HyperGeometric Continuous Calculus

Julian Del Bel

January 22, 2025

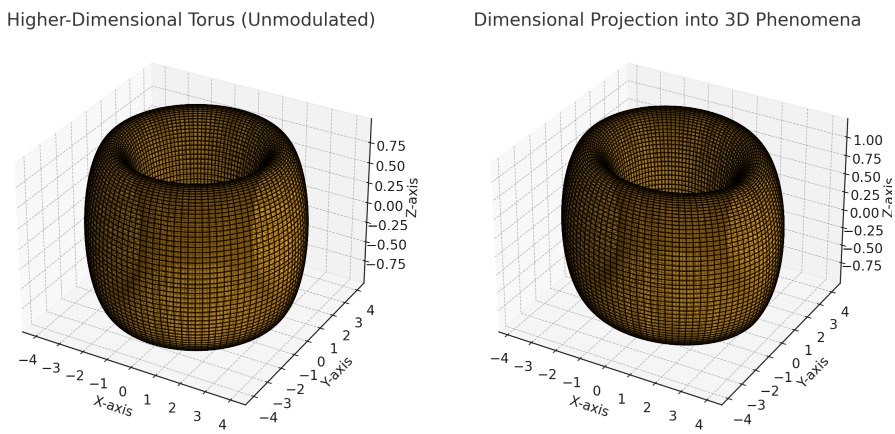
Abstract

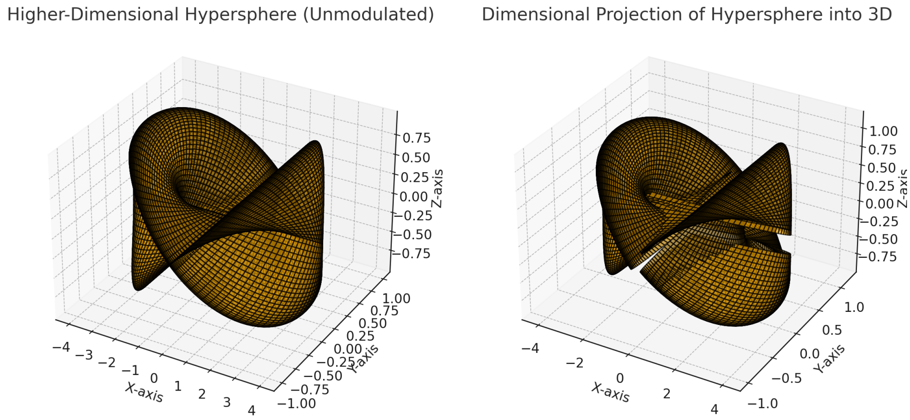
Hypergeometrical Continuous Calculus (HGCC) is an advanced framework that redefines the traditional notion of continuous calculus by integrating concepts from recursion, holography, nonlocality, and scale-dependent dynamics. While classical calculus focuses on the evolution of fields or systems based on immediate, local interactions, HGCC introduces a deeper, more interconnected view of the universe, where the evolution of any given point in spacetime is influenced not only by its current state but also by past and future events. In HGCC, influence, energy, or information can propagate recursively through spacetime, with the system's evolution dependent on both local and nonlocal influences. This framework allows for a more comprehensive understanding of complex phenomena that cannot be fully captured by traditional calculus.

The key strength of HGCC lies in its ability to model systems where time, space, and causality behave in a nontraditional manner, such as in quantum entanglement, gravitational wave propagation, and the cosmological evolution of the system. These phenomena often exhibit behaviors that span vast temporal and spatial scales, with recursive feedback loops and nonlocal interactions, challenging the assumptions of classical models. HGCC provides the necessary mathematical tools to describe these complex systems, introducing concepts like recursive influence fields, holographic memory, and retrocausality to account for the intricate interdependence of events across spacetime.

0.1 Additional Versors

The structure of HGCC allows understanding of influence dynamics, integrating both local interactions and nonlocal influences, while accounting for the scale-dependent nature of spacetime geometry. The combination of these elements enables HGCC to extend the reach of traditional calculus, offering new avenues for understanding and modeling phenomena at the intersection of quantum mechanics, general relativity, and cosmology.





