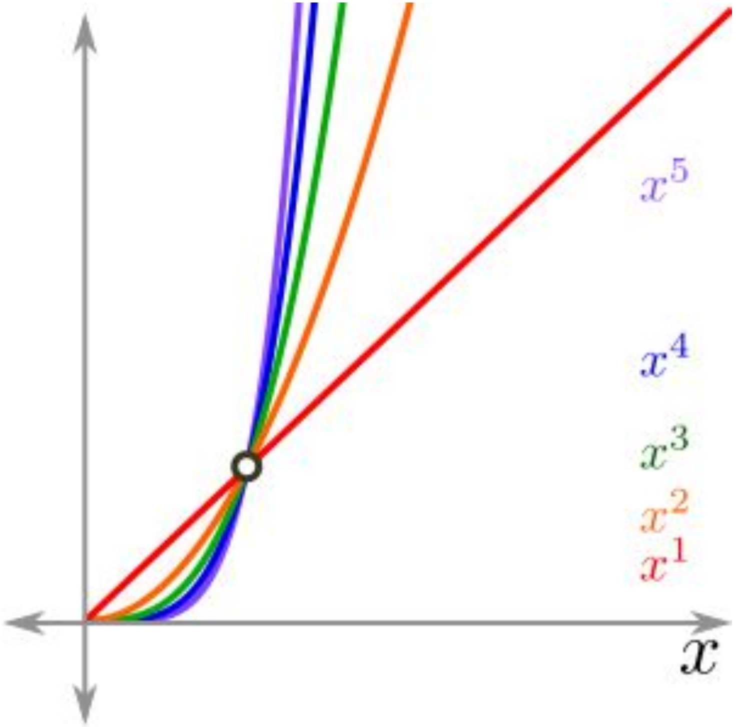


Big-Oh

- ➔ To show $f(n) = O(g(n))$ only need to find a pair of constants c and n_0 such that $|f(x)| \le c \cdot |g(x)|$, if $n > n_0$
  - Not unique
  - Infinite pairs of c and n_0
- ➔ For our RAM Model, define $T(n) \le O(f(n))$ and say that a program whose running time is $O(f(n))$ is said to have a growth rate of $f(n)$

Power Functions

- Definition: Let *a* be any nonnegative real number. Define p_a the power function with exponent *a* as follows:
  - $p_a(x) = x^a$ for each nonnegative real number x



Orders of Power Functions

- ➔ Observe if $1 < x$
  - Then $x < x^2$, since $x > 0$
  - And $x^2 < x^3$
  - And we can order $1 < x < x^2 < x^3$
- ➔ So for any rational numbers r and s, if $x > 1$ and $r < s$, then $x^r < x^s$
- ➔ In terms of Big-Oh this can translates to
  - For any rational number r and s, if $r < s$, then x^r is $O(x^s)$

Orders of Power Functions

- Example: Show if $x > 1$ then $3x^3+2x+7 \leq 12x^3$
  - $2x < 2x^3$ and $7 < 7x^3$, so
  - $3x^3+2x+7 \leq 3x^3+2x^3+7x^3 = 12x^3$
- Example: Show $f(x)$ is $O(g(x))$ when $f(x)=2x^4+3x^3+5$ and $g(x) = x^4$
  - Use the above technique
  - $2x^4+3x^3+5 \leq 2x^4+3x^4+5x^4 = 10x^4$
  - $c = 10$, and $n_0 = 1$
  - $f(x) = O(x^4)$

Orders of Power Functions: More Examples

- Show $3x^3-1000x-200$ is $O(x^3)$
  - Triangle inequality $|a+b| \le |a| + |b|$ for all real numbers a and b
    - Show that $|a-b| \le |a| + |b|$
    - $|a-b| = |a+(-b)| \le |a| + |-b| = |a| + |b|$
  - So now $3x^3-1000x-200 \le 3x^3+1000x+200 \le 3x^3+1000x^3+200x^3 = 1203x^3$
  - So $c = 1203$, and $n_0 = 1$

Orders of Polynomials: Your Turn

- ➔ Show that $7x^4-95x^3+3$ is $O(x^4)$

- ➔ What is the order of $\frac{n(n+1)(2n+1)}{6}$

Orders of Polynomials: Your Turn

- ➔ Show that $7x^4-95x^3+3$ is $O(x^4)$
  - $7x^4-95x^3+3 \le 7x^4+95x^3+3 \le 7x^4+95x^4+3x^4 = 105x^4$
  - Thus $c = 105$, $n_0 = 1$

- ➔ What is the order of $\frac{n(n+1)(2n+1)}{6}$
  - Can be rewritten as $\frac{1}{6}[n(n+1)(2n+1)] = 2n^3+3n^2+n$
  - Now our problem looks familiar and we can apply the above technique
  - $2n^3+3n^2+n \le 2n^3+3n^3+n^3 \le 6n^3$
  - $c = 6$ and $n_0 = 1$
  - Thus, $f(n) = \frac{1}{6}[n(n+1)(2n+1)] = O(n^3)$