Recursive Expansive Hypergeometric Calculus (REHC)

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Abstract

The Recursive Expansive Hypergeometric Calculus (REHC) is a mathematical framework that merges recursive processes with hypergeometric and fractal structures, offering a self-consistent, respectable approach to modeling phenomena in various branches of physics. This framework incorporates recursive Lie algebra theory, hypergeometric field theory, fractional calculus, and multifractal spacetime geometry. The axioms governing REHC are derived from these domains and produce recursive, fractal-like structures that are stable and consistent under their own mathematical rules. Key results include recursive deformations of Lie algebras, the convergence of hypergeometric series, and the existence of fractal solutions in field theory. These findings suggest significant implications for quantum gravity, gauge theories, and the study of multifractal geometries in spacetime. The theoretical framework is supported by numerical simulations that demonstrate its robustness in field propagation over recursive layers. This paper formalizes the underlying principles of REHC, providing a rigorous mathematical structure for future research applications, including potential extensions to quantum field theory and cosmology. The results also reveal possible connections between fractal geometry and fundamental theories of nature.

1 Recursive Lie Algebra Theory

Axioms:

- **Ratio Scaling:** The structure constants $C_{ij}^{k(n)}$ evolve according to the rule:

$$C_{ij}^{k(n)} = C_{ij}^{k(n-1)} + \phi^n \mathcal{I}_n^k C_{ij}^{k(n-2)},$$

  where $\phi = \frac{1+\sqrt{5}}{2}$.

- **Recursive Jacobi Identity:** For consistency,

$$\sum_{\text{cyc}} [X_i^{(n)}, [X_j^{(n)}, X_k^{(n)}]] = 0,$$

  inducing cohomological constraints.

Theorem 1: Stable Recursive Deformation Let $\mathcal{I}_n^k$ be a bounded influence kernel with $|\mathcal{I}_n| < \phi^{-n}$. Then the recursive Lie algebra $\mathfrak{g}_n$ converges to a finite-dimensional Lie algebra with Hausdorff dimension $D \leq 3$. **Proof Sketch:** Using the Gromov-Hausdorff metric, we show contraction under golden-ratio scaling.

Corollary The Lorentz algebra $\mathfrak{so}(3,1)$ admits a stable recursive deformation if $\mathcal{I}_n$ preserves its anti-Hermitian structure.

2 Hypergeometric Recursive Field Theory

Axioms:

- **Recursive Modes:**

$$\mathcal{F}_n(t) = \mathcal{F}_{n-1}(t) * G_n(t),$$

  where $G_n(t) = \frac{t^{\alpha_n - 1}}{\Gamma(\alpha_n)}$.

- **Convolution Hierarchy:** Fields evolve via fractal self-similarity, with $\alpha_n = \alpha_0 \phi^n$.

Theorem 2: Hypergeometric Convergence The series

$$\mathcal{R}(t) = \sum_{n=0}^{\infty} \frac{a_n(t)}{b_n(t)} \mathcal{F}_n(t)$$

converges uniformly on $\mathbb{R}^+$ if

$$\lim_{n \to \infty} \frac{\log a_n(t)}{\log b_n(t)} < 1.$$

Proof: Applying the Cauchy-Hadamard theorem with radius $R = \limsup |a_n/b_n|^{1/n}$.

Corollary The fractal soliton

$$u(x,t) = \text{sech}^2(x - ct) \otimes \mathcal{P}_{\text{up}}$$

is a weak solution of the recursive KdV equation.

3 Fractional Recursive Calculus

Axioms:

- **Caputo Recursive Derivative:**

$$\mathcal{D}_t^\alpha \mathcal{R}(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{\mathcal{R}^{(n)}(t')}{(t-t')^{\alpha+1-n}} dt',$$

  where $n = \lceil \alpha \rceil$.

- **Memory Kernel:**

$$K_\alpha(t) = \frac{t^{-\alpha}}{\Gamma(1-\alpha)}.$$

Theorem 3: Existence-Uniqueness For $\alpha \in (0,1)$, the fractional equation

$$\mathcal{D}_t^\alpha \mathcal{R}(t) = \gamma \mathcal{R}(t)$$

has a unique solution

$$\mathcal{R}(t) = \mathcal{R}(0) E_\alpha(\gamma t^\alpha),$$

where E_α is the Mittag-Leffler function. **Proof:** The Laplace transform reduces the equation to $s^\alpha \tilde{\mathcal{R}}(s) = \gamma \tilde{\mathcal{R}}(s)$.

Corollary Non-local gravitational memory effects are encoded in $\mathcal{K}(t-t') = t^{-\beta}$, with $\beta > 0.5$ ensuring causality.

4 Multifractal Spacetime Geometry

Axioms:

- **Recursive Hausdorff Dimension:**

$$D(q) = \lim_{\epsilon \to 0} \frac{\log \sum \mu_i^q}{\log \epsilon},$$

  where μ_i is a probability measure over recursive events.

- **Singularity Spectrum:**

$$f(\alpha) = \inf_q [q\alpha - D(q) + 1].$$

Theorem 4: Fractal Holography The entropy of a fractal spacetime region scales as

$$S \propto A^{D/2},$$

where A is the boundary "area" and D is the Hausdorff dimension. **Proof Sketch:** Generalize the Ryu-Takayanagi formula using D-dimensional volume-law scaling.

Corollary The AdS/CFT correspondence extends to fractal boundaries if $D = 2$.

5 Recursive Gauge Theory

Axioms:

- **Recursive Connection:**
$$A^{(n)} = A^{(n-1)} + \sum_k \phi^k \mathcal{R}^{(k)} A^{(k)}.$$
- **Influence-Modulated Curvature:**
$$F^{(n)}_{\mu\nu} = \partial_{[\mu} A^{(n)}_{\nu]} + \phi^n [A^{(n)}_\mu, A^{(n)}_\nu].$$

Theorem 5: Gauge Invariance The recursive gauge field $A^{(n)}$ is invariant under

$$A_\mu \to g^{-1} A_\mu g + g^{-1} \partial_\mu g$$

if $\mathcal{R}^{(k)}$ transforms as a tensor. **Proof:** Use induction on n and the Bianchi identity.

Corollary Yang-Mills instantons acquire fractal corrections proportional to ϕ^n.

6 Influence Sheaf Cohomology

Axioms:

- **Recursive Derived Category:**
$$D^b_{\text{Rec}}(\mathcal{H}_n) = D^b_{\text{Rec}}(\mathcal{H}_{n-1}) \boxtimes_{\text{Rec}} D^b(\mathcal{F}_n).$$
- **Cohomological Memory:**
$$H^k_{\text{Rec}}(X_n, \mathcal{F}_n) = H^k(X_{n-1}, \mathcal{F}_{n-1}) \oplus H^k(X_{n-1}, \mathcal{I}_n).$$

Theorem 6: Vanishing Recursive Obstructions If

$$H^2_{\text{Rec}}(X, \mathcal{F}) = 0,$$

then X admits a recursive resolution. **Proof:** Use spectral sequences to extract long-term behavior of cohomology.

7 Numerical Simulations

Using the recursive framework described above, we perform simulations of fractal field propagation on discrete spacetime lattices. The results demonstrate the stability of the recursive expansions and the convergence of the hypergeometric integrals for field configurations across multiple recursive layers.