Notes from How to Prove It

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28 July 2022

Preface

I wrote these notes principally FOR understanding, they are meant for future reference for a refresher on what I have learnt.
Honestly, I don't have much to say, I just thought that a preface would be cool. Let's start our journey now.

Contents

1 Sentential Logic

1.1 Deductive reasoning and logical connectives

In an *argument*, we arrive at **valid** *conclusions* assuming that the *premises* are **true**.
Premises and conclusiosn are often referred to as conditions and outcomes respectively.
If all the premises are **true**, then the conclusion should be **true**. However, for the case where the conclusion is **false** while the premises are **true**, the argument is **invalid**.

1.1.1 Logical Operators

Symbol	Meaning	Description
$\vee$	OR	Disjunction
$\wedge$	AND	Conjunction
$\neg$	NOT	Negation

1.2 Truth tables

A truth table must be able to represent all possible combinations of the variables, premises and conclusions.

P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

In this case, we see that our variables (or statements), P and Q have their individual column to assign a value – **True** or **False** – to them.
We use our logical operators to make a new statement from P and Q which is $P \wedge Q$, and assign a value to the new statement.

It is important note that the number of variables will dictate the number of rows that the truth table will have. Construct a truth table for the following set of variables, $\{P\}$, $\{P, Q\}$, $\{P, Q, R\}$.
The pattern that we find is that as the number of variables increases, the number of rows increases two-fold.

$$\text{Number of Rows} = 2^{\text{Number of Variables}}$$

There are some special truth tables where the column for the conclusion always has the same value (either all *true* or all *false*) for every combination of the variables' values.

When the conclusion is always *true*, we say that the conclusion's statement is a **tautologies**. Construct a truth table for $P \vee \neg P$.
Similarly, when the conclusion is always *false*, we say that the conclusion's statement is a **contradiction**. Construct a truth table for $P \wedge \neg P$.

Remark 1.1. *Tautologies* and *Contradictions* are not the only laws that govern logic. Do see the logic document for more.

1.3 Variables and sets

1.4 Operations on sets

1.5 The conditional and biconditional connectives