CSE 6331 – Algorithms – Spring, 2015 – Prof. Supowit

Homework 2 – Due: Wednesday, January 28

1. Design and analyze an efficient algorithm for the following problem: given is an adjacency list for a digraph $G = (V, E)$ and three distinguished vertices *a*, *b*, and $c \in V$. The output is a shortest path from *a* to *c* that passes through *b*.

By this phrase "design and analyze an algorithm" (a phrase which we'll see again in future homeworks and exams), we mean (1) Describe your algorithm as clearly as you can, with ordinary sentences and perhaps also with heavily commented pseudo-code, (2) Argue that your algorithm is correct, and (3) Analyze its worst-case running time. By "efficient," we mean that the lower the asymptotic complexity of your algorithm, the higher will be your grade.

2. Design and analyze an $O(|V| + |E|)$ time algorithm for the following problem: given is an adjacency list for a digraph $G = (V, E)$ and two subsets $A, B \in V$. The output is the number

$$\min_{a \in A,\, b \in B} \{\text{dist}(a, b)\},$$

or "∞" if this number doesn't exist.

3. Modify the BFS algorithm so as to compute, for each vertex *v* in the graph, not only *v.dist* (the length of a shortest path from the source vertex *s* to *v*), but also the number of different paths of that length. Store this information in a field called *v.num_paths*.

Write up a clear description of your algorithm, using both prose and commented pseudo-code.

Make sure that your algorithm still runs in time $\Theta(|V| + |E|)$.