1 Core Features of HGCC

1.1 Recursion

At the heart of HGCC lies the concept of recursion. In traditional calculus, the evolution of a field or system is determined by its local state and interactions at any given point in spacetime. In HGCC, however, the influence at a point is determined not only by its current condition but also by past and future states of the system. This introduces a recursive structure in which each point in spacetime is affected by a feedback loop that spans across time and space. The recursive nature of the system means that the present state of the system is shaped by an infinite cycle of influences that both propagate forward in time and loop back upon themselves, creating an interconnected lattice of events.

This recursive interaction allows for self-similarity at various scales and underpins much of the structure of the HGCC framework. The behavior of influence is no longer described by a single trajectory, but instead by a system of intertwined cycles, in which each event or influence can be both a cause and an effect, depending on its temporal and spatial context. The recursive structure enables the modeling of systems where retrocausality (the influence of future events on the present) is naturally integrated into the framework, a feature that classical calculus cannot accommodate. In essence, recursion introduces a cyclical, non-linear evolution of influence that reflects the inherent complexity of natural processes.

1.2 Holography

In HGCC, holography plays a central role in defining the relationship between local and global phenomena across spacetime. The holographic principle suggests that the entire history of influences within a system is encoded within every part of the system, creating a self-referential, interconnected structure. This means that, in HGCC, each part of the spacetime continuum—whether small or large, local or global—contains the information of the whole. This holistic view of spacetime allows for the representation of the system's dynamics in a unified structure, where influences propagate not only locally but are also reflected and stored across all scales.

The concept of holographic memory is introduced, where each point in spacetime retains a record of past interactions, encoding information about previous, present, and even future states. This memory is stored in recursive cycles and is essential for modeling nonlocal influences that may not be immediately adjacent in space or time but still have a significant effect on the system's evolution. In this sense, holography in HGCC is not just a passive recording of past events, but an active encoding mechanism that facilitates the propagation of influence across time and space. This memory structure underpins the recursive influence field, ensuring that the system's evolution remains interconnected and that past and future interactions are not treated as separate, isolated events.

1.3 Nonlocality

A foundational concept in HGCC is nonlocality, which challenges the classical notion of influence being confined to adjacent or immediate points in spacetime. In HGCC, influence at any given point (x, t)

can be influenced by events that are not only separated spatially but also temporally. This nonlocal interaction is a crucial feature that distinguishes HGCC from traditional models, enabling the framework to account for phenomena like quantum entanglement, where distant particles instantaneously affect each other, and gravitational waves, which propagate across the lattice of spacetime, influencing distant regions.

In the context of HGCC, nonlocality is represented by the recursive influence kernel, which integrates over influences from not just the immediate surroundings but from past and future events. This kernel enables the modeling of systems where the evolution of a point in spacetime is influenced by both local sources and distant, nonlocal events. This is particularly important for phenomena that span vast distances and timescales, such as the propagation of gravitational waves through the cosmos, or the interdependence of quantum particles in an entangled state.

By allowing influences to propagate across vast distances in spacetime and even across time itself, HGCC provides a comprehensive framework for modeling the entangled nature of the system, where local events are never truly isolated but are in constant communication with other parts of the system.

1.4 Scale-Dependent Dynamics

In HGCC, the behavior of influence across spacetime is scale-dependent, meaning that the way influence propagates or decays is not uniform across all scales. This feature is modeled using exotic logarithmic functions, such as the POG (Pi-based logarithm) and PHOG (Phi-based logarithm), which capture the attenuation or amplification of influence based on the geometrical structure of spacetime. These functions allow for the modeling of fractal-like behavior, where influence may amplify at certain scales or attenuate at others, reflecting the recursive and cyclical nature of spacetime.

