Algebra; A Brief Overview

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1 Groups

The integers under the operation of addition, $(\mathbb{Z}, +)$, are a key example of what is known as a *group*. A group consists of an underlying set, which in the case of the integers is $\mathbb{Z}$, coupled with an operation, addition $(+)$ in this case, which allows two members of the set a and b to interact and produce another member, $a + b$, of the same set.

A group operation must be a *binary operation*, like addition, meaning that it involves two elements of the set. What is more, for a binary operation to be a group operation.

We insist further that the operation satisfies three particular conditions, all of which hold for integer addition: the operation, $+$, must be *associative*, i.e. $a+(b+c) = (a+b)+c$ for any three members a, b, c of the set.

There must be an *identity element*, denoted by 0, which has the property that $a + 0 = 0 + a = a$ always holds.

Finally, each member a of the set must have an *inverse* element, denoted here by $-a$, that reverses the effects of adding a in the sense that $a + (-a) = (-a) + a = 0$, the identity element.

> **Remark 1.1** (Abelian Group)**.** Integer addition also satisfies the *commutative* law in that $a + b = b + a$. Commutativity is not part of the general definition of a group. However, when the operation of a group G does respect the commutative law, we say that G is an *abelian group*.