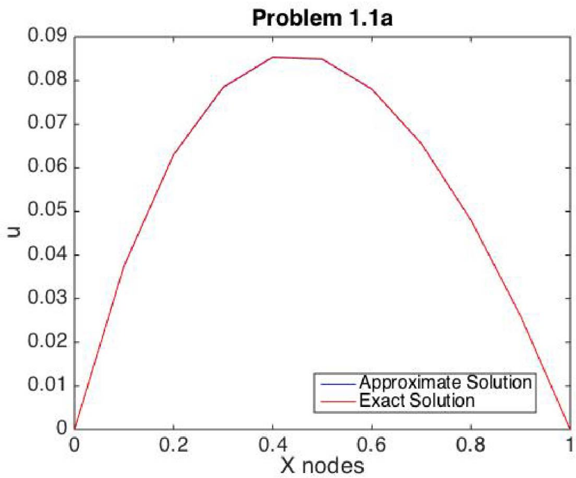
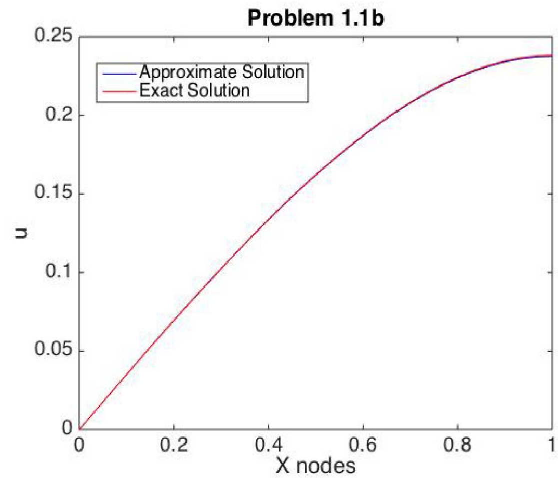
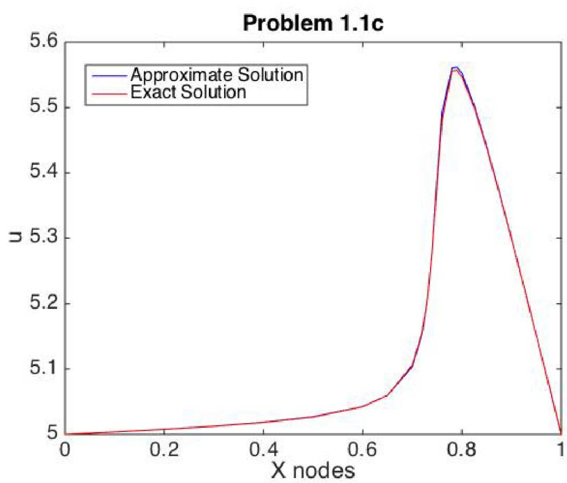
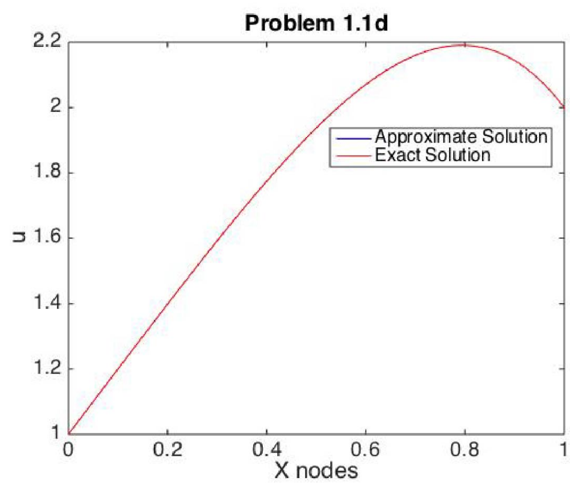
Problem One Point One









Problem One Point Two

(a) The strong form is given by: *Find $u \in C^2$ where*

$$-\frac{d}{dr}(\kappa r \frac{du}{dr}) = rf$$
$$u(1) = 100, \quad u(10) = 0$$

The weak form is derived at the end of the following steps:

$$-\frac{d}{dr}(\kappa r \frac{du}{dr}) = rf$$
$$-\frac{d}{dr}(\kappa r \frac{du}{dr})\phi = rf\phi$$
$$-\int_1^{10} \frac{d}{dr}(\kappa r \frac{du}{dr})\phi dr = \int_1^{10} rf\phi dr$$

$$-\kappa\left(\phi r u_r\Big|_1^{10} - \int_1^{10} r u_r \phi_r dr\right) = f\int_1^{10} r\phi dr$$
$$\kappa\int_1^{10} r u_r \phi_r dr = f\int_1^{10} r\phi dr$$

(b) If we are referring to *relative error*, we get the desired accuracy with `Mesh 1`. The relative error at the indicated point is given by