For instance, at certain scales, influence may become self-similar, reflecting the influence of larger structures or cycles in the system. These scaling behaviors are essential for modeling systems with fractal or recursive geometries, where events at different scales are connected through the recursive influence structure. The nonuniform scaling of influence reflects the inherent complexity and interconnectedness of the system, allowing HGCC to accurately describe phenomena that involve feedback loops, such as those found in cosmological evolution or quantum dynamics.

The scale-dependent dynamics in HGCC provide a tool capturing the multi-scale nature of physical phenomena, enabling the modeling of systems where influence behaves differently depending on the spatial or temporal scale under consideration.

1.5 Recursive Influence Field

In Hypergeometrical Continuous Calculus (HGCC), the influence field $\mathcal{I}(x,t)$ at a point (x,t) is a dynamic quantity that evolves over spacetime in response to both local and nonlocal interactions. The governing equation for the evolution of the influence field takes into account recursive feedback loops, meaning that the influence at any given point depends not only on the present state but also on past and future events, encapsulating both retrocausality and nonlocality in its formulation.

The equation governing the evolution of the influence field is given by:

$$\frac{\partial \mathcal{I}(x,t)}{\partial t} = \mathcal{F}(x,t) + \int_S \mathcal{R}(x',t')\,\mathrm{d}S(x',t')$$

where:

- $\mathcal{I}(x,t)$ is the influence field at point (x,t), representing the total influence experienced by the system at this specific point in spacetime.
- $\mathcal{F}(x,t)$ is the local source term, representing contributions from immediate, local interactions at the point (x,t). This could correspond to, for example, local gravitational sources or quantum emitters, which affect the influence field directly at the location.
- $\mathcal{R}(x',t')$ is the recursive influence kernel, which captures influences that propagate from both past and future states of the system. This kernel introduces the concept of recursive interactions, where

the influence at any point (x, t) depends on the influences from other points across spacetime, including nonlocal effects from retrocausal and cyclic contributions.

The recursive nature of the influence field arises from the inclusion of the term $\int_S \mathcal{R}(x', t')\, \mathrm{d}S(x', t')$, which represents an integral over the influence of past and future states. This integral encapsulates the holographic memory of the spacetime system, meaning that influences from past states and events, as well as those that will occur in the future, are recorded and used to influence the system at every point in spacetime. This is not a conventional instantaneous response but a continuous, recursive influence that creates a feedback loop across spacetime, which is characteristic of HGCC.

The recursive influence kernel $\mathcal{R}(x', t')$ is the key element that differentiates this calculus from conventional models. It represents a generalized Green's function that accounts for influences that span across time and space, not only from current sources but also from previous or future sources, thereby creating a nonlocal interaction. This recursive component of the equation introduces a memory effect, which is crucial for modeling phenomena like retrocausality, where future events can influence the present state, or holographic effects, where the entirety of spacetime is interconnected through recursive cycles.

A critical aspect of the recursive influence field is its incorporation of nonlocality and retrocausality. The influence at point (x, t) is not solely determined by the local interactions or sources at that point but is influenced by events in the past and future, as encoded in the recursive influence kernel $\mathcal{R}(x', t')$.

1. Nonlocality: The recursive kernel $\mathcal{R}(x', t')$ integrates over a region S of spacetime, meaning that the influence felt at (x, t) depends on events at distant points (x', t'), even if they are not directly adjacent to (x, t). This is a manifestation of spacetime entanglement, where the interconnectedness of spacetime geometry ensures that distant points can influence one another, even through recursive loops.

2. Retrocausality: The recursive influence kernel also implies that events in the future may have a direct influence on the present. As the system evolves, the feedback loop created by the recursive interactions means that future states contribute to the ongoing dynamics, forming a causal loop that operates both forwards and backwards in time.

The presence of both nonlocal and retrocausal influences in the recursive equation leads to an enriched mathematical structure that mirrors the complex, interconnected nature of spacetime. It reflects the idea that local events cannot be fully understood without accounting for the influence of distant or even future events, and that time itself may not be as linear or unidirectional as traditionally assumed.

