Problem 2

The **X** and **Y** computations are identical, so I will just do the **X** computation and the result will be the same for **Y** with the variables swapped. Remember $\int dA = \frac{1}{2}$.

$$
\begin{aligned}
k^e_{11x} &= \frac{1}{2} * \frac{k_0}{2A_e}\left[((x_3 - x_1)(-1) - (x_2 - x_1)(-1))((x_3 - x_1)(-1) - (x_2 - x_1)(-1))\right] \\
&= \frac{k_0}{4A_e}(x_2 - x_3)^2 \\
k^e_{12x} &= \frac{1}{2} * \frac{k_0}{2A_e}\left[((x_3 - x_1)(-1) - (x_2 - x_1)(-1))((x_3 - x_1)(1) - (x_2 - x_1)(0))\right] \\
&= \frac{1}{2} * \frac{k_0}{2A_e}\left[(x_1 - x_3 + x_2 - x_1)(x_3 - x_1)\right] \\
&= \frac{1}{2} * \frac{k_0}{2A_e}(x_2 - x_3)(x_3 - x_1) \\
k^e_{13x} &= \frac{1}{2} * \frac{k_0}{2A_e}\left[((x_3 - x_1)(-1) - (x_2 - x_1)(-1))((x_3 - x_1)(0) - (x_2 - x_1)(1))\right] \\
&= \frac{1}{2} * \frac{k_0}{2A_e}(-x_3 + x_1 + x_2 - x_1)(x_1 - x_2) \\
&= \frac{1}{2} * \frac{k_0}{2A_e}\left[(-x_3 + x_2)(x_1 - x_2)\right] \\
k^e_{22x} &= \frac{1}{2} * \frac{k_0}{2A_e}\left[((x_3 - x_1)(1) - (x_2 - x_1)(0))((x_3 - x_1)(1) - (x_2 - x_1)(0))\right] \\
&= \frac{1}{2} * \frac{k_0}{2A_e}(x_3 - x_1)^2 \\
k^e_{23x} &= \frac{1}{2} * \frac{k_0}{2A_e}\left[((x_3 - x_1)(1) - (x_2 - x_1)(0))((x_3 - x_1)(0) - (x_2 - x_1)(1))\right] \\
&= \frac{1}{2} * \frac{k_0}{2A_e}(x_3 - x_1)(x_1 - x_2) \\
k^e_{33x} &= \frac{1}{2} * \frac{k_0}{2A_e}\left[((x_3 - x_1)(0) - (x_2 - x_1)(1))((x_3 - x_1)(0) - (x_2 - x_1)(1))\right] \\
&= \frac{1}{2} * \frac{k_0}{2A_e}(x_1 - x_2)^2
\end{aligned}
$$

The sub-diagonal elements can be filled with their corresponding symmetric values. Summing the **X** and **Y** contributions gives the result.