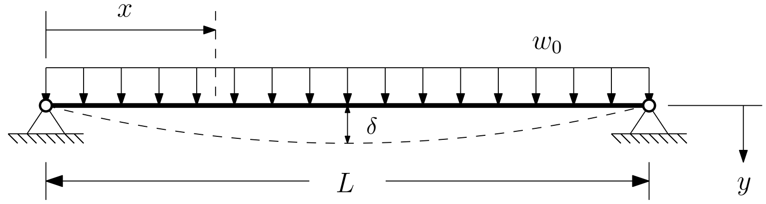
# HOMEWORK 4

Please include this sheet as the cover of your homework assignment.

Due Tuesday, March 31

Name: ______________________________

Department: ______________________________




**Problem 1.**

Consider the Euler–Bernoulli equation, which describes the relationship between a beam's deflection, $y$ and an applied moment $M$:

$$-\frac{d^2}{dx^2}\Big(EIy\Big) \;=\; M,$$

where the product $EI$ (the elastic modulus, $E$, times the second moment of area, $I$) is called the flexural rigidity of the beam.

For the case of a simply supported beam of constant flexural rigidity that is carrying a uniformly distributed load, $w_0$, as shown above, the moment as a function of $x$ is $M = \frac{w_0}{2}x\,(L-x)$. In this case, it can be easily shown that the deflection, $\delta$, at the middle of the beam is given by $\delta = \frac{5w_0L^4}{384EI}$.

Using your 1D finite element code:

1.) Determine the midspan deflection, $\delta$, using two linear elements of equal length (for simplicity take $EI = 1$, $L = 10$, and $w_0 = 38.4$).

2.) Compute the relative error of your approximation to $\delta$.

3.) Starting with the two-element mesh used above, generate a series of uniformly refined meshes (i.e., meshes with $4, 8, 16, \ldots$ elements of equal length), until the relative error of your approximation is below 1%.

**Problem 2.**

Consider the case of the two-dimensional model problem with $k(x, y) = k_0 =$ constant and $b(x, y) = 0$. Starting with the equation on slide 22 of lecture 10, show that the stiffness matrix for an arbitrary triangular element with linear shape functions is given by

$$\mathbf{k}^e = \frac{k_0}{4A_e}(\mathbf{X} + \mathbf{Y})$$

where

$$\mathbf{X} = \begin{bmatrix} (x_2 - x_3)^2, & (x_2 - x_3)(x_3 - x_1), & (x_2 - x_3)(x_1 - x_2) \\ (x_2 - x_3)(x_3 - x_1), & (x_3 - x_1)^2, & (x_3 - x_1)(x_1 - x_2) \\ (x_2 - x_3)(x_1 - x_2), & (x_3 - x_1)(x_1 - x_2), & (x_1 - x_2)^2 \end{bmatrix}$$

and $\mathbf{Y}$ is the matrix obtained by replacing $x_i$ by $y_i$ in the definition of $\mathbf{X}$.