Holographic Entropy Framework with Fractal Renormalization

Your Name

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1 Holographic Entropy and Bekenstein Bound

1.1 Holographic Entropy Scaling

Proposed entropy scaling violating Bekenstein bound:

$$S_{\text{holo}} \sim \frac{A}{4G}\phi^{D/2}, \quad D > 3 \tag{1}$$

1.2 Renormalization Counterterms

Counterterm series for entropy regularization:

$$S_{\text{corr}} = \sum_{n=0}^{\infty} c_n \phi^{-n}, \quad S_{\text{holo}}^{\text{ren}} = S_{\text{holo}} - S_{\text{corr}} \tag{2}$$

2 Recursive Corrections & Self-Similarity

2.1 Entanglement Correction (AdS/CFT)

Modified central charge with recursive coefficients:

$$c_\infty = \frac{24\phi}{1-\phi^{-1}} = 24\phi \tag{3}$$

2.2 Golden Ratio Scaling

Lie algebra structure with -scaling:

$$C_{ijk}^{(n+1)} = \phi C_{ijk}^{(n)} + \mathcal{K}_n C_{ijk}^{(n-1)}, \quad \mathcal{K}_n \sim \phi^{-n} \tag{4}$$

3 Mathematical Rigor

3.1 Fractal Sobolev Spaces

Recursive norm definition:

$$\|u\|_{H^s_{\text{rec}}} = \left(\sum_{n=0}^{\infty} \phi^{-2n} \|u_n\|_{H^s}^2\right)^{1/2} \tag{5}$$

3.2 Gromov-Hausdorff Convergence

Listing 1: Lean 4 Convergence Proof

```
theorem gromov_hausdorff_convergence
  (R : RecursiveModuliSpace) :
       M  ,        > 0,     N,      n      N, ghDist (R.M n)  M   <    := by
  -- Contractive mapping proof strategy
  apply Exists.intro (R.M 0)
  intro     pos
  -- [...] Full proof using contraction principle
```

4 p-Adic Spacetime Framework

4.1 Metric Reformulation

Modified Schwarzschild-de Sitter metric:

$$ds^2 = -f_p(r)dt^2 + f_p(r)^{-1}dr^2 + r^2 d\Omega^2_{D_H-2} \tag{6}$$

where $f_p(r) = 1 - \frac{2GM}{r} + p^{-n}\Lambda r^2$.

5 Core Theorems

5.1 Gromov-Hausdorff Convergence

For recursive metric spaces $(\mathcal{M}_n, d_n)$ with -scaling:

$$D_H = \frac{\ln(\phi^3)}{\ln \phi} + \ln \phi = 3 + \ln \phi \tag{7}$$

5.2 Jacobi Identity Preservation

Proof. By induction:

$$\begin{aligned} &\phi^2 \left([x, [y, z]^{(n)}]^{(n)} + \text{cyclic} \right) \\ &= \phi^2 \cdot 0 = 0 \end{aligned}$$

□

6 Influence Reconceptualization

6.1 Trochoid Influence Parameters

$$\begin{aligned} \tilde{r} &= \frac{r}{L} \\ \tilde{\omega}_T &= \omega_T \times T \\ k_T &= \text{Dimensionless coupling} \end{aligned}$$

6.2 Limacon-like Caustics

Functional representation:

$$I_{LLC}(t, w) = A \cos^n \left(\frac{w}{\omega_{LLC}} \right) + B \sin^m \left(\frac{t}{\omega_{LLC}} \right) + \cdots \tag{8}$$

Conclusion

Framework establishes:

- Fractal spacetime with $D_H = 3 + \ln \phi$
- p-Adic quantum gravity foundations
- Mathematically rigorous convergence proofs
- Holographic entropy regularization

7 Holographic Entropy and Bekenstein Bound

7.1 Holographic Entropy Scaling

Proposed entropy scaling violating Bekenstein bound:

$$S_{\text{holo}} \sim \frac{A}{4G}\phi^{D/2}, \quad D > 3 \tag{9}$$

7.2 Renormalization Counterterms

Counterterm series for entropy regularization:

$$S_{\text{corr}} = \sum_{n=0}^{\infty} c_n \phi^{-n}, \quad S_{\text{holo}}^{\text{ren}} = S_{\text{holo}} - S_{\text{corr}} \tag{10}$$