The evolution of the influence field is governed by the dynamics encoded in both the local source term $\mathcal{F}(x, t)$ and the recursive influence kernel $\mathcal{R}(x', t')$. The solution to the equation provides a way to compute how the influence at each point in spacetime evolves, taking into account not only the immediate effects of local sources but also the ongoing, recursive propagation of influences through past and future states.

The combination of these terms gives rise to a recursive evolution of the influence field, which incorporates both causal and noncausal components. This recursive framework allows for the modeling of complex systems, such as:

- Gravitational waves propagate across spacetime in a manner that is influenced not only by the current gravitational sources but also by past events and potentially future states of the system.
- Quantum fields exhibit recursive behaviors, where particles interact not only with present quantum states but also with past and future quantum fluctuations.
- The large-scale structure of the universe is shaped by recursive processes that involve interactions between matter and energy at various points in the cosmos, spanning vast temporal and spatial scales.

It evolves according to both local source terms and recursive feedback from past and future states, effectively modeling retrocausality and nonlocality within the spacetime lattice. The recursive feedback loops ensure that the system's evolution is intertwined with its past and future, providing a framework to model complex, recursive, and interconnected systems across different scales and times.

1.6 Exotic Logarithmic Functions

HGCC utilizes two special logarithmic functions—POG (Pi-based Logarithm) and PHOG (Phi-based Logarithm)—to describe the scale-dependent dynamics of influence.

1.6.1 PHOG (Phi-based Logarithm)

HGCC utilizes two special logarithmic functions—POG (Pi-based Logarithm) and PHOG (Phi-based Logarithm)—to describe the scale-dependent dynamics of influence.

The PHOG function is a key component in modeling scale-dependent dynamics in Hypergeometrical Continuous Calculus (HGCC). It is defined as:

$$\text{PHOG}_b(x) = \log_b(x), \quad b = \phi \cdot \kappa$$

where:

- b is the base of the logarithm, which incorporates the golden ratio ϕ and a scaling factor κ.
- κ is a dynamic scaling factor that adjusts based on epicykloidal influences, which are linked to the recursive cycles in the system's geometry and its fractal-like nature.

1.6.2 The Golden Ratio ϕ and Its Role in Scaling

The inclusion of the golden ratio ϕ in the base of the logarithmic function introduces a profound self-similarity property that is foundational to the structure of fractal systems. The golden ratio $\phi = \frac{1+\sqrt{5}}{2}$ is renowned for its appearance in natural patterns, geometric structures, and recursive phenomena. In the context of the PHOG function, ϕ serves to impart a resonant quality to the scaling behavior of influence propagation across spacetime, especially as the system exhibits recursive cycles.

The golden ratio governs the scaling behavior of influence by ensuring that the dynamics at different scales are proportionally consistent with each other. This introduces a level of fractality into the system, where the influence at one scale mirrors that at another, allowing for the self-similar propagation of effects. This behavior is particularly useful when modeling systems with intricate recursive cycles or geometries that repeat at different scales, such as those governed by epicykloidal dynamics.

1.6.3 The Scaling Factor κ and Epicykloidal Influences

The scaling factor κ is crucial in determining how influence is modified across different recursive cycles. It adjusts dynamically based on epicykloidal influences, which are tied to the recursive nature of the system and reflect the influence of cyclic interactions at various scales.

The epicykloidal geometry in HGCC introduces recursive cycles that affect the propagation of influence across spacetime. These cycles are influenced by a combination of both local interactions and larger, cosmic-scale dynamics, where smaller-scale interactions are recursively embedded within larger cycles. The scaling factor κ acts as a modulator, adjusting how influence is amplified or attenuated depending on the scale of the recursion, which allows for a more nuanced representation of recursive, cyclical behavior across spacetime.

By incorporating the golden ratio ϕ and scaling factor κ, the PHOG function governs the attenuation or amplification of influence in a way that accounts for both local and nonlocal recursive dynamics. The result is a system that naturally exhibits fractal-like, scale-invariant properties, making the PHOG function particularly well-suited for modeling systems where self-similarity and resonance are key features.

