Machine Learning for Agricultural Applications

Assignment 1

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Task 1 – Calculate Derivatives [10 points]

Calculate the derivatives of the following real-valued functions of a real variable:

a) $f(x) = u(x)\,v(x)$,

b) $g(x) = \dfrac{u(x)}{v(x)}$,

c) $h(x) = u(v(x))$,

d) $f(x) = x^2 \sin x$,

e) $g(x) = \ln(1 + x^2)$,

f) $h(x) = \tanh\left(\dfrac{(x-1)^2}{x^2 + 1 + \cos x}\right)$.

Hints: $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$, $\sinh(x) = \frac{e^x - e^{-x}}{2}$, $\ln(x) = \log_e(x)$ natural logarithm.

Task 2 – Calculate Gradients [10 points]

Calculate the gradients of the following real-valued functions of three real variables:

a) $f(x, y, z) = x\,z^2 e^y \cos y$,

b) $g(x, y, z) = \ln(\sqrt{x^2 + y^2 + z^2})$.

Task 3 – Calculate Minimum [10 points]

a) Calculate the gradient of the following function:

$$f(x, y) = (a - x)^2 + b(y - x^2)^2, \quad \text{with } a, b > 0 \text{ and } x, y \in \mathbb{R}$$

b) Calculate the extremum of the function $f(x, y)$ analytically and check if it is a minimum.

The function $f(x, y)$ is the so-called Rosenbrock function, which is often used as a performance test problem for optimization algorithms.

Task 4 – Gradient Descent [20 points]

Write a python program, that calculates the minimum of the Rosenbrock functions $f(x, y)$ given above with the gradient descent method. You need the gradients from task 3.

Compare the analytical result from task 3 with the result from gradient descent. Also think about useful plots.