Stable Hot Spot Analysis (Draft)

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Abstract: The Urban Heat Island (UHI) effect describes the difference in temperature between cities and their surrounding areas. However, temperature differences within city limits, so-called Intra Urban Heat Islands (IUHI), affect human health as well as the energy demands in local areas. In order to anticipate and mitigate the resulting impacts of heat trough urban planing, a method to reliably detect these local areas is needed. Existing methods from the geo-statistical field can identify those areas. But these statistics can be unstable in their detection of hot spots, in particular for temperature hot spots, depending on their parametrization. In this paper, we propose a modification of the well known Getis-Ord (G^*) statistic, called the *Focal* G^* statistic. This modification replaces the computation of the global mean and standard deviation with their focal counterparts. We define the stability of our approach by introducing a stability metric called *Stability of Hot spot* (SoH), which requires that hot spots have to be in similar areas regardless of the chosen weight matrix. The results are evaluated on real world temperature data for the city of Karlsruhe.

1 Introduction

For urban city planers, the detection of Intra Urban Heat Islands (IUHI) is of high interest as high temperatures impact energy consumption [**?**] as well as human health [**?**]. The effect that the temperatures between an urban area and its surroundings differ, called the Urban Heat Island effect [**?**], has long been the subject of research. Most historical studies conducted had to rely on few, fixed weather stations, low spatio-temporal resolution of satellites or small-scale mobile measurements by car, preventing the modelling of finer-grained temperature differences within an urban area itself. With the advent of inexpensive mobile sensors, higher spatio-temporal resolution of satellites and volunteered geographic informations, to name just a small selection, it is now feasible to focus on the temperature differences in a city as the subject of interest.

Hot spot analysis is a tool which is suited to detect such areas. In the context of IUHI we can detect those areas where the temperature is significantly different from the mean temperature of our study area. This enables us to identify points of interest without the need to pre-process the data.

Although existing methods are independent of concrete values, their results are highly dependent on the size of the study area and their parametrization such as the weight matrix in the case of the Getis-Ord statistic. This dependency can lead to unstable hot spots, where the identified hot spots only appear in one specific combination of parameter. The generalization of insights gained from unstable hot spots is suboptimal. A city planer who has to rely on those insights will most likely prioritize the wrong area to invest his limited resources. Given the increasing importance to detect extreme local temperatures in cites, see e.g. [**?**, **?**, **?**], means to reliably detect and mitigate the effect of temperature extremes in a proactive fashion are needed.

To solve this problem, we first propose a metric to measure the stability of a hot spot analysis regarding its parametrization of the weight matrix called the *Stability of Hot spot* (SoH). This metric measures whether a hot spot found for a given weight matrix is carried over to the found hot spots with a different weight matrix. This enables us to quantify the stability of any hot spot analysis. Based on our reasoning behind the instability of existing hot spot analyses, we propose a modification of the well known Getis-Ord statistic (G^*): the *Focal Getis-Ord* statistic (Focal G^*). Instead of the global mean and variance used by G^*, it only uses the mean and variance of a predefined region around each point. This region is a subset of the whole study area. By doing this, the instability is contained within a smaller region and thereby independent of the parametrization of the weight matrix. We test our stability metric as well as this modified approach on data given by two temperature snapshots of the city of Karlsruhe taken in 2008.

2 Related Work

2.1 Urban Heat Island

The scientific interest in the phenomena of UHI is well established. One of the earliest known overviews of the scientific literature of the city climates is given by Albert Kratzer 1937 [**?**]. At that time already, relations between temperature, humidity, human heat fluxes and air pollution are investigated.

A more recent overview comes from Arnfied [**?**]. The focus here lies on the development in the field of climatology between 1980 and 2003. In particular, the rise of simulations and modelling is lauded, but, to quote Arnfield, "simple methods are still needed to estimate UHI intensity within urban areas" [**?**].

Schwarz et al. [**?**] compare 11 different Surface Urban Heat Island (SUHI) indicators on a dataset of 263 European cities with monthly mean temperatures. They show that the selection of indicators is important for the detection of UHI due to possible instabilities of each indicator.