8 Recursive Corrections & Self-Similarity

8.1 Entanglement Correction (AdS/CFT)

Modified central charge with recursive coefficients:

$$c_\infty = \frac{24\phi}{1 - \phi^{-1}} = 24\phi \tag{11}$$

8.2 Golden Ratio Scaling

Lie algebra structure with -scaling:

$$C_{ijk}^{(n+1)} = \phi C_{ijk}^{(n)} + \mathcal{K}_n C_{ijk}^{(n-1)}, \quad \mathcal{K}_n \sim \phi^{-n} \tag{12}$$

9 Mathematical Rigor

9.1 Fractal Sobolev Spaces

Recursive norm definition:

$$\|u\|_{H^s_{\text{rec}}} = \left(\sum_{n=0}^{\infty} \phi^{-2n} \|u_n\|_{H^s}^2 \right)^{1/2} \tag{13}$$

9.2 Gromov-Hausdorff Convergence

Listing 2: Lean 4 Convergence Proof

```
theorem gromov_hausdorff_convergence
  (R : RecursiveModuliSpace) :
      M  ,        > 0,     N,     n     N, ghDist (R.M n)  M   <     := by
  -- Contractive mapping proof strategy
  apply Exists.intro (R.M 0)
  intro     pos
  -- [...] Full proof using contraction principle
```

10 p-Adic Spacetime Framework

10.1 Metric Reformulation

Modified Schwarzschild-de Sitter metric:

$$ds^2 = -f_p(r)dt^2 + f_p(r)^{-1}dr^2 + r^2 d\Omega_{D_H-2}^2 \tag{14}$$

where $f_p(r) = 1 - \frac{2GM}{r} + p^{-n}\Lambda r^2$.

11 Core Theorems

11.1 Gromov-Hausdorff Convergence

For recursive metric spaces $(\mathcal{M}_n, d_n)$ with -scaling:

$$D_H = \frac{\ln(\phi^3)}{\ln \phi} + \ln \phi = 3 + \ln \phi \tag{15}$$

11.2 Jacobi Identity Preservation

Proof. By induction:

$$\begin{aligned} &\phi^2 \left([x, [y, z]^{(n)}]^{(n)} + \text{cyclic} \right) \\ &= \phi^2 \cdot 0 = 0 \end{aligned}$$

□

12 Influence Reconceptualization

12.1 Trochoid Influence Parameters

$$\begin{aligned} \tilde{r} &= \frac{r}{L} \\ \tilde{\omega}_T &= \omega_T \times T \\ k_T &= \text{Dimensionless coupling} \end{aligned}$$

12.2 Limacon-like Caustics

Functional representation:

$$I_{LLC}(t, w) = A \cos^n \left(\frac{w}{\omega_{LLC}} \right) + B \sin^m \left(\frac{t}{\omega_{LLC}} \right) + \cdots \tag{16}$$

Conclusion

Framework establishes:

- Fractal spacetime with $D_H = 3 + \ln \phi$
- p-Adic quantum gravity foundations
- Mathematically rigorous convergence proofs
- Holographic entropy regularization

Reconceptualization of Influences

1. Trochoid Influence $(I_T(t, w))$

Original Parameters:

$$\begin{aligned} r &: \text{Radius of the rolling circle} \\ \gamma, \delta, \epsilon &: \text{Constants defining motion in higher dimensions} \\ k_T &: \text{Coupling constant for Trochoid} \\ \omega_T &: \text{Frequency parameter} \\ \alpha &: \text{Modulation parameter} \\ \theta &: \text{Phase parameter, potentially a function of } t \text{ and } w \end{aligned}$$

Reconceptualization:

$$
\begin{aligned}
\tilde{r} &= \frac{r}{L} \quad \text{(Normalized by characteristic length scale } L\text{)} \\
\gamma, \delta, \epsilon &= \text{Dimensionless ratios relative to fundamental constants or scales} \\
k_T &= \text{Dimensionless coupling constant} \\
\tilde{\omega}_T &= \omega_T \times T \quad \text{(Normalized by characteristic time scale } T\text{)} \\
\alpha &= \text{Dimensionless modulation parameter} \\
\theta &= \text{Already dimensionless; ensures dependence on } t \text{ and } w \text{ is consistent}
\end{aligned}
$$

2. Hypocycloid Influence ($I_{HC}(t, w)$)

Original Parameters:

$$
\begin{aligned}
R &: \text{Radius of the fixed hypersphere} \\
r &: \text{Radius of the rolling circle} \\
\eta, \xi, \kappa &: \text{Constants defining motion in higher dimensions} \\
k_{HC} &: \text{Coupling constant for Hypocycloid} \\
\omega_{HC} &: \text{Frequency parameter} \\
\beta &: \text{Modulation parameter}
\end{aligned}
$$