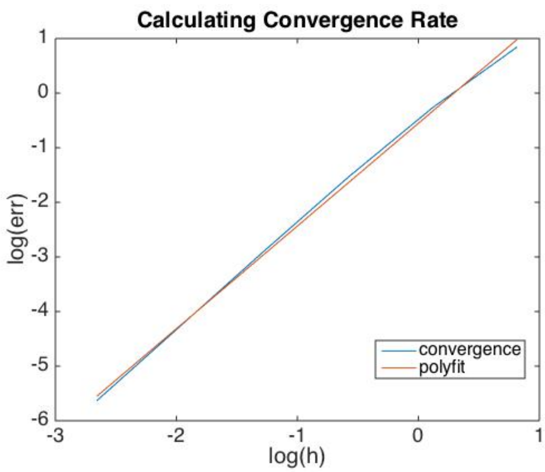
$$\frac{|51.1219 - 48.8117|}{|51.1219|} = 0.0473 < \frac{1}{10}$$

There are 4 elements in this mesh. I'm quite sure that we are after the absolute error, however. we get the desired accuracy for `Mesh 4`.

$$|(u - u_h)(3.25)| = |48.8679 - 48.8117| = 0.0562 < \frac{1}{10}$$

There are 32 elements in this mesh.

(c) Here is the plot:



Here is the code for plotting:

```
err = fliplr(log([2.3102 0.7667 0.2162 0.0562 0.0142 0.0036]));
h = fliplr(log([2.25 1.125 0.5625 0.281250 0.140625 0.070312]));
p = polyfit(h, err, 1); % Fit to a line
disp(p(1));
P = polyval(p, h);
set(gcf, 'color', 'w')
plot(h, err, h, P)
 title('Calculating Convergence Rate')
xlabel('log(h)')
ylabel('log(err)')
legend('convergence', 'polyfit')
```

The slope of the line given by `polyfit()` is $1.8811 \approx 2$.

(d) The absolute error in this example is $0.0459 < \frac{1}{10}$ at the indicated point. There are, however, only 10 elements, which is fewer than the 32 used in the structured mesh in part (b). The nodes are spaced more closely around $r = 3.25$ in the unstructured mesh than in some of the structured meshes with more elements, and therefore gives a better approximation in this neighborhood.

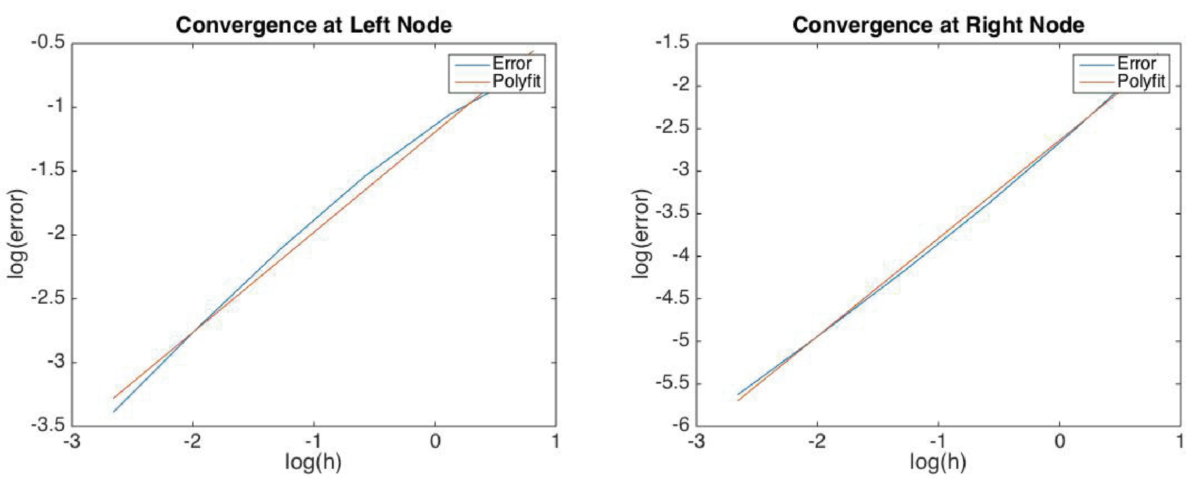
Also, the solution is nearly linear in the right half of the domain, and so the solutions rapidly converge point-wise for all meshes in this subdomain. The unstructured mesh has nodes at only $x = 5.5$ and $x = 7.375$ and one of the structured meshes has nodes at $x = 6.625$, $x = 7.1875$, $x = 7.75$, and $x = 8.31$, yet both have similar accuracies in this neighborhood.

(e) For this problem, I will use a finite difference to approximate u_h' at the end points. That is, the derivative is approximated as

$$u_h' = \frac{u_l^{(e)} - u_r^{(e)}}{h^{(e)}}$$

where e denotes 'element', l denotes 'left' and r denotes 'right' for the left and right endpoints of the element. Since we are looking for 10% error, I assume that in this case we are referring to relative error and not absolute error. At the left endpoint I get the desired accuracy with `Mesh 5`. At the right endpoint I get the desired accuracy with `Mesh 2`.

(f) The convergence of the flux is slower than the convergence for the solution. The rates of convergence are about 0.7847 and 1.1516 at the left and right endpoints, respectively.



Problem Two Point One

Problem Two Point Two