PS4 AR - Process.

$$Y_t = c + \phi Y_{t-1} + \varepsilon_t, \quad (|\phi|<1, \ \varepsilon_t \sim WN(0, \sigma_\varepsilon^2))$$

(a). $E[Y_t | Y_{t-1}, Y_{t-2}, \dots] = c + \phi Y_{t-1}$

$Var[Y_t | Y_{t-1}] = \sigma_\varepsilon^2$.

Interpretation of $E[Y_t | \mathcal{F}_{t-1}]$: is the best estimator of Y_t.

(b) $Y_t = c + \phi Y_{t-1} + \varepsilon_t$

$Y_{t-1} = c + \phi Y_{t-2} + \varepsilon_{t-1}$

$Y_t = c \sum_{j=0}^{k-1} \phi^j + \sum_{j=0}^{k-1} \phi^j \varepsilon_{t-j} + \phi^k Y_{t-k}$.

$|\phi|<1$, if we let $k \to \infty$.

$Y_t = \frac{c}{1-\phi} + \sum_{j=0}^{\infty} \phi^j \varepsilon_{t-j}$, $\mu = \frac{c}{1-\phi}$.

(c) ① $E[Y_t] = E[E(Y_t | Y_{t-1})] = E(c + \phi Y_{t-1}) = c + E[\phi Y_{t-1}]$

$E[Y_t] = \mu$

② $Var[Y_t] = E[Var(Y_t | Y_{t-1})] + Var[E(Y_t | Y_{t-1})] = \sigma_\varepsilon^2 + \phi^2 Var[Y_{t-1}]$

$Var[Y_{t-1}] = \sigma_\varepsilon^2 / (1-\phi^2)$.

③ Yes, AR(1) is weakly stationary,

$\gamma_j = E[(Y_t - \mu)(Y_{t-j} - \mu)] = \frac{\phi^j \sigma_\varepsilon^2}{1-\phi^2}$, $\rho_j = \frac{\gamma_j}{Var[Y_t]} = \frac{\gamma_j}{\gamma_0} = \phi^j$

According to def. in "Time Series analysis" of James . D. Hamilton , page 45.

(d)

Log returns : $p<0.05$, not i.d., they exhibit serial correlation.

AR(1) residuals : $p>0.05$, AR(1) residuals are i.d. (independently distributed