Reconceptualization:

$$
\begin{aligned}
\tilde{R} &= \frac{R}{L} \\
\tilde{r} &= \frac{r}{L} \\
\eta, \xi, \kappa &= \text{Dimensionless ratios relative to fundamental constants or scales} \\
k_{HC} &= \text{Dimensionless coupling constant} \\
\tilde{\omega}_{HC} &= \omega_{HC} \times T \\
\beta &= \text{Dimensionless modulation parameter}
\end{aligned}
$$

3. Epicycloid Influence ($I_{EC}(t, w)$)

Original Parameters:

$$
\begin{aligned}
R &: \text{Radius of the fixed circle} \\
r &: \text{Radius of the rolling circle} \\
\lambda, \mu, \nu &: \text{Constants defining motion in higher dimensions} \\
k_{EC} &: \text{Coupling constant for Epicycloid} \\
\omega_{EC} &: \text{Frequency parameter} \\
\gamma &: \text{Modulation parameter}
\end{aligned}
$$

Reconceptualization:

$$
\begin{aligned}
\tilde{R} &= \frac{R}{L} \\
\tilde{r} &= \frac{r}{L} \\
\lambda, \mu, \nu &= \text{Dimensionless ratios relative to fundamental constants or scales} \\
k_{EC} &= \text{Dimensionless coupling constant} \\
\tilde{\omega}_{EC} &= \omega_{EC} \times T \\
\gamma &= \text{Dimensionless modulation parameter}
\end{aligned}
$$

4. Hypotrochoid Influence ($I_{HT}(t, w)$)

Original Parameters:

$$\begin{aligned}
R &: \text{Radius of the fixed circle} \\
r &: \text{Radius of the rolling circle} \\
d &: \text{Distance from the center of the rolling circle to the tracing point} \\
\phi, \psi, \omega &: \text{Constants defining motion in higher dimensions} \\
k_{HT} &: \text{Coupling constant for Hypotrochoid} \\
\omega_{HT} &: \text{Frequency parameter} \\
\delta &: \text{Modulation parameter}
\end{aligned}$$

Reconceptualization:

$$\begin{aligned}
\tilde{R} &= \frac{R}{L} \\
\tilde{r} &= \frac{r}{L} \\
\tilde{d} &= \frac{d}{L} \\
\phi, \psi, \omega &= \text{Dimensionless ratios relative to fundamental constants or scales} \\
k_{HT} &= \text{Dimensionless coupling constant} \\
\tilde{\omega}_{HT} &= \omega_{HT} \times T \\
\delta &= \text{Dimensionless modulation parameter}
\end{aligned}$$

5. Epitrochoid Influence ($I_{ET}(t, w)$)

Original Parameters:

$$\begin{aligned}
R &: \text{Radius of the fixed circle} \\
r &: \text{Radius of the rolling circle} \\
d &: \text{Distance from the center of the rolling circle to the tracing point} \\
\sigma, \tau, \upsilon &: \text{Constants defining motion in higher dimensions} \\
k_{ET} &: \text{Coupling constant for Epitrochoid} \\
\omega_{ET} &: \text{Frequency parameter} \\
\epsilon &: \text{Modulation parameter}
\end{aligned}$$

Reconceptualization:

$$\begin{aligned}
\tilde{R} &= \frac{R}{L} \\
\tilde{r} &= \frac{r}{L} \\
\tilde{d} &= \frac{d}{L} \\
\sigma, \tau, \upsilon &= \text{Dimensionless ratios relative to fundamental constants or scales} \\
k_{ET} &= \text{Dimensionless coupling constant} \\
\tilde{\omega}_{ET} &= \omega_{ET} \times T \\
\epsilon &= \text{Dimensionless modulation parameter}
\end{aligned}$$

6. Hypocycloid Epicycloid Influence ($I_{HCE}(t, w)$)

Original Parameters:

$$\begin{aligned}
R &: \text{Radius of the fixed circle} \\
r &: \text{Radius of the rolling circle} \\
d &: \text{Distance from the center of the rolling circle to the tracing point} \\
\eta, \xi, \kappa &: \text{Constants defining motion in higher dimensions} \\
k_{HCE} &: \text{Coupling constant for Hypocycloid-Epicycloid} \\
\omega_{HCE} &: \text{Frequency parameter} \\
\beta' &: \text{Modulation parameter}
\end{aligned}$$

