Clayton Cópula

$$C = \left(\max\left\{u^{-\theta} + v^{-\theta} - 1; 0\right\}\right)^{-1/\theta}$$

$C \sim unif(0,1) \rightarrow C = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$

$\frac{dc}{du} \Leftrightarrow C' = -\frac{1}{\theta}(u^{-\theta} + v^{-\theta} - 1)^{-\left(\frac{1+\theta}{\theta}\right)}(-\theta)u^{-\theta-1}$

$C' = w$

$$w = (u^{-\theta} + v^{-\theta} - 1)^{-\left(\frac{1+\theta}{\theta}\right)} u^{-\theta-1}$$

$$w u^{\theta+1} = (u^{-\theta} + v^{-\theta} - 1)^{-\left(\frac{1+\theta}{\theta}\right)}$$

$$w^{\frac{-\theta}{1+\theta}} \cdot u^{-\theta} = u^{-\theta} + v^{-\theta} - 1$$

$$w^{-\frac{\theta}{1+\theta}} \cdot u^{-\theta} - u^{-\theta} + 1 = v^{-\theta}$$

$$\left(u^{-\theta}\left(w^{-\frac{\theta}{1+\theta}} - 1\right) + 1\right)^{-1/\theta} = v$$

Gumbel - Hougard

$$C = e^{-\left[(-\log(u))^{\theta} + (-\log(v))^{\theta}\right]^{\frac{1}{\theta}}}$$

$C \sim unif(0,1)$ $u \sim unif(0,1)$ $v \sim unif(0,1)$

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Frank Copula:

$$C(u,v) = -\frac{1}{\theta}\log\left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1}\right)$$

$w = C'(u,v) \sim unif(0,1)$ $u \sim unif(0,1)$ $v \sim F^{-1}_{v|u}(w)$

$F_{v|u}(v) = C'(u,v)$

$$w = -\frac{1}{\theta}\log\left(\frac{(e^{-\theta} - 1) + (e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1}\right) \cdot \frac{e^{-\theta u}(e^{-\theta v} - 1)}{e^{-\theta} - 1} \cdot -\theta$$

$$w = \frac{(e^{-\theta} - 1)}{(e^{-\theta} - 1) + (e^{-\theta u} - 1)(e^{-\theta v} - 1)} \cdot \frac{e^{-\theta u}(e^{-\theta v} - 1)}{e^{-\theta} - 1}$$

$$w = \frac{e^{-\theta u}(e^{-\theta v} - 1)}{(e^{-\theta} - 1) + (e^{-\theta u} - 1)(e^{-\theta v} - 1)} = w(e^{-\theta} - 1) + w(e^{-\theta u} - 1)(e^{-\theta v} - 1) = e^{-\theta u}(e^{-\theta v} - 1)$$

$$w(e^{-\theta} - 1) = e^{-\theta u}(e^{-\theta v} - 1) - w(e^{-\theta u} - 1)(e^{-\theta v} - 1)$$

$$w(e^{-\theta} - 1) = (e^{-\theta u} - w(e^{-\theta u} - 1))(e^{-\theta v} - 1)$$

$$\frac{w(e^{-\theta} - 1)}{e^{-\theta u} - w(e^{-\theta u} - 1)} = e^{-\theta v} - 1$$

$$\left(1 + \frac{w(e^{-\theta} - 1)}{e^{-\theta u} - w(e^{-\theta u} - 1)}\right) = e^{-\theta v} \quad \Leftrightarrow \quad \ln\left(1 + \frac{w(e^{-\theta} - 1)}{e^{-\theta u} - w(e^{-\theta u} - 1)}\right) = -\theta v \theta$$

$$-\frac{1}{\theta}\ln\left(1 + \frac{w(e^{-\theta} - 1)}{e^{-\theta u} - w(e^{-\theta u} - 1)}\right) = v = F^{-1}_{v|u}(w)$$

FGM Copula:

$$C(u,v) = uv(1+\theta(1-u)(1-v))$$

$w = C^1(u,v) = F_{V|u}(v) \sim unif(0,1)$
$u \sim unif(0,1)$
$v = F^{-1}_{V|u}(w) \sim unif(0,1)$

$$w = \frac{d}{du}\left[uv + \theta(u-u^2)(v-v^2)\right]$$

$$w = v + \theta(1-2u)(v-v^2) \Leftrightarrow w - v = \theta(1-2u)(v-v^2)$$

$$\theta\left(\frac{w-v}{1-2u}\right) = v - v^2$$

$$\frac{w}{\theta(1-2u)} - \frac{v}{\theta(1-2u)} = v - v^2$$

$$\frac{w}{\theta(1-2u)} = v\left(1 + \frac{1}{\theta(1-2u)}\right) - v^2$$

$-1 \le \theta \le 1$
$0 \le w \le 1 \quad 0 \le v \le 1$
$0 \le u \le 1$

$$0 = v^2 + v\left(1 + \frac{1}{\theta(1-2u)}\right) - \frac{w}{\theta(1-2u)}$$

$$\frac{-\left(1 + \frac{1}{\theta(1-2u)}\right) \pm \sqrt{\left(1+\frac{1}{\theta(1-2u)}\right)^2 + 4\frac{w}{\theta(1-2u)}}}{2}$$

$$\frac{-\left(\frac{\theta(1-2u)+1}{\theta(1-2u)}\right) \pm \sqrt{\left(\frac{\theta(1-2u)+1}{\theta(1-2u)}\right)^2 + \frac{4w}{\theta(1-2u)}}}{2}$$

$$v = \begin{cases} \dfrac{-\left(\frac{\theta(1-2u)+1}{\theta(1-2u)}\right) + \sqrt{\left(\frac{\theta(1-2u)+1}{\theta(1-2u)}\right)^2 + \frac{4w}{\theta(1-2u)}}}{2} & si \end{cases}$$