1.6.4 Self-Similarity and Resonance

The PHOG function's incorporation of ϕ introduces self-similarity in the influence dynamics, which means that the scaling of influence follows patterns that repeat at multiple scales. This fractal behavior is central to the way recursive influence is modeled in HGCC. The system exhibits resonance at different scales, meaning that specific points in spacetime may experience an amplification of influence due to constructive feedback from the recursive cycles.

This self-similar scaling is particularly relevant for modeling fractal structures, such as cosmic filaments, quantum fluctuations, or gravitational waves. At every recursive cycle, the system's influence field maintains a consistent scaling pattern, where each level of recursion has a similar form, but potentially with modified intensity, governed by the scaling factor κ.

The resonance created by the golden ratio also introduces a form of symmetry in the system's scaling. This symmetry is not only geometrically significant but also carries profound implications for the influence dynamics, where specific scales resonate and create feedback loops that amplify or attenuate the influence in particular regions of spacetime. This recursive resonance helps to explain phenomena where smaller-scale events can lead to large-scale consequences and vice versa.

1.6.5 Mathematical Implications of the PHOG Function

The PHOG function modifies the logarithmic behavior of influence propagation, introducing scale-dependent attenuation or amplification. The mathematical formulation:

$$\mathrm{PHOG}_b(x) = \log_{\phi\cdot\kappa}(x)$$

defines the relationship between the influence x and its scale $b = \phi \cdot \kappa$. By adjusting the base b dynamically through the scaling factor κ, the function accounts for the influence of both local and global recursive cycles, thus enabling the propagation of influence in a manner that is consistent with the recursive and self-similar structure of spacetime.

The behavior of the PHOG function can be characterized by its scaling properties:

- Attenuation: As x increases, the influence weakens logarithmically. This is governed by the recursive cycles and the self-similarity encoded by ϕ.
- Amplification: At certain scales, the influence may be amplified, due to constructive feedback from recursive resonance. This amplification occurs when the influence reaches resonant points in the recursive cycles.
- Fractal Dynamics: The function's scaling behavior reflects the fractal nature of the system, where influence behaves similarly at different scales, but with varying intensity.

1.6.6 Applications in Recursive Systems

The PHOG function's role in modeling recursive, fractal-like systems is invaluable in HGCC. It is especially well-suited for scenarios where influence propagates through a system that exhibits recursive cycles, such as:

- Gravitational Wave Propagation: In systems like gravitational waves, where influence propagates through spacetime in recursive cycles, the PHOG function provides a means to model the scaling of wave amplitudes across different scales.
- Quantum Field Theory: The self-similar and resonant properties of the PHOG function make it an ideal candidate for modeling influence propagation in quantum fields, where interactions occur at multiple scales and involve recursive cycles.
- Cosmological Models: In large-scale cosmological systems, such as the formation of galaxies or the distribution of dark matter, the PHOG function can model how influences across different scales interact and resonate to shape the structure of the universe.

1.6.7 Summary

The PHOG function introduces a powerful tool for modeling scale-dependent influence in HGCC. By utilizing the golden ratio ϕ and the dynamic scaling factor κ, it encapsulates the self-similarity and resonance inherent in recursive systems. This logarithmic function is fundamental for describing the propagation of influence in fractal-like structures, recursive cycles, and resonant systems, where the behavior at one scale can influence and reflect behavior at other scales. Through its ability to capture attenuation, amplification, and fractal dynamics, the PHOG function plays a critical role in the modeling of complex systems within the HGCC framework.

1.7 Holographic Memory and Ledger

In Hypergeometrical Continuous Calculus (HGCC), the concept of a holographic memory is integral to the recursive propagation of influence across spacetime. The ledger in HGCC serves as a dynamic, multi-dimensional repository that stores the record of influences as they traverse through hyperspherical spacetime. This ledger captures not only the past and present influences but also encodes future influences, which is crucial for maintaining the retrocausal and nonlocal properties of the theory. The ledger allows for a cohesive representation of spacetime interactions, which are not confined to local causes and effects but rather span across temporal and spatial dimensions in a recursive manner.

