HOMEWORK 2

Please include this sheet as the cover of your homework assignment.

Due Thursday, February 12th

Name: ______________________________

Department: ______________________________

Problem 1.

Use piecewise linear finite elements to solve the following boundary value problem:

$$
\begin{gathered}
-\frac{d}{dx}\left((1+x)\frac{du}{dx}\right) = 1, \qquad 0 < x < 1 \\
u(0) = u(1) = 0.
\end{gathered} \tag{1}
$$

Solve the problem using (a) two elements of equal length and (b) four elements of equal length. (Do not use the master element concept for this problem. Follow the example of lecture 4). In both cases, plot the exact ($u(x) = \ln(1+x)/\ln 2 - x$) and approximate solutions together.

Problem 2.

Using the procedure discussed in class, compute the element shape functions over the master element for a cubic ($k = 3$) Lagrange element and plot the results.

Problem 3. (Part of project)

Write a MATLAB function of the form

```
psi = lagrange_poly(k)
```

that takes as input the degree k of the shape functions desired and returns a structure `psi` of size 1 by $k+1$ with two fields, `psi.fun` and `psi.der` that store the polynomial coefficients, in descending powers, of the $k+1$ Lagrange shape functions and their derivatives, respectively. Use the MATLAB function `polyfit`, as discussed in class, to generate the polynomials.

For example, in class, we derived the the quadratic Lagrange shape functions. In MATLAB, we would represent these polynomials in the following form:

$$\hat{\psi}_1 = \frac{1}{2}\xi^2 - \frac{1}{2}\xi \quad \Rightarrow \quad \texttt{psi(1).fun = [0.5000 -0.5000 0]},$$

$$\hat{\psi}_2 = -\xi^2 + 1 \quad \Rightarrow \quad \texttt{psi(2).fun = [-1 0 1]},$$

$$\hat{\psi}_3 = \frac{1}{2}\xi^2 + \frac{1}{2}\xi \quad \Rightarrow \quad \texttt{psi(3).fun = [0.5000 0.5000 0]},$$

and their derivatives would be represented in the form:

$$\hat{\psi}_1' = \xi - \frac{1}{2} \quad \Rightarrow \quad \texttt{psi(1).der = [1 -0.5000]},$$

$$\hat{\psi}_2' = -2\xi \quad \Rightarrow \quad \texttt{psi(2).der = [-2 0]},$$

$$\hat{\psi}_3' = \xi + \frac{1}{2} \quad \Rightarrow \quad \texttt{psi(3).der = [1 0.5000]}.$$

Test your code by verifying the results for $k = 3$ as determined in Problem 2. Please provide your source code, which should include comments explaining the function.