

## VR tests

### Lo and MacKinlay (1988):

They formulate two test statistics for the random walk properties that are applicable under different sets of null hypothesis assumptions.

1. To test for i.i.d. Gaussian with constant variance
2. To test for a heteroskedastic random walk hypothesis where they weaken the *i.i.d.* assumption and allow for fairly general forms of conditional heteroskedasticity and dependence.

Then the variance ratio test statistic is given by

$$\frac{\widehat{VR}(q) - 1}{\sqrt{\widehat{\theta}(q)}} \stackrel{a}{\sim} N(0, 1)$$

(This formula is given in the slide Chap2 - Page 111)

### Chow and Denning (1993):

Since the variance ratio restriction holds for every difference  $q$ , it is common to evaluate the statistic at several selected values of  $q$ .

To control the size of the joint test, Chow and Denning (1993) propose a (conservative) test statistic that examines the maximum absolute value of a set of multiple variance ratio statistics.

The *p-value* for the Chow-Denning statistic using  $m$  variance ratio statistics is bounded from above by the probability for the Studentized Maximum Modulus (SMM) distribution with parameter  $m$  and  $T$  degrees-of-freedom.

For a detailed discussion of these joint VR tests, see Fong, Koh, and Ouliaris (1997).

## Wild Bootstrap in VR

Kim (2006) offers a wild bootstrap approach to improving the small sample properties of variance ratio tests.

Basic idea: The approach involves computing the individual (Lo and MacKinlay) and joint (Chow and Denning, Wald) variance ratio test statistics on samples of  $T$  observations formed by weighting the original data by mean 0 and variance 1 random variables, and using the results to form bootstrap distributions of the test statistics. The bootstrap *p-values* are computed directly from the fraction of replications falling outside the bounds defined by the estimated statistic.

Reference:

[http://www.eviews.com/help/helpintro.html#page/content/advtimeser-Variance\\_Ratio\\_Test.html](http://www.eviews.com/help/helpintro.html#page/content/advtimeser-Variance_Ratio_Test.html)