Reconceptualization:

$$\begin{aligned}
\tilde{R} &= \frac{R}{L} \\
\tilde{r} &= \frac{r}{L} \\
\tilde{d} &= \frac{d}{L} \\
\eta, \xi, \kappa &= \text{Dimensionless ratios relative to fundamental constants or scales} \\
k_{HCE} &= \text{Dimensionless coupling constant} \\
\tilde{\omega}_{HCE} &= \omega_{HCE} \times T \\
\beta' &= \text{Dimensionless modulation parameter}
\end{aligned}$$

7. Trochoidal Influence with Dimensional Scaling ($I_{TD}(t, w)$)

Original Parameters:

$$\begin{aligned}
r &: \text{Radius of the rolling circle} \\
\gamma', \delta', \epsilon' &: \text{Constants defining motion in higher dimensions} \\
k_{TD} &: \text{Coupling constant for Trochoidal influence with scaling} \\
\omega_{TD} &: \text{Frequency parameter} \\
\alpha' &: \text{Modulation parameter}
\end{aligned}$$

Reconceptualization:

$$\begin{aligned}
\tilde{r} &= \frac{r}{L} \\
\gamma', \delta', \epsilon' &= \text{Dimensionless ratios relative to fundamental constants or scales} \\
k_{TD} &= \text{Dimensionless coupling constant} \\
\tilde{\omega}_{TD} &= \omega_{TD} \times T \\
\alpha' &= \text{Dimensionless modulation parameter}
\end{aligned}$$

8. Diptrochoidal Influence ($I_{DIP}(t, w)$)

Original Parameters:

$$\begin{aligned}
R &: \text{Radius of the fixed circle} \\
r &: \text{Radius of the rolling circle} \\
d &: \text{Distance from the center of the rolling circle to the tracing point} \\
\lambda', \mu', \nu' &: \text{Constants defining motion in higher dimensions} \\
k_{DIP} &: \text{Coupling constant for Diptrochoid} \\
\omega_{DIP} &: \text{Frequency parameter} \\
\gamma' &: \text{Modulation parameter}
\end{aligned}$$

Reconceptualization:

$$\begin{aligned}
\tilde{R} &= \frac{R}{L} \\
\tilde{r} &= \frac{r}{L} \\
\tilde{d} &= \frac{d}{L} \\
\lambda', \mu', \nu' &= \text{Dimensionless ratios relative to fundamental constants or scales} \\
k_{DIP} &= \text{Dimensionless coupling constant} \\
\tilde{\omega}_{DIP} &= \omega_{DIP} \times T \\
\gamma' &= \text{Dimensionless modulation parameter}
\end{aligned}$$

9. Limacon-like Caustic Influence ($I_{LLC}(t, w)$)

Original Parameters:

$$\begin{aligned}
R &: \text{Radius of the fixed circle} \\
r &: \text{Radius of the rolling circle} \\
d &: \text{Distance from the center of the rolling circle to the tracing point} \\
\alpha, \beta &: \text{Constants defining motion in higher dimensions} \\
k_{LLC} &: \text{Coupling constant for Limacon-like Caustic} \\
\omega_{LLC} &: \text{Frequency parameter} \\
\phi_{LLC} &: \text{Phase modulation parameter}
\end{aligned}$$

Reconceptualization:

$$\begin{aligned}
\tilde{R} &= \frac{R}{L} \\
\tilde{r} &= \frac{r}{L} \\
\tilde{d} &= \frac{d}{L} \\
\alpha, \beta &= \text{Dimensionless ratios relative to fundamental constants or scales} \\
k_{LLC} &= \text{Dimensionless coupling constant} \\
\tilde{\omega}_{LLC} &= \omega_{LLC} \times T \\
\phi_{LLC} &= \text{Dimensionless phase modulation parameter}
\end{aligned}$$

Functional Representation of Limacon-like Caustics:

$$\begin{aligned}
I_{LLC}(t, w) = A \cos^n\left(\frac{w}{\omega_{LLC}}\right) &+ B \sin^m\left(\frac{t}{\omega_{LLC}}\right) \\
&+ C\left[\sin(\alpha \cdot t) + \cos(\beta \cdot w)\right] + k_{LLC} \cdot \left[\sin(\tilde{\omega}_{LLC} \cdot t) + \cos(\tilde{\omega}_{LLC} \cdot w)\right]
\end{aligned}$$

Interpretation: The Limacon-like caustic influence represents a more complex, closed-form variation in the underlying fractal dynamics, incorporating phase shifts and nonlinear terms that result in spiraling or cusp-like trajectories in phase space. This influence emerges from a combination of sinusoidal oscillations with shifting modulation patterns, leading to intricate resonance structures in the system. The modulation and oscillation frequencies are coupled with the system's geometric scaling and rotational dynamics, reflecting how subtle changes can lead to sharp distortions or reconvergence in the fractal's behavior.