A more recent development is the focus on IUHI. The definition for IUHI comes from Martin et al. [**?**]. By defining thresholds with respect to spatial reference, these enable the detection of hot spots in a city, which Martin et al. call surface intra-UHI. This boils down to five steps, i.e. essentially a comparison of absolute deviation from the mean temperature given a survey area. The results can then be used to detect areas of interest in a city and potentially trigger alerts for a much finer spatial granularity.

While the proposition to examine urban micro-climates can be traced back as far as 1937 [**?**], we have found only few works in this field. Schwarz et al. [**?**] state that the reduction of an UHI to a single value for a whole city is questionable regarding its explanatory power. But also that there is currently no other way to quantify the differences amongst cities.

The difficulties presented by comparing the UHI values between different cities are the topic of Stewart and Oke [**?**]. They propose the use of local climate zones (LCZ) to standardise the methodology and terminology.

Another area where urbanizations affects the temperature distribution are subsurface temperatures. Menberg et al. [**?**] look into the distribution of the temperatures below the surface – the subsurface temperature. They find local hot spots of heat with deviations of up to +20°K in some areas related to local heat sources.

2.2 Hot Spot Analysis

The goal of an analysis of temperatures in a city is to find the most interesting, significant areas: Hot spots [**?**]. This goal is similar to the hot spot analysis in the field of geo-statistics. One of the most fundamental approach is Moran´s I [**?**]. There it is tested whether or not a spatial dependency exists. This gives the information on global dependencies in a data set. Upon this hypothesis test several geo-statistical tests are based. The most well known are the Getis-Ord statistic [**?**] and LISA [**?**]. In both cases the general, the global statistic of Moran´s I is applied in a local context. The goal is to detect not only global values, but instead to focus on local hot spots and to measure the significance of those local areas.

The local Getis-Ord statistic [**?**] is defined as follow:

Definition 1 (Getis-Ord G_i^* statistic)**.** *Assuming a study area with n measurements, let $X = [x_1, \ldots, x_n]$ be all values measured in this area. Let $w_{i,j}$ be a spatial weight between two points i and j for all $i, j \in \{1, \ldots, n\}$. The Getis-Ord G_i^* statistic is given as:*

$$G_i^* = \frac{\sum_{j=1}^{n} w_{i,j} x_j - \bar{X} \sum_{j=1}^{n} w_{i,j}}{S \sqrt{\frac{n \sum_{j=1}^{n} w_{i,j}^2 - (\sum_{j=1}^{n} w_{i,j})^2}{n-1}}} \tag{1}$$

where:

- *$\bar{X}$ is the mean of all measurements,*
- *S is the standard deviation of all measurements.*

As it is known, this statistic creates a z-score, which denotes the significance of an area in relation to its surrounding areas.

LISA is quite similar, as it is the local statistic for Moran's I [**?**], but the z-score has a different meaning. Apart from G_i^*, LISA does not distinguish between cold spots and hot spots as it assigns high z-score to most similar areas.

Two other well known method are the kernel density estimation [**?**] and kriging [**?**]. These do not provide significance levels. Instead, they estimate the values for each location based on the rest of the study area and a threshold value [**?**]. Therefore, results for different areas are not comparable, especially in the case of differing temperature distributions. Kriging was developed for the estimation of ore deposits [**?**], but today, applications for geo-temporal forecasts with this approach can be found, e.g. for the city of Zurich. [1]

All of the aforementioned methods use weights between pairs of points, usually based on their geographical distance. However, in real applications, the points are aggregated into rasters and the weights are represented as a weight matrix. This allows for expressing the algorithms in terms of map algebra operations, a term first coined by Dana Tomlin [**?**] and computed in a distributed fashion (e.g. using Geotrellis framework running on Apache Spark [**?**]).

In this work, we focus on the Getis-Ord statistic applied to rasters of land surface temperature. This enables us to transform the formula for the G^* statistic into a computationally more efficient form. We then propose a modification of the standard G^* statistics, to increase the stability of the hot spots found.

3 Stability of Hot Spot Analysis

Existing methods to determine hot spots are dependent on the parametrization of the weight matrix as well as on the size of the study area.

Consider the real-world example depicted in Fig. 1. The temperature map of a morning thermal flight dataset (Fig. 1a) has been processed using G^*.

