Solución Parcial 1 Señales y Sistemas

a) Sea la distancia media entre 2 señales periodicas $x_1(t)\in\mathbb{R},\mathbb{C}$ y $x_2(t)\in\mathbb{R},\mathbb{C}$ definida como:

$$d^2(x_1,x_2)=\bar{P}_{x_1-x_2}=\lim_{T\to\infty}\frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\,dt$$ con $x_1(t)=Ae^{j\omega_0 t}$ y $x_2(t)=Be^{j5\omega_0 t}$

con $w_0=\frac{2\pi}{T}$ con $T, A, B\in\mathbb{R}^+$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\frac{1}{T}\int_T |Ae^{jw_0t}-Be^{j5w_0t}|^2\,dt$$ recordemos que: $|x(t)|^2=x(t)\,x^*(t)$

luego $$d^2(x_1,x_2)=\lim_{T\to\infty}\frac{1}{T}\int_T (Ae^{jw_0t}-Be^{j5w_0t})(Ae^{jw_0t^*}-Be^{j5w_0t^*})\,dt$$ el operador conjugado es lineal por ende $(A-B)(A-B)^*=(A-B)(A^*-B^*)$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\frac{1}{T}\int_T [A^2e^{jw_0t}e^{jw_0t^*}-ABe^{jw_0t}e^{j5w_0t^*}-BAe^{j5w_0t}e^{jw_0t^*}+B^2e^{j5w_0t}e^{j5w_0t^*}]\,dt$$

$$-Ae^{jw_0t}Be^{j5w_0t^*}-Be^{j5w_0t}Ae^{jw_0t^*}=-x_1(t)x_2^*(t)-x_2(t)x_1^*(t)=-2x_1(t)x_2^*(t)$$

Por tanto: $$d^2(x_1,x_2)=\lim_{T\to\infty}\frac{1}{T}\int_T (A^2e^{jw_0t}e^{jw_0t^*}-2ABe^{jw_0t}e^{j5w_0t^*}+B^2e^{j5w_0t}e^{j5w_0t^*})\,dt$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\frac{1}{T}\int_T A^2e^{jw_0t}e^{jw_0t^*}dt-\frac{1}{T}\int_T 2ABe^{jw_0t}e^{j5w_0t^*}dt+\frac{1}{T}\int_T B^2e^{j5w_0t}e^{j5w_0t^*}dt$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\frac{A^2}{T}\int_T e^{jw_0t}e^{-jw_0t}dt-\frac{2AB}{T}\int_T e^{jw_0t}e^{-j5w_0t}dt+\frac{B^2}{T}\int_T e^{j5w_0t}e^{-j5w_0t}dt$$

$$d^2(x_1,x_2)\doteq\lim_{T\to\infty}\frac{A^2}{T}\int_T e^{jw_0t-jw_0t}dt-\frac{2AB}{T}\int_T e^{jw_0t-j5w_0t}dt+\frac{B^2}{T}\int_T e^{j5w_0t-j5w_0t}dt$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\frac{A^2}{T}\int_T \cancel{e^0}^{1}dt-\frac{2AB}{T}\int_T e^{-j4w_0t}dt+\frac{B^2}{T}\int_T \cancel{e^0}^{1}dt$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\frac{A^2}{T}\int_T 1\,dt-\frac{2AB}{T}\int_T e^{-j4w_0t}dt+\frac{B^2}{T}\int_T 1\,dt$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\frac{A^2}{T}t\Big|_0^T-\frac{2AB}{T}\frac{e^{-j4w_0t}}{-j4w_0}\Big|_0^T+\frac{B^2}{T}t\Big|_0^T$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\left[\left(\frac{A^2}{\cancel{T}}\cancel{T}-\frac{A^2}{\cancel{T}}\cancel{(0)}^{0}\right)-\frac{2AB}{T}\left(\frac{e^{-j4w_0T}}{-j4w_0}+\frac{\cancel{e^{-j4w_0 0}}^{1}}{j4w_0}\right)+\left(\frac{B^2}{\cancel{T}}\cancel{T}-\cancel{\frac{B^2}{T}(0)}^{0}\right)\right]$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\left[A^2-\frac{2AB}{T}\left(\frac{\cos(4w_0T)-j\,\mathrm{Sen}(4w_0T)}{-j4w_0}+\frac{1}{j4w_0}\right)+B^2\right]$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\left[A^2-\frac{2AB}{T}\left(\frac{\cos(4\frac{2\pi}{\cancel{T}}\cancel{T})-j\,\mathrm{Sen}(4\frac{2\pi}{\cancel{T}}\cancel{T})}{-j4\frac{2\pi}{T}}+\frac{1}{j4\frac{2\pi}{T}}\right)+B^2\right]$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\left[A^2-\frac{2AB}{T}\left(\frac{\cancel{\cos(8\pi)}^{1}-j\,\cancel{\mathrm{Sen}(8\pi)}^{0}}{-j\frac{8\pi}{T}}+\frac{1}{j\frac{8\pi}{T}}\right)+B^2\right]$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\left[A^2-\frac{2AB}{T}\left(\frac{1}{-j\frac{8\pi}{T}}-\cancel{\frac{j0}{j\frac{8\pi}{T}}}^{0}+\frac{1}{j\frac{8\pi}{T}}\right)+B^2\right]$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\left[A^2-\frac{2AB}{T}\left[\frac{T}{j8\pi}-\frac{T}{j8\pi}\right]+B^2\right]$$