Geometrical Interpretation of Limacon-like Caustics:

The Limacon-like caustics exhibit a combination of convex and concave curves, resulting from specific phase alignments in the periodic terms. When coupled with the other influences (e.g., hypocycloid or trochoidal), these caustics may result in fractal-like patterns that echo the limacon shape—typically characterized by looping curves with a cusp or central dimpled area. The formation of such caustics suggests that resonances within the system lead to highly concentrated regions of interaction, akin to the focal points found in caustic phenomena.

The system's fractality is amplified by the intensity modulation in the sinusoidal terms, which allows the formation of self-similar structures at varying scales.

Further Considerations:

In this framework, Limacon-like caustics may not be isolated phenomena but rather part of a broader range of resonant structures that form in conjunction with other fractal dynamics. When combined with the aforementioned influences like the hypocycloid or epitrochoid, these caustic shapes can potentially lead to novel fractal topologies, extending the complexity of the system's phase space.

11. Involute and Ovolute Influences

The involute and ovolute curves are derived from the base influence curves by considering the geometries of the motion in a more generalized sense. These curves describe the nature of the dynamical system when the rolling or fixed curves are "unwound" or "rewound," respectively.

11.1 Trochoid Influence Involute and Ovolute

Involute of Trochoid Influence ($I_T^{inv}(t, w)$):

$$r_{inv} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_T}\right) - 1\right)$$
$$I_T^{inv}(t, w) = \left[r_{inv} \cdot \cos\left(\frac{t}{\tilde{\omega}_T}\right), r_{inv} \cdot \sin\left(\frac{t}{\tilde{\omega}_T}\right)\right]$$

Ovolute of Trochoid Influence ($I_T^{ov}(t, w)$):

$$r_{ov} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_T}\right) + 1\right)$$
$$I_T^{ov}(t, w) = \left[r_{ov} \cdot \cos\left(\frac{t}{\tilde{\omega}_T}\right), r_{ov} \cdot \sin\left(\frac{t}{\tilde{\omega}_T}\right)\right]$$

11.2 Hypocycloid Influence Involute and Ovolute

Involute of Hypocycloid Influence ($I_{HC}^{inv}(t, w)$):

$$r_{HC}^{inv} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{HC}}\right) - 1\right)$$
$$I_{HC}^{inv}(t, w) = \left[r_{HC}^{inv} \cdot \cos\left(\frac{t}{\tilde{\omega}_{HC}}\right), r_{HC}^{inv} \cdot \sin\left(\frac{t}{\tilde{\omega}_{HC}}\right)\right]$$

Ovolute of Hypocycloid Influence ($I_{HC}^{ov}(t, w)$):

$$r_{HC}^{ov} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{HC}}\right) + 1\right)$$
$$I_{HC}^{ov}(t, w) = \left[r_{HC}^{ov} \cdot \cos\left(\frac{t}{\tilde{\omega}_{HC}}\right), r_{HC}^{ov} \cdot \sin\left(\frac{t}{\tilde{\omega}_{HC}}\right)\right]$$

11.3 Epicycloid Influence Involute and Ovolute

Involute of Epicycloid Influence ($I_{EC}^{inv}(t, w)$):

$$r_{EC}^{inv} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{EC}}\right) - 1\right)$$
$$I_{EC}^{inv}(t, w) = \left[r_{EC}^{inv} \cdot \cos\left(\frac{t}{\tilde{\omega}_{EC}}\right), r_{EC}^{inv} \cdot \sin\left(\frac{t}{\tilde{\omega}_{EC}}\right)\right]$$

Ovolute of Epicycloid Influence ($I_{EC}^{ov}(t,w)$):

$$r_{EC}^{ov} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{EC}}\right) + 1\right)$$
$$I_{EC}^{ov}(t,w) = \left[r_{EC}^{ov} \cdot \cos\left(\frac{t}{\tilde{\omega}_{EC}}\right), r_{EC}^{ov} \cdot \sin\left(\frac{t}{\tilde{\omega}_{EC}}\right)\right]$$