[1] https://r-video-tutorial.blogspot.de/2015/08/spatio-temporal-kriging-in-r.html

(a) Morning temperatures

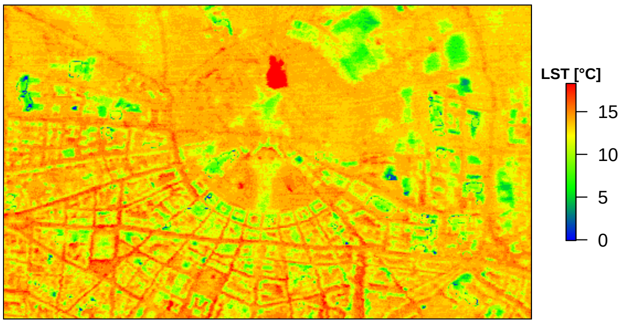


Figure 1: Karlsruhe city center. Selected area of $2.4 \times 1.4\,km$. Pixel size $5 \times 5\,m$.

As we can see, hot spots are oftentimes disappearing or appearing unrelated to previously found hot spots. While these computations indeed show hot spots and the results are correct, they lack stability.

For a data analyst, when exploring the data interactively by choosing different filter sizes (in form matrices), it is important that the hot spot's position and size changes in a predictable manner. This intuition is the basis for our stability metric.

We define a hot spot found in comparably more coarse resolutions as *parent* (larger weight matrix) and in finer resolutions as *child* (smaller weight matrix). To be stable, one assumes that every parent has at least one child and that each child has one parent. For a perfectly stable interaction, it can be easily seen that the connection between parent and child is a injective function and between child and parent a surjective function. To measure the closeness of connection, we propose a metric called the *Stability of Hot spot* (SoH). It measures the deviation from a perfectly stable transformation of resolutions.

In its downward property (from parent to child, injective) it is defined as:

$$SoH^{\downarrow} = \frac{ParentsWithChildNodes}{Parents} = \frac{|Parents \cap Children|}{|Parents|} \tag{2}$$

And for its upward property (from child to parent, surjective):

$$SoH^{\uparrow} = \frac{ChildrenWithParent}{Children} = 1 - \frac{|Children - Parents|}{|Children|} \tag{3}$$

where $ParentsWithChildNodes$ is the number of parents that have at least one *child*, $Parents$ is the total number of *parent*, $ChildrenWithParent$ is the number of children and $Children$ as the total number of children. The SoH is defined for a range between 0 and 1, where 1 represents a perfectly stable transformation while 0 would be a transformation with no stability at all.

4 Focal Getis-Ord

4.1 Dataset

The two datasets (morning and evening flights) depicted in Fig. **??** and Fig. **??** were obtained from a thermal flight over the city of Karlsruhe on 26.09.2008 at 6:30–7:45 and 20:00–21:30. The flights were executed by the Nachbarschaftsverband Karlsruhe[2]. A single pixel in the raster represents an area of approximately $5 \times 5m$. The whole dataset of size $35 \times 25\,km$ was cropped into the inner city area of $2.4 \times 1.4\,km$. The temperatures in our dataset range from -1.7°C to 18.3°C. Missing values in the dataset were interpolated using a focal median function with a square matrix of 11x11 pixels, mainly for speeding up further computations and to avoid special handling of NA values.

4.2 Method

In the following text, we use the notation $R \overset{\text{op}}{\circ} M$ to denote a focal operation *op* applied on a raster R with a focal window determined by a matrix M. This is rougly eqivalent to a command `focal(x=R, w=M, fun=op)` from package *raster* in the R programming language [**?**].

Definition 2 (G^* function on rasters)**.** *The function G^* can be expressed as a raster operation:*

$$G^*(R, W, st) = \frac{R \overset{\text{sum}}{\circ} W - M * \sum_{w \in W} w}{S\sqrt{\frac{N * \sum_{w \in W} w^2 - (\sum_{w \in W} w)^2}{N-1}}}$$

where:

- *R is the input raster.*
- *W is a weight matrix of values between 0 and 1.*
- *$st = (N, M, S)$ is a parametrization specific to a particular version of the G^* function. (Def. 3 and 4).*

Definition 3 (Standard G^* parametrization)**.** *Computes the parametrization st as global statistics for all pixels in the raster R:*

- *N represents the number of all pixels in R.*
- *M represents the global mean of R.*
- *S represents the global standard deviation of all pixels in R.*