$$d^2(x_1,x_2)=\lim_{T\to\infty}\left[A^2-\cancel{\frac{2AB}{T}(0)}^{0}+B^2\right]$$

$$\boxed{d^2(x_1,x_2)=A^2+B^2}$$

b) Señal Continua

$$x_{(t)} = 3\cos(1000\pi t) + 5\operatorname{Sen}(2000\pi t) + 10\cos(11000\pi t) \qquad F_S = 5KH_z$$

revisamos primero que cumpla con la relación de Nyquist

$w_1 = 1000\pi\, rad/s \qquad F_1 = \frac{w_1}{2\pi} = \frac{1K\pi}{2\pi} = 500 \qquad$ Según Nyquist $F_S \geq 2\,Max(F_1, F_2, F_3)$

$w_2 = 2000\pi\, rad/s \qquad F_2 = \frac{w_2}{2\pi} = \frac{2K\pi}{2\pi} = 1000 \qquad F_S \geq 2F_3$

$w_3 = 11000\pi\, rad/s \qquad F_3 = \frac{w_3}{2\pi} = \frac{11K\pi}{2\pi} = 5500 \qquad 5000\,H_z \geq 2(5500)$

$5000\,H_z \geq 11000\,H_z$ falso, No cumple Nyquist

Realizando la discretización

para $t = \frac{n}{F_S}$ $\quad x\left(t = \frac{n}{F_S}\right) = 3\cos\left(1000\pi \frac{n}{F_S}\right) + 5\operatorname{Sen}\left(2000\pi \frac{n}{F_S}\right) + 10\cos\left(11000\pi \frac{n}{F_S}\right)$

$$x[n] = 3\cos\left[\frac{1000\pi}{5000} n\right] + 5\operatorname{Sen}\left[\frac{2000\pi n}{5000}\right] + 10\cos\left[\frac{11000\pi n}{5000}\right]$$

$$x[n] = 3\cos\left[\frac{1}{5}\pi n\right] + 5\operatorname{Sen}\left[\frac{2}{5}\pi n\right] + 10\cos\left[\frac{11}{5}\pi n\right]$$

* Ahora miramos el valor del omega max si está dentro del intervalo $[-\pi, \pi]$ para saber si es una Señal copia

$\Omega_3 = \frac{11\pi}{5} > \pi$ Por ende es un aliasing o Copia

$\Omega_{3ori} = \Omega_{3copia} - 2\pi = \frac{11}{5}\pi - 2\pi = \frac{11\pi}{5} - \frac{10\pi}{5} = \frac{\pi}{5} \longrightarrow \Omega_{original}$

$\Omega_{ori} = 2\pi f_{ori} \qquad f_{ori} = \frac{\Omega_{ori}}{2\pi} = \frac{\frac{\pi}{5}}{\frac{2\pi}{1}} = \frac{\pi}{10\pi} = \frac{1}{10}$ ahora cual es la frecuencia de muestreo adecuada?

$f_{ori} = \frac{F_{3ori}}{F_S} \qquad F_{3ori} = f_{ori} F_S = \frac{1}{10} 5000 = 500\,H_z$ $\quad F_{3ori}$ si cumple con NyQuist

luego la Nueva señal es: $x[n] = 3\cos\left[\frac{\pi}{5} n\right] + 5\operatorname{Sen}\left[\frac{2}{5}\pi n\right] + 10\cos\left[\frac{\pi}{5} n\right]$

donde resulta $x[n] = 13\cos\left[\frac{\pi}{5} n\right] + 5\operatorname{Sen}\left[\frac{2\pi}{5} n\right]$

c) Señal analoga corriente

$$x(t) = 20\cos\left(\frac{t}{3}\right) + 20\cos\left(\frac{t}{4}\right) \ [A]$$

hacemos la relación de frecuencias ω parahallar su respectivo periodo para la discretización.

$\frac{\omega_1}{\omega_2} = \frac{\frac{1}{3}}{\frac{1}{4}} = \frac{4}{3} \in Q$ por lo tanto existe periodo T para esta señal cuasiperiodica

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\frac{1}{3}} = 3(2\pi) = 6\pi$ $\quad T = T_1 \ell = T_2 K \quad K, \ell \in \mathbb{Z}$

$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\frac{1}{4}} = 8\pi$ $\quad T = 6\pi \ell = 8\pi K \quad MCM = 24$

$T = 24\pi \ [S]$

con ello se calcula la F_s con F_1 y F_2

$F_1 = \frac{\frac{1}{3}}{2\pi} = \frac{1}{6\pi}$ $\quad F_2 = \frac{\frac{1}{4}}{2\pi} = \frac{1}{8\pi}$ $\quad$ con $F_{1max} = \frac{1}{6\pi}$ $\quad F_s \geq 2\left(\frac{1}{6\pi}\right)$

$F_s \geq \frac{1}{3\pi}$