This holographic ledger is essential for the propagation of influence according to the recursive and nonlocal principles of HGCC. The ledger stores influence trajectories at various points in spacetime, incorporating influences that originate from past states as well as those that may affect the future. The ledger is therefore a unified structure where all influences coexist in a synchronized, multi-temporal fashion. It serves as the foundation for memory recall and the recursive nature of influence propagation, allowing influences to be traced and modified across recursive cycles.

The ledger's interaction with the recursive geometry of spacetime is encoded through the kernel projection operator $\mathcal{K}(x,t)$, which facilitates the tracing of influence over time and space, ensuring that every influence leaves a record that can be recalled and utilized to modify future states.

$$\mathcal{K}(x,t) = \int_S \Phi(x,t;x',t') \cdot G(x,t;x',t') \, \mathrm{d}S(x',t')$$

1.7.1 Components of the Projection Operator

The projection operator $\mathcal{K}(x,t)$ functions as the primary mechanism through which influence is propagated across spacetime and stored in the ledger. It combines two key components:

- $\Phi(x,t;x',t')$ is the influence kernel, which acts as a memory tracker. This kernel encodes the influence history of past interactions between spacetime points. It captures the recursive feedback loops inherent in the geometry of HGCC, which allows influences to be not only local but also extendable backward and forward in time. The influence kernel is responsible for maintaining the temporal coherence of the influence field, ensuring that previous events and future potentialities are both accounted for.
- $G(x,t;x',t')$ is the modified Green's function, which reflects the influence of spacetime curvature and holographic geometry. This function is adapted to the recursive nature of spacetime, ensuring that influences are propagated along the cykloid curves and modified by epicykloidal influences. The Green's function also serves to account for nonlocal interactions—that is, influences that propagate across spacetime in a non-contiguous manner, reflecting the retrocausal feedback loops that govern the system.

The integrand, $\Phi(x,t;x',t') \cdot G(x,t;x',t')$, represents the convolution of past interactions (encoded by the influence kernel) with the spacetime propagation dynamics (described by the Green's function). The result is a summation of all influences that have propagated through spacetime and are stored at each point (x,t), encapsulating both local and nonlocal contributions.

1.7.2 Recursive Nature of the Ledger

The recursive quality of the ledger lies in its ability to recurse through spacetime. At every point (x,t), the ledger contains the influence history not only of the past but also of the future—an essential feature for modeling retrocausality in HGCC. The recursive interaction between past, present, and future influences ensures that no influence is ever truly lost, but rather, it is continuously reincorporated into the ongoing evolution of the system. This recursive mechanism establishes an intricate feedback loop where each influence contributes to and shapes the future state of the system.

The recursive memory function is reflected in the following expression:

$$\mathcal{M}(x,t) = \mathcal{K}(x,t) + \int_S \mathcal{K}(x,t) \cdot \mathcal{K}(x',t') \, \mathrm{d}S(x',t')$$

where $\mathcal{M}(x,t)$ represents the memory at point (x,t), incorporating both past influences and recursively propagated future states. The recursive term $\mathcal{K}(x',t') \cdot \mathcal{K}(x,t)$ represents the feedback mechanism where the current state of influence is integrated with prior interactions to continuously adjust and modify the ledger's contents.

1.7.3 Nonlocality and Retrocausality in the Ledger

A critical aspect of HGCC is its nonlocal nature, which is inherently captured by the holographic ledger. The nonlocal property of the ledger arises from its ability to store spatially and temporally distant influences in a unified framework. This means that influences do not need to originate from immediately neighboring points in spacetime, but rather can be propagated from distant points across both space and time.

Retrocausality is another key feature embedded within the ledger. It ensures that future states can influence past events, in addition to the standard causal progression from past to future. This principle is fundamental to the recursive nature of HGCC, where the future is not entirely independent of the past but rather interacts with and shapes it in a feedback-driven manner.

Mathematically, retrocausality is encoded by the noncommutative nature of the ledger's memory, which enables influences to propagate both forwards and backwards in time. This is represented by the integral expression for the kernel projection operator $\mathcal{K}(x,t)$, where both past and future states are captured in a holistic manner.

1.7.4 Dynamic Update of the Ledger

The ledger in HGCC is not a static entity but rather undergoes continuous updates as new influences are introduced. As influence propagates through spacetime, the ledger dynamically updates the stored interactions, ensuring that new information is incorporated while preserving the recursive nature of influence propagation. This continuous updating mechanism is described by:

$$\mathcal{L}(x,t) = \mathcal{L}(x,t-1) + \mathcal{K}(x,t)$$

where $\mathcal{L}(x,t)$ represents the ledger's state at time t, incorporating the current influence, $\mathcal{K}(x,t)$, added to the prior state $\mathcal{L}(x,t-1)$.

1.7.5 Summary

The holographic ledger in HGCC plays a critical role in the theory's framework by providing a unified record of influences across spacetime. The ledger tracks influences recursively, encoding both past and future events, and ensures that nonlocal and retrocausal interactions are correctly modeled. The projection operator $\mathcal{K}(x,t)$ serves as the core mathematical tool for encoding memory, while the integration of the influence kernel $\Phi(x,t;x',t')$ and the Green's function $G(x,t;x',t')$ ensures that the propagation of influence respects the nonlocal and recursive properties of HGCC. The ledger's continuous update ensures that the system remains coherent and that influences from all points in spacetime contribute to the ongoing evolution of the system.

2 Applications of HGCC

HGCC is highly applicable to a wide range of advanced physical systems that exhibit recursion, nonlocality, and scale-dependent dynamics. Some of its potential applications include:

2.1 Quantum Mechanics

HGCC can model retrocausal effects and quantum entanglement, providing a mathematical framework for understanding time-symmetric quantum field theories and nonlocal interactions.

2.2 Cosmology

In cosmology, HGCC can be used to model gravitational wave echoes, cosmic inflation, and the propagation of dark matter. The recursive nature of HGCC allows for the modeling of cosmic-scale dynamics over vast timescales.

2.3 General Relativity

HGCC provides a framework for understanding the curvature of spacetime and its influence on gravitational waves, black hole dynamics, and spacetime singularities. The nonlocal and recursive interactions in HGCC are ideal for describing the complex geometry of general relativistic systems.

3 Bridge

Hypergeometrical Continuous Calculus represents an extension of traditional calculus, designed to model systems with recursive, holographic, scale-dependent, and nonlocal influences. Through the use of recursive influence fields, exotic logarithmic functions, and holographic memory, HGCC provides a framework for understanding complex systems in quantum mechanics, cosmology, and general relativity.

4 Geometrical Foundations of Curves in HGCC

To fully understand the mathematical framework of Hypergeometrical Continuous Calculus (HGCC), it is essential to establish the foundational geometrical concepts underpinning the theory. Central to this theory is the behavior and propagation of curves across a hyperspherical spacetime. These curves are not merely the standard Euclidean or Riemannian structures but reflect the intricate, recursive, and holographic properties of the space itself.

4.1 Curves in Spacetime and Recursive Cykloid Geometry

In HGCC, curves are modeled within a non-Euclidean, hyperspherical geometry that accounts for the cyclical nature of spacetime. These curves are defined not only by their local tangents but also by the recursive feedback loops that govern their evolution. The foundational geometry incorporates cykloid structures, which are curves generated by a point on the circumference of a circle rolling along a straight line, and epicykloidal structures, which arise when the rolling circle itself is moving along the perimeter of another circle. These recursive geometries lead to fractal-like curves, forming the basis of influence propagation across spacetime.

The curve $\gamma(t)$ representing an influence propagating through the hyperspherical spacetime is expressed as:

$$\gamma(t) = [x(t), y(t), z(t), \dots]$$

where t represents the temporal evolution of the curve, and $x(t), y(t), z(t), \dots$ are the spatial coordinates as the influence propagates. The recursive nature of the curve is governed by the relationship:

$$\gamma(t) = \gamma(t - \Delta t) + \delta\gamma(t)$$

where $\delta\gamma(t)$ is a small increment in the influence based on past or future states, reflecting recursion.