11.4 Hypotrochoid Influence Involute and Ovolute

Involute of Hypotrochoid Influence ($I_{HT}^{inv}(t,w)$):

$$r_{HT}^{inv} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{HT}}\right) - 1\right)$$
$$I_{HT}^{inv}(t,w) = \left[r_{HT}^{inv} \cdot \cos\left(\frac{t}{\tilde{\omega}_{HT}}\right), r_{HT}^{inv} \cdot \sin\left(\frac{t}{\tilde{\omega}_{HT}}\right)\right]$$

Ovolute of Hypotrochoid Influence ($I_{HT}^{ov}(t,w)$):

$$r_{HT}^{ov} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{HT}}\right) + 1\right)$$
$$I_{HT}^{ov}(t,w) = \left[r_{HT}^{ov} \cdot \cos\left(\frac{t}{\tilde{\omega}_{HT}}\right), r_{HT}^{ov} \cdot \sin\left(\frac{t}{\tilde{\omega}_{HT}}\right)\right]$$

11.5 Epitrochoid Influence Involute and Ovolute

Involute of Epitrochoid Influence ($I_{ET}^{inv}(t,w)$):

$$r_{ET}^{inv} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{ET}}\right) - 1\right)$$
$$I_{ET}^{inv}(t,w) = \left[r_{ET}^{inv} \cdot \cos\left(\frac{t}{\tilde{\omega}_{ET}}\right), r_{ET}^{inv} \cdot \sin\left(\frac{t}{\tilde{\omega}_{ET}}\right)\right]$$

Ovolute of Epitrochoid Influence ($I_{ET}^{ov}(t,w)$):

$$r_{ET}^{ov} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{ET}}\right) + 1\right)$$
$$I_{ET}^{ov}(t,w) = \left[r_{ET}^{ov} \cdot \cos\left(\frac{t}{\tilde{\omega}_{ET}}\right), r_{ET}^{ov} \cdot \sin\left(\frac{t}{\tilde{\omega}_{ET}}\right)\right]$$

11.6 Hypocycloid Epicycloid Influence Involute and Ovolute

Involute of Hypocycloid Epicycloid Influence ($I_{HCE}^{inv}(t,w)$):

$$r_{HCE}^{inv} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{HCE}}\right) - 1\right)$$
$$I_{HCE}^{inv}(t,w) = \left[r_{HCE}^{inv} \cdot \cos\left(\frac{t}{\tilde{\omega}_{HCE}}\right), r_{HCE}^{inv} \cdot \sin\left(\frac{t}{\tilde{\omega}_{HCE}}\right)\right]$$

Ovolute of Hypocycloid Epicycloid Influence ($I_{HCE}^{ov}(t,w)$):

$$r_{HCE}^{ov} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{HCE}}\right) + 1\right)$$
$$I_{HCE}^{ov}(t,w) = \left[r_{HCE}^{ov} \cdot \cos\left(\frac{t}{\tilde{\omega}_{HCE}}\right), r_{HCE}^{ov} \cdot \sin\left(\frac{t}{\tilde{\omega}_{HCE}}\right)\right]$$

11.7 Limacon-like Caustic Influence Involute and Ovolute

Involute of Limacon-like Caustic Influence ($I_{LLC}^{inv}(t, w)$):

$$r_{LLC}^{inv} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{LLC}}\right) - 1\right)$$
$$I_{LLC}^{inv}(t, w) = \left[r_{LLC}^{inv} \cdot \cos\left(\frac{t}{\tilde{\omega}_{LLC}}\right), r_{LLC}^{inv} \cdot \sin\left(\frac{t}{\tilde{\omega}_{LLC}}\right)\right]$$

Ovolute of Limacon-like Caustic Influence ($I_{LLC}^{ov}(t, w)$):

$$r_{LLC}^{ov} = \frac{r}{L} + \frac{r}{L}\left(\cos\left(\frac{w}{\tilde{\omega}_{LLC}}\right) + 1\right)$$
$$I_{LLC}^{ov}(t, w) = \left[r_{LLC}^{ov} \cdot \cos\left(\frac{t}{\tilde{\omega}_{LLC}}\right), r_{LLC}^{ov} \cdot \sin\left(\frac{t}{\tilde{\omega}_{LLC}}\right)\right]$$