

## Solución Parcial 1 Señales y Sistemas

a) Sea la distancia media entre 2 señales periódicas  $x_1(t) \in \mathbb{R}, \mathbb{C}$  y  $x_2(t) \in \mathbb{R}, \mathbb{C}$  definida como:

$$d^2(x_1, x_2) = \overline{|x_1 - x_2|^2} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt \quad \text{con } x_1(t) = A e^{j\omega_0 t}, x_2(t) = B e^{j\omega_0 t}$$

$$\text{y } \omega_0 = \frac{2\pi}{T} \quad \text{con } T, A, B \in \mathbb{R}^+$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T |A e^{j\omega_0 t} - B e^{j\omega_0 t}|^2 dt \quad \text{recordemos que: } |x(t)|^2 = x(t) x^*(t)$$

$$\text{luego } d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T (A e^{j\omega_0 t} - B e^{j\omega_0 t})(A e^{j\omega_0 t} - B e^{j\omega_0 t})^* dt \quad \text{el operador conjugado es lineal por ende}$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T (A e^{j\omega_0 t} - B e^{j\omega_0 t})(A e^{j\omega_0 t} - B e^{j\omega_0 t})^* dt$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T [A^2 e^{j\omega_0 t} e^{j\omega_0 t^*} - A B e^{j\omega_0 t} e^{j\omega_0 t^*} - B A e^{j\omega_0 t} e^{j\omega_0 t^*} + B^2 e^{j\omega_0 t} e^{j\omega_0 t^*}] dt$$

$$- A e^{j\omega_0 t} e^{j\omega_0 t^*} - B e^{j\omega_0 t} e^{j\omega_0 t^*} = -x_1(t) x_2^*(t) - x_2(t) x_1^*(t) = -2x_1(t) x_2^*(t)$$

$$\text{por tanto: } d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T (A^2 e^{j\omega_0 t} e^{j\omega_0 t^*} - 2AB e^{j\omega_0 t} e^{j\omega_0 t^*} + B^2 e^{j\omega_0 t} e^{j\omega_0 t^*}) dt$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T A^2 e^{j\omega_0 t} e^{j\omega_0 t^*} dt - \frac{1}{T} \int_T 2AB e^{j\omega_0 t} e^{j\omega_0 t^*} dt + \frac{1}{T} \int_T B^2 e^{j\omega_0 t} e^{j\omega_0 t^*} dt$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{A^2}{T} \int_T e^{j\omega_0 t} e^{-j\omega_0 t} dt - \frac{2AB}{T} \int_T e^{j\omega_0 t} e^{-j\omega_0 t} dt + \frac{B^2}{T} \int_T e^{j\omega_0 t} e^{-j\omega_0 t} dt$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{A^2}{T} \int_T e^{j\omega_0 t - j\omega_0 t} dt - \frac{2AB}{T} \int_T e^{j\omega_0 t - j\omega_0 t} dt + \frac{B^2}{T} \int_T e^{j\omega_0 t - j\omega_0 t} dt$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{A^2}{T} \int_T e^{0} dt - \frac{2AB}{T} \int_T e^{-j4\omega_0 t} dt + \frac{B^2}{T} \int_T e^{0} dt$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{A^2}{T} \int_T 1 dt - \frac{2AB}{T} \int_T e^{-j4\omega_0 t} dt + \frac{B^2}{T} \int_T 1 dt$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{A^2}{T} t \Big|_0^T - \frac{2AB}{T} \frac{e^{-j4\omega_0 t}}{-j4\omega_0} \Big|_0^T + \frac{B^2}{T} t \Big|_0^T$$

$$d^2(x_1, x_2) = \lim_{T \rightarrow \infty} \frac{A^2}{T} \Big|_0^T + \frac{2AB}{T} \frac{e^{-j4\omega_0 t}}{j4\omega_0} \Big|_0^T + \frac{B^2}{T} \Big|_0^T$$

b) Señal Continua

$$X(t) = 3\cos(1000\pi t) + 5\sin(2000\pi t) + 10\cos(11000\pi t) \quad F_s = 5\text{ KHz}$$