Definition 4 (Focal G^* parametrization)**.** *Let F be a boolean matrix such that: $all(dim(F) \geq dim(W))$. This version uses focal operations to compute per-pixel statistics given by the focal neighbourhood F as follows:*

- *N is a raster computed as a focal operation $R \overset{\text{sum}}{\circ} F$. Each pixel represents the number of pixels from R convoluted with the matrix F.*

[2] http://www.nachbarschaftsverband-karlsruhe.de/

- *M is a raster computed as a focal mean* $R \overset{\texttt{mean}}{\circ} F$*, thus each pixel represents a mean value of its* F*-neighbourhood.*

- *S is a raster computed as a focal standard deviation* $R \overset{\texttt{sd}}{\circ} F$*, thus each pixel represents a standard deviation of its* F*-neighbourhood.*

5 Results

5.1 Heatmap

5.2 Clumping

5.3 Zoom

Blueline is Focal G^*. Redline is G^*

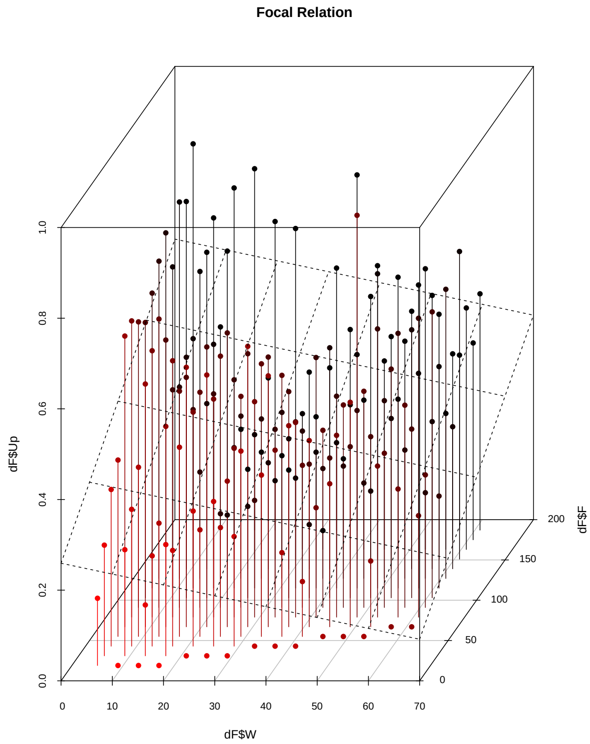
5.4 Blur

5.5 Results and Discussion

The results for the hot spot analysis are found in Fig. **??** and Fig. **??** for a comparison of G^* and the Focal G^* statistic. It can easily be seen that both versions produce similar results, but the focal versions produces a more differentiated picture for larger weight matrices. Small differences on a global scale are more pronounced on a regional scale and result in smaller and finer areas for hot spots. This enables the detection of additional hot spots and interesting areas which are most easily observable for the weight matrix of size 7×7 in the evening (Fig. **??**). This enables the detection of significant deviations from the surrounding area. In contrast the Standard G^* statistic shows larger areas as important. Therefore, depending on the need of a planner, the Focal G^* statistic is more helpful to identify individual areas of interest whereas the standard G^* statistic gives a more broad overview. For the identification of IUHI this is quite important. As a city planer wishes to detect critical areas, it is important to detect not only general hot areas, but also those points where the most extreme differences in a local context exist. Finding those areas can help identify the underlying reasons or plan individual solutions.

Based on these images one can also see that the hot spots found by the Focal G^* statistic seem to be more stable. We compare their stability using only the $SoH^{\uparrow}$ for lack of space. A plot of the results can be found in Fig. **??**. Fig. **??** compares the $SoH^{\uparrow}$ between each increasing size of the weight matrix W. At a first glance, one can see that the typical implementation with a square weight matrix is the most unstable hot spot analysis, regardless of time of day. This is to be expected as the binary weights increase the dependence on the weight matrix. The use of a decreasing weight matrix perform better. As the more outlying data points get less weight, this reduces the dependence on the weight matrix and leads therefore to more stable results. Our proposed Focal G^* statistic achieves the

