HOMEWORK 1

Please include this sheet as the cover of your homework assignment.

Due Thursday, January 29th

Name: ______________________________

Department: ______________________________

Problem 1.

Consider the following boundary value problem

$$
\begin{aligned}
-\frac{d^2u}{dx^2} + u = x, \qquad 0 < x < 1 \\
u(0) = u(1) = 0.
\end{aligned} \tag{1}
$$

(a) Demonstrate that

$$u(x) \;=\; x - \frac{\sinh x}{\sinh 1}$$

is the exact solution of the boundary value problem (note: you do not need to solve the DE).

(b) Derive the weak form of the problem.

(c) Apply the Galerkin method to construct approximate solutions in the subspaces V_N for $N = 1, 2,$ and 3 using the following trigonometric basis functions

$$\left[\ \phi_1 = \sin(\pi x),\ \ \phi_2 = \sin(2\pi x),\ \ \phi_3 = \sin(3\pi x)\ \right]$$

Plot the exact and approximate solutions together, and calculate the percentage error in the approximations at $x = 0.25,\ 0.50,$ and 0.75.

(d) Repeat part (c) using the following polynomial basis functions

$$\left[\ \phi_1 = x(1-x),\quad \phi_2 = x\left(\frac{1}{2} - x\right)(1-x),\ \ \phi_3 = x\left(\frac{1}{3} - x\right)\left(\frac{2}{3} - x\right)(1-x)\ \right]$$

Problem 2.

In class we looked at the following boundary value problem

$$-\frac{d^2u}{dx^2} = x, \qquad 0 < x < 1 \tag{2}$$
$$u(0) = u(1) = 0.$$

(a) Given that the Galerkin method produces a diagonal stiffness matrix for this problem with the trigonometric basis functions used in class, show that the i-th solution component α_i is given by the general expression

$$\alpha_i = \frac{2(-1)^{i+1}}{i^3\pi^3}$$

(b) Apply the Galerkin method using the polynomial basis functions of problem 1, part d with $N = 2$ for this problem. Do you notice anything special about u_N in this case?

(c) Explain why the monomial basis

$$\left[\ \phi_1 = 1, \quad \phi_2 = x,\ \phi_3 = x^2, \ldots,\ \phi_N = x^N\ \right]$$

could not be used as a basis for this problem (or any of the problems considered here).