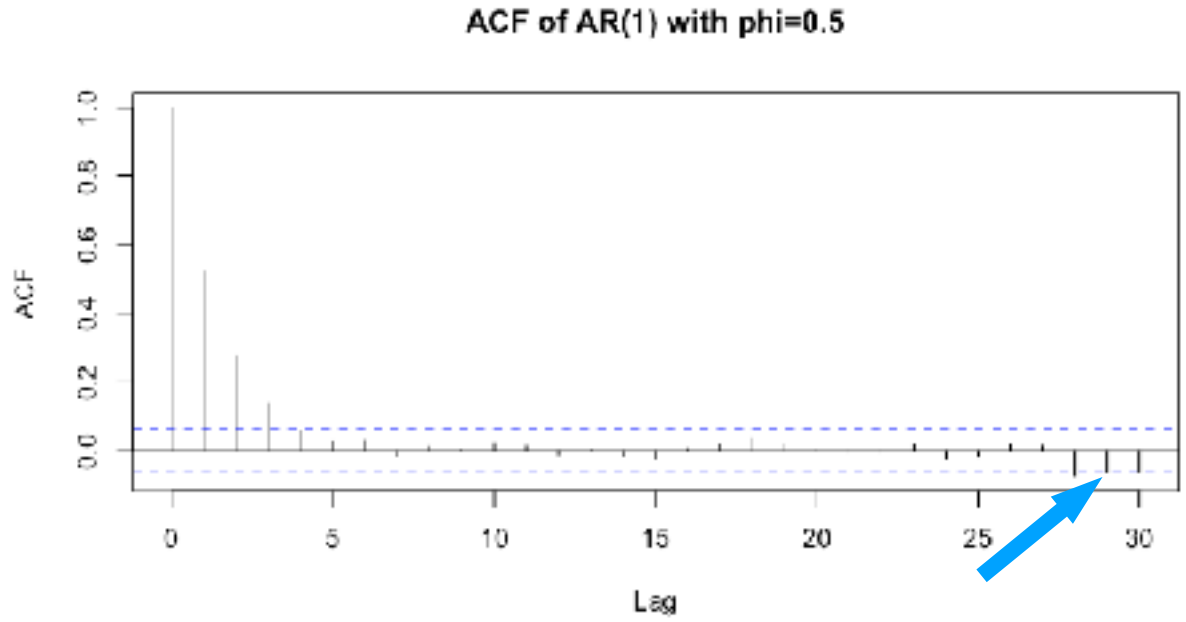

Q&A: For AR(1) with $\phi = 0.5$, ACF is given by $\rho_k = \phi^k$. We observe negative values of ACF in the above figure, Why?

- The true correlation $\rho_k = \phi^k$ is the expectation of the $Y_t Y_{t-k}$, which is the population value. It is always positive in this example.

- But the above plot shows the sample correlation, rather than the population value, i.e., the ACF in the plot is calculated by the formula $\frac{1}{T}\sum_t Y_t Y_{t-k}$.

- The sample value converges to the population value as the sample size goes to infinity.

- However, when the population value is quite close to zero (ϕ^k will decrease when k increases), the noise in the model may contaminate the calculation and thus make the ACF negative when ρ_k is around zero (see blue arrow).

- Although there are negative values in the plot, which contradict the fact $\rho_k = \phi^k > 0$, this situation only happens when ρ_k is quite close to zero. Besides the negative value for sample ACF is also near zero.