(a) FocalRelation



(b) WeightRealtion

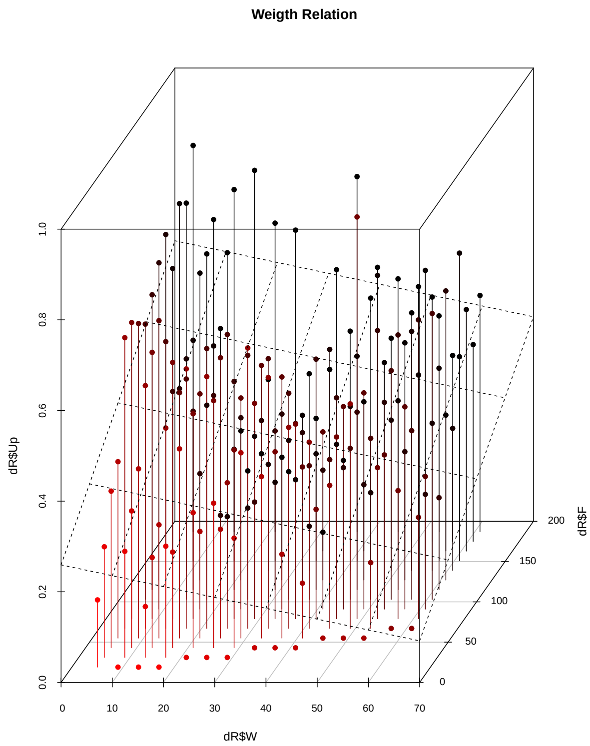
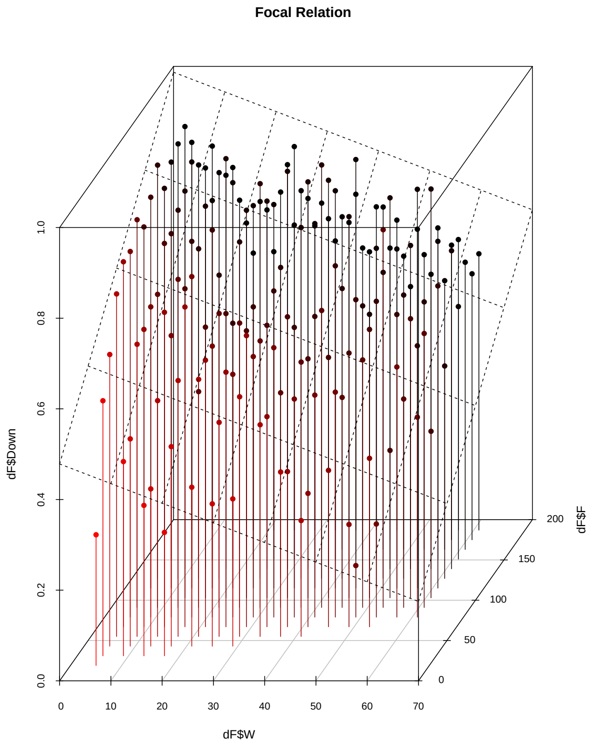


