Assignment

Modeling and Control of Hybrid Systems (SC42075)
2022-2023

Delft Center for Systems and Control, Delft University of Technology

General remarks

- This assignment consists of two parts. In the second part, you will see several steps that are outlined in a road map. You should follow this road map and present the results in a clear and concise report. In this report, you should clearly explain and motivate all the choices you have made while solving the practical exercise.

  Moreover, you should include an evaluation and conclusions section of 1–1.5 pages in your report, in which you briefly outline the main insights you have obtained while doing this assignment.

  You should also add the MATLAB files you have written in an appendix to the report.

- The deliverable of this assignment is a written report of max. 50 pages (including cover pages and appendices) about the assignment that also contains the *appendix* with your m files. Make sure to mention your group number and student number on the cover page. The report is to be uploaded as a single **pdf** file to **brightspace**.

  The *hard* deadline for submitting the report is Monday, June 19, 2023 at 17:00 p.m.

- You will be graded based on the contents and the presentation of the report, on the originality[1] of your answers, and on the correctness, *efficiency*, and readability of the MATLAB files (i.e., do not forget to include explanatory comments in your MATLAB files).

- We suggest you to keep the computations symbolic or analytical as long as possible and not to hard-code any of the parameters in your MATLAB programs (instead, write one separate MATLAB function or script that defines the parameters) so that you can easily take other parameter values, longer control horizons, other reference signals, etc. into account.

  Furthermore, since each step of this assignment depends on the preceding ones, we recommend that after Steps 2.2, 2.5, and 2.8 of the road map, you check your intermediate results with the teaching assistant (during the question hours).

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[1]I.e., a correct answer that differs from the answers given by the other groups will be graded higher than a correct answer that is an almost literal copy of the answer of another group.

1 Part 1: Hybrid system example

Select a system in your own field of research or interests that can be considered as a hybrid system (and that is not yet discussed in the lecture notes).

Step 1.1: Give a description of the system.

Step 1.2: Describe the system as a hybrid automaton; if needed, extend the definition of the (autonomous) hybrid automaton given in the lecture notes with inputs.

2 Part 2: Adaptive cruise control

Set-up

We consider an adaptive cruise control (ACC) application in which 2 cars are driving one after the other (see Figure 1).
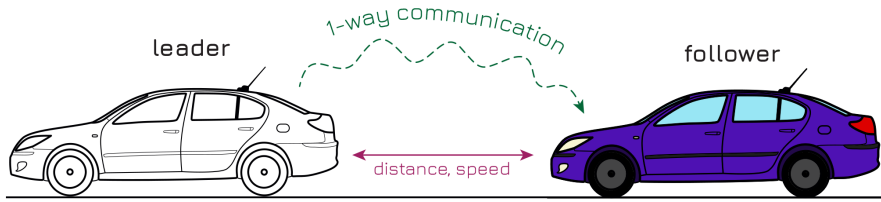

Figure 1: ACC set-up considered in the assignment.

In general, the aim of ACC is to ensure a minimal separation between the vehicles (i.e., distance keeping) and a speed adaptation (i.e., the speed differences between the vehicles should be kept as small as possible). In this exercise we will — for the sake of simplicity — only consider the speed adaptation control and we assume that the leading vehicle communicates its speed to the following vehicle, which then has to track this speed as well as possible.

For the vehicle dynamics we consider a simplified model in which at time t the following forces act on the vehicle (which has mass m):

- The "driving" force $F_{\text{drive}}(t)$, which is related to the throttle input $u(t)$ and the gear ratio $g(t)$ by the following equation:

$$F_{\text{drive}}(t) = \frac{b}{1 + \gamma r(t)} u(t),$$

  where b and γ are two constants and $r(t) \in \{1, 2\}$ indicates the gear ratio at time t. The gear ratio is taken to depend on the speed of the vehicle in the following way:

$$r(t) = \begin{cases} 1 & \text{if } v(t) < v_{\text{g}} \\ 2 & \text{if } v_{\text{g}} \leqslant v(t) \end{cases}$$

  The throttle input $u(t)$ is the control input.

- The road gradient resistance force $F_{\text{gradient}}(t)$ is a function of the slope of the road θ:

$$F_{\text{gradient}}(t) = mg \sin \theta(x(t)).$$

- A dynamic friction force $F_{\text{friction}}(t)$ that is proportional to the square of the speed $v(t)$ of the vehicle:

$$F_{\text{friction}}(t) = cv^2(t).$$

The parameters of the vehicle are given in Table 1.

Braking will be simulated by applying a negative throttle. Therefore, the gear ratio influences both the acceleration and deceleration forces. We will assume that the vehicles drive in the forward direction, so the speed will always be non-negative. For passenger comfort we also include a maximal acceleration/deceleration:

$$|a(t)| \leqslant a_{\text{comf,max}}.$$

Parameter	Value	Units
m	700	kg
g	9.8	m/s^2
c	0.5	kg/m
b	3600	N
$u_{\max}$	1.6	—
$u_{\min}$	−1.4	—
$a_{\text{comf,max}}$	2.5	m/s^2
γ	1.8	—
v_g	18	m/s
h	10	m
w	50	m

Table 1: Parameters of the problem.

Tasks & Road map

Step 2.1: Note that as we are only considering the speed adaptation and as the leading vehicle communicates its speed to the follower, we only have to consider the following vehicle. The continuous-time model for the position $x(t)$ and speed $v(t)$ of the following vehicle in the state-space form is formulated as:

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} v(t) \\ \dfrac{F_{\text{drive}} - F_{\text{gradient}} - F_{\text{friction}}}{m} \end{bmatrix} = \begin{bmatrix} v(t) \\ \underbrace{\dfrac{b/m}{1+\gamma r(t)}u(t)}_{F_{\text{drive}}} - \underbrace{g\sin\theta(x(t))}_{F_{\text{gradient}}} - \underbrace{\dfrac{c}{m}v^2(t)}_{F_{\text{friction}}} \end{bmatrix}$$

Given the maximal throttle input $u_{\max}$ and the maximal braking input $u_{\min}$, determine the maximal speed $v_{\max}$ and the maximal acceleration $a_{\text{acc,max}}$ and deceleration $a_{\text{dec,max}}$ of the vehicle for $\theta = 0$.

Step 2.2: Construct a piecewise-affine (PWