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1) Se tiene la señal $x(t) = 0{,}3\cos(1000\pi t - \frac{\pi}{4}) + 0{,}6\,\text{Sen}(2000\pi t) + 0{,}1\cos(11000\pi t - \pi)$

Se procede a hacer la discretización

$$x(t) = 0{,}3\cos(1000\pi t - \tfrac{\pi}{4}) + 0{,}6\,\text{Sen}(2000\pi t) + 0{,}1\cos(11000\pi t - \pi)$$

Usando la identidad: $\cos(a \pm b) = \cos(a)\cos(b) \mp \text{Sen}(a)\text{Sen}(b)$

- $0{,}3\cos(1000\pi t - \frac{\pi}{4}) = 0{,}3\left[\cos(1000\pi t)\cos(\frac{\pi}{4}) + \text{Sen}(1000\pi t)\,\text{Sen}(\frac{\pi}{4})\right]$

$$= 0{,}3\cos(1000\pi t)\underbrace{\cancel{\cos(\tfrac{\pi}{4})}}_{\frac{\sqrt{2}}{2}} + 0{,}3\,\text{Sen}(1000\pi t)\underbrace{\cancel{\text{Sen}(\tfrac{\pi}{4})}}_{\frac{\sqrt{2}}{2}}$$

$$0{,}3\cos(1000\pi t - \tfrac{\pi}{4}) = \frac{0{,}3\sqrt{2}}{2}\cos(1000\pi t) + \frac{0{,}3\sqrt{2}}{2}\,\text{Sen}(1000\pi t)$$

- $0{,}1\cos(11000\pi t - \pi) = 0{,}1\left[\cos(11000\pi t)\underbrace{\cancel{\cos(\pi)}}_{-1} + \text{Sen}(11000\pi t)\underbrace{\cancel{\text{Sen}(\pi)}}_{0}\right]$

$$0{,}1\cos(11000\pi t - \pi) = -0{,}1\cos(11000\pi t)$$

Luego mi $x(t)$ será:

$$x(t) = \frac{3}{10}\frac{\sqrt{2}}{2}\cos(1000\pi t) + \frac{3}{10}\frac{\sqrt{2}}{2}\text{Sen}(1000\pi t) + 0{,}6\,\text{Sen}(2000\pi t) - 0{,}1\cos(11000\pi t)$$

ahora empezamos la discretización $t = nT_s$ con $T_s = \frac{1}{F_s}$ $F_s = 5\text{KHz}$

$$x(t = n/F_s) = \frac{3\sqrt{2}}{20}\cos\left[\frac{1000\pi n}{5000}\right] + \frac{3\sqrt{2}}{20}\text{Sen}\left[\frac{1000\pi}{5000}n\right] + 0{,}6\,\text{Sen}\left[\frac{2000\pi}{5000}n\right] - 0{,}1\cos\left[\frac{11000\pi}{5000}n\right]$$

$$x[n] = \frac{3\sqrt{2}}{20}\cos\left[\frac{\pi}{5}n\right] + \frac{3\sqrt{2}}{20}\text{Sen}\left[\frac{\pi}{5}n\right] + 0{,}6\,\text{Sen}\left[\frac{2\pi}{5}n\right] - 0{,}1\cos\left[\frac{11\pi}{5}n\right]$$

Forma expandida usando identidades ↑

Forma compacta: $x[n] = 0{,}3\cos\left[\frac{\pi}{5}n^{-\pi/4}\right] + 0{,}6\,\text{Sen}\left[\frac{2\pi}{5}n\right] + 0{,}1\cos\left[\frac{11\pi}{5}n - \pi\right]$

Ahora como nuestra señal está discretizada revisaremos que la Frecuencia de muestreo cumpla con Nyquist.

$\omega_1 = 1000\pi$ con $\omega = 2\pi f$ $F = \frac{\omega}{2\pi}$

$\omega_2 = 2000\pi$

$\omega_3 = 11000\pi$

$F_1 = \frac{1000\cancel{\pi}}{2\cancel{\pi}} = 500\,\text{Hz}$

$F_2 = \frac{2000\cancel{\pi}}{2\cancel{\pi}} = 1000\,\text{Hz}$

$F_3 = \frac{11000\cancel{\pi}}{2\cancel{\pi}} = 5500\,\text{Hz}$

$F_{Nq} \geq 2F_{max}$

$F_s \geq 2F_3$

$5000 \geq 2(5500)$

$5000 \ngeq 11000$

No Cumple con Nyquist

Por ende como $\Omega_2 = \frac{2\pi}{5}$ y $\Omega_1 = \frac{\pi}{5}$ No son copias

pero $\Omega_3 = \frac{11\pi}{5} > [-\pi, \pi] \longrightarrow \Omega_3$ Copia   luego la frecuencia original

$$\Omega_{3or1} = \frac{11\pi}{5} - 2\pi = \frac{11\pi}{5} - \frac{10}{5}\pi = \frac{\pi}{5} \qquad f_{or1} = \frac{\Omega_{3or1}}{2\pi} = \frac{\frac{\cancel{\pi}}{5}}{2\cancel{\pi}} = \frac{1}{10}$$