

CSE 6331 – Algorithms – Spring, 2015 – Prof. Supowit

Homework 9 – Due: Friday, April 10

1. In lecture we discussed the following version of the LP problem:

Problem  $A$ : Given a matrix  $A \in \mathbb{R}^{m \times n}$  and vectors  $\mathbf{b} \in \mathbb{R}^m$  and  $\mathbf{c} \in \mathbb{R}^n$ , find a vector  $\mathbf{x} \in \mathbb{R}^n$  that maximizes  $\mathbf{c}^T \mathbf{x}$  subject to  $A\mathbf{x} \geq \mathbf{b}$ .

Now consider

Problem  $B$ : Given a matrix  $A \in \mathbb{R}^{m \times n}$  and a vector  $\mathbf{b} \in \mathbb{R}^m$ , find a vector  $\mathbf{x} \in \mathbb{R}^n$  such that  $A\mathbf{x} \geq \mathbf{b}$ .

Assuming that you have a subroutine that efficiently solves problem  $B$ , how could you efficiently solve problem  $A$ ?

2. Describe a way to solve the following problem using linear programming:

Input: a weighted digraph  $G = (V, E, \text{dist})$ , where  $\text{dist} : E \rightarrow \mathbb{R}^+$  and some  $s \in V$ .

Output: for each  $v \in V$ , the number

$d_v =$  the length of a shortest path from  $s$  to  $v$ .

Your LP should have  $O(|E|)$  constraints.