4.2 Cyclic and Epicyclic Curves

The influence curves in HGCC do not follow traditional geodesic paths; rather, they embody a cyclical nature that loops through spacetime. These curves are influenced by cyclical spacetime curvatures, described by cykloid and epicykloid geometries.

4.2.1 Cykloid Curves

A cykloid curve is the trajectory traced by a point on the circumference of a circle as it rolls along a straight line. Mathematically, the position of a point on the circle is given by:

$$x(t) = r\,(t - \sin(t))$$
$$y(t) = r\,(1 - \cos(t))$$

where r is the radius of the circle, and t represents the parameter corresponding to time or evolution. These curves are essential for modeling the propagation of influence through cyclical spacetime structures, as they reflect both local and global recursive interactions.

4.2.2 Epicykloid Curves

An epicykloid curve is a more complex curve traced by a point on a circle rolling along the outside of another fixed circle. The general parametric equations for an epicykloid are:

$$x(t) = (R + r)\cos(t) - r\cos\left(\frac{R + r}{r}t\right)$$
$$y(t) = (R + r)\sin(t) - r\sin\left(\frac{R + r}{r}t\right)$$

where R is the radius of the fixed circle, r is the radius of the rolling circle, and t represents the evolution of time. Epicykloidal curves introduce an additional layer of recursion by incorporating the rolling dynamics of two interacting cycles, allowing them to model more complex recursive and holographic interactions in spacetime.

4.3 Geodesic Curves in HGCC

In traditional geometry, geodesics represent the shortest paths between two points in a curved space. However, in HGCC, due to the recursive and nonlocal nature of spacetime, the concept of geodesics is modified. Instead of following a simple curve that minimizes local distance, geodesic curves in HGCC must account for the influence of both local curvature (hypotrochoidal effects) and global recursive cycles (epicykloidal effects).

The modified geodesic equation in HGCC can be expressed as:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = \mathcal{R}^\mu(x, t)$$

where:

- $\frac{d^2x^\mu}{d\tau^2}$ represents the second derivative of the position x^μ with respect to the spacetime parameter τ,
- $\Gamma^\mu_{\alpha\beta}$ are the Christoffel symbols, representing the spacetime curvature,
- $\mathcal{R}^\mu(x, t)$ is the recursive influence function that modifies the path to account for past and future influences.

This equation describes how influence propagates along a curve in a recursive, non-Euclidean spacetime where influences from past and future states affect the present path.

4.4 Influence Curves and Scaling

One of the most critical aspects of HGCC is the scale-dependent propagation of influence. The exotic logarithmic functions such as POG and PHOG govern how influence decays or amplifies as it propagates across different scales in spacetime.

The attenuation or amplification of influence along a curve is modeled by the following scaling function:

$$\mathcal{S}(x, t) = \log_b(x) \quad \text{where} \quad b = \pi \cdot \kappa \quad \text{or} \quad b = \phi \cdot \kappa$$

This logarithmic function encapsulates the relationship between the scale of influence and the geometric properties of the curve, accounting for both local and global scaling effects.

4.5 Curvature of Influence Curves

The curvature of influence curves in HGCC is influenced by the recursive, holographic, and nonlocal properties of spacetime. The curvature is defined as:

$$\mathcal{C} = \frac{d^2\gamma(t)}{dt^2} + \mathcal{R}(x, t)$$

where $\mathcal{R}(x, t)$ represents the recursive influence term, which modulates the curvature based on past and future states.

4.6 Summary

In HGCC, the fundamental geometrical objects—curves—are deeply intertwined with the recursive, holographic, and nonlocal properties of spacetime. Cykloid and epicykloid geometries provide the foundation for influence propagation, while the modified geodesic equation allows for a generalized understanding of influence curves under non-Euclidean conditions. The influence curves themselves are subject to scaling laws and curvature modulations dictated by the logarithmic functions of POG and PHOG, which enable the modeling of complex recursive dynamics across spacetime. These foundational geometrical concepts lay the groundwork for more advanced calculations and applications of HGCC.