Figure 2: Heatmap of Relation between size of weight matrix W and focal matrix F

(a) FocalRelation



(b) WeightRealtion

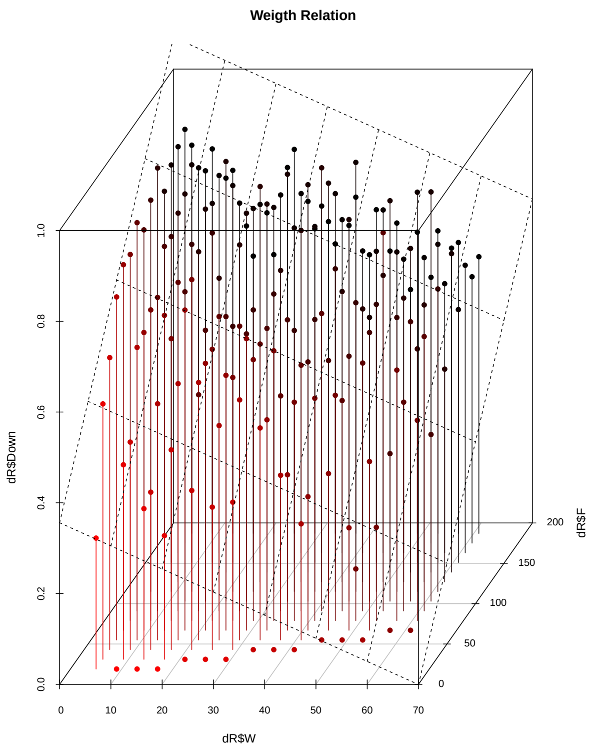
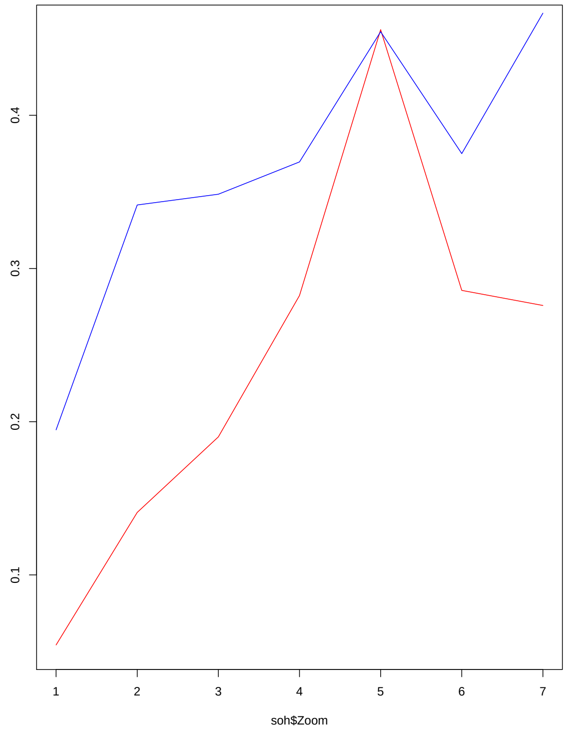


Figure 3: Heatmap of Relation between size of weight matrix W and focal matrix F

(a) Upward



(b) Downward

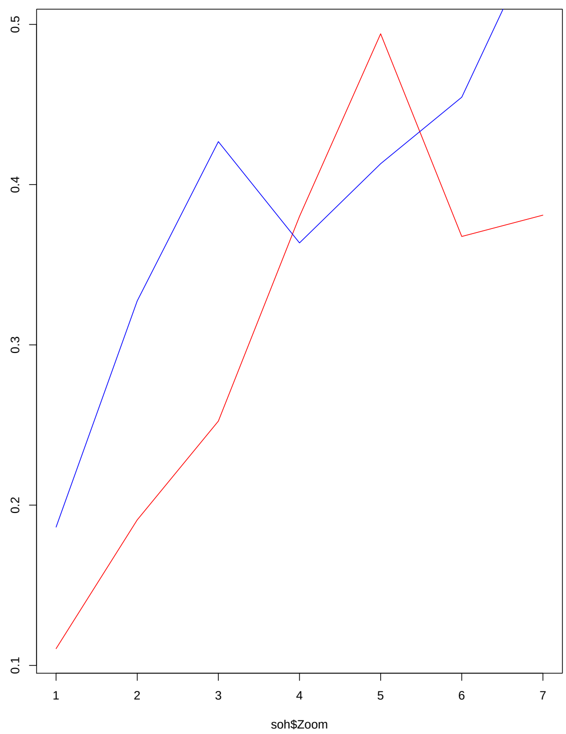
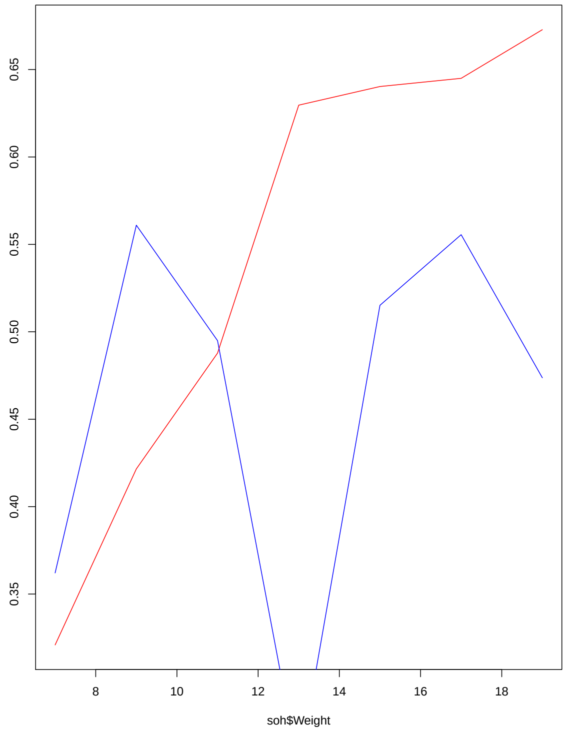