Revisemos primero que cumpla con la relación de Nyquist

$$\begin{aligned} \omega_1 &= 1000\pi \text{ rad/s} & F_1 &= \frac{\omega_1}{2\pi} = \frac{1\text{KR}}{2\pi} = 500 & \text{Según Nyquist } F_s &\geq 2\text{Max}(F_1, F_2, F_3) \\ \omega_2 &= 2000\pi \text{ rad/s} & F_2 &= \frac{\omega_2}{2\pi} = \frac{2\text{KR}}{2\pi} = 1000 & F_s &\geq 2F_2 \\ \omega_3 &= 11000\pi \text{ rad/s} & F_3 &= \frac{\omega_3}{2\pi} = \frac{11\text{KR}}{2\pi} = 5500 & 5000\text{ Hz} &\geq 2(5500) \\ & & & & 5000\text{ Hz} &\geq 11000\text{ Hz} \quad \text{falso, No cumple Nyquist} \end{aligned}$$

Realizando la discretización

$$\text{para } t = \frac{n}{F_s} \quad X(t = \frac{n}{F_s}) = 3\cos\left(\frac{1000\pi n}{F_s}\right) + 5\sin\left(\frac{2000\pi n}{F_s}\right) + 10\cos\left(\frac{11000\pi n}{F_s}\right)$$

$$X[n] = 3\cos\left[\frac{1000\pi n}{5000}\right] + 5\sin\left[\frac{2000\pi n}{5000}\right] + 10\cos\left[\frac{11000\pi n}{5000}\right]$$

$$X[n] = 3\cos\left[\frac{1}{5}\pi n\right] + 5\sin\left[\frac{2}{5}\pi n\right] + 10\cos\left[\frac{11}{5}\pi n\right]$$

\* Ahora miramos el valor del omega max si está dentro del intervalo  $[-\pi, \pi]$  para saber si es una Señal copia

$$\Omega_3 = \frac{11\pi}{5} > \pi \quad \text{Por ende es un aliasing o copia}$$

$$\Omega_{3on} = \Omega_{3copia} - 2\pi = \frac{11\pi}{5} - 2\pi = \frac{11\pi}{5} - \frac{10\pi}{5} = \frac{\pi}{5} \rightarrow \Omega_{original}$$

$$\Omega_{on} = 2\pi f_{on} \quad f_{on} = \frac{\Omega_{on}}{2\pi} = \frac{\frac{\pi}{5}}{2\pi} = \frac{1}{10} = \frac{1}{10} \quad \text{ahora cual es la frecuencia de muestreo adecuada?}$$

$$f_{on} = \frac{F_{3on}}{F_s} \quad F_{3on} = f_{on} F_s = \frac{1}{10} 5000 = 500\text{ Hz} \quad F_{3on} \text{ si cumple con Nyquist}$$

$$\text{luego la Nueva Señal es: } X[n] = 3\cos\left[\frac{\pi}{5}n\right] + 5\sin\left[\frac{2}{5}\pi n\right] + 10\cos\left[\frac{\pi}{5}n\right]$$

$$\text{donde resulta } X[n] = 13\cos\left[\frac{\pi}{5}n\right] + 5\sin\left[\frac{2}{5}\pi n\right]$$

c) Señal analógica corriente

$$x(t) = 20 \cos\left(\frac{t}{3}\right) + 20 \cos\left(\frac{t}{4}\right) \text{ [A]}$$

hacemos la relación de frecuencias  $\omega$  para hallar su respectivo periodo para la discretización.

$\frac{\omega_1}{\omega_2} = \frac{\frac{1}{3}}{\frac{1}{4}} = \frac{4}{3} \in \mathbb{Q}$  por lo tanto existe periodo  $T$  para esta señal cuasiperiódica

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\frac{1}{3}} = 3(2\pi) = 6\pi \quad T = T_1 \cdot l = T_2 \cdot k \quad k, l \in \mathbb{Z}$$

$$T = 6\pi \cdot l = 8\pi \cdot k \quad \text{MCM} = 24$$

$$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\frac{1}{4}} = 8\pi$$

$$T = 24\pi \text{ [s]}$$

con ello se calcula la  $F_s$  con  $F_1$  y  $F_2$

$$F_1 = \frac{\frac{1}{3}}{2\pi} = \frac{1}{6\pi}$$

$$F_2 = \frac{\frac{1}{4}}{2\pi} = \frac{1}{8\pi}$$

$$\text{con } F_{\max} = \frac{1}{6\pi}$$

$$F_s \geq 2\left(\frac{1}{6\pi}\right)$$

$$F_s \geq \frac{1}{3\pi}$$