Figure 4: TODO

(a) Upward



(b) Downward

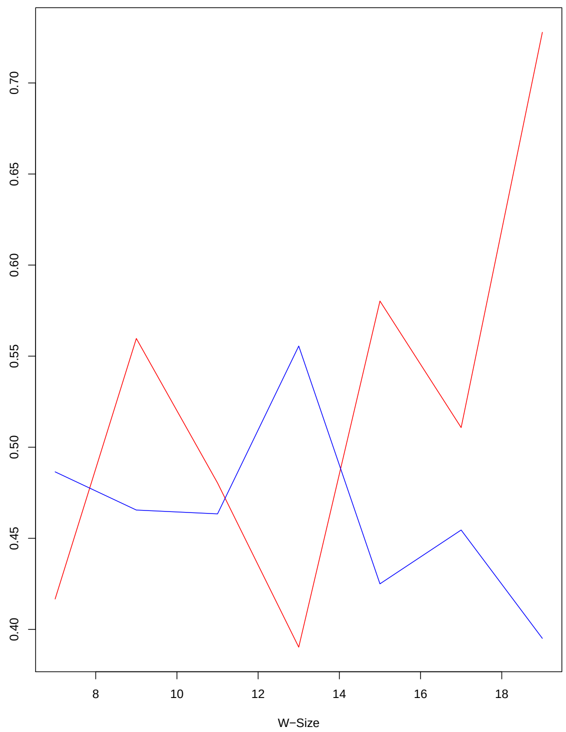


Figure 5: TODO

most stable results in almost all cases. Only data points in a restricted region around the area of interest may influence the significance result. Through this restriction high values at key points gains more weight regardless of the weight matrix and are therefore more independent of the weight matrix. This increases the stability. The decrease in stability for the largest weight matrices is most likely a result of the parametrization of the focal matrix. With increasing size of the weight matrix in relation to the focal matrix the value of each pixel is approaching to the mean of the area of the weight matrix. As can be easily seen from Def. 2 then the value for every pixel would be zero.

6 Conclusions and Future Work

In this work, we generalized the Getis-Ord statistic to deal with the problem of stability inherent in hot spot analysis. We identified possible underlying reasons for this instability: The weight matrix as well as the size of the study area. We developed a modified approach that deals with those two factors. The result is a modified G^* statistic called the *Focal* G^* *statistic*. It reduces the study area used for comparison into regions and achieves through this an increase in stability. To determine the effectiveness of our approach, we propose a stability metric for hot spots called *SoH*. To our knowledge no such metric existed before this work. The SoH computes the ratio of dependence of hot spots for different parametrizations of weight matrices. It enables to express the stability between each parametrization using single value restricted between zero and one. Based on this number one can decide which parametrization to use and researchers can compare the stability of their methods for unsupervised hot spot analyses. In particular, for temperature values one wishes to detect those areas which have high differences regardless of a particular parametrization. If a hot spot only appears for one parametrization, the information gained for general use is quite small and can even lead to an inefficient allocation of resources.

This research has several restrictions which have to be taken into account. First, we only tested the $SoH^{\uparrow}$ metric. While we assume, based on our graphical analysis, that the $SoH^{\downarrow}$ stability should be similar, we have no hard results. The results themselves are tested on two events in time for a fixed area of the city of Karlsruhe. We have not tested it on smaller or larger study areas, but we assume that the stability of the Focal G^* would stay the same whereas the stability of the G^* statistic would increase with a smaller study area and decrease with a larger study area. This follows the reasoning that the impact of a singular point increases with a decrease of the study area. To test this dependency is an interesting task for future work. The last restriction is the fixed size in this work of the focal matrix for the Focal G^* approach. We only tested one size in this work, but it is highly probable that the size of the focal matrix has an impact on the stability as could be seen in

Fig. **??**. While an overall trend can be seen in this work when the size of the weight matrix W and the focal matrix F are almost identical, the exact ratio is beyond the scope of this work. The optimal ratio as well as when the stability suffers from a too similar size are interesting question for future work.

7 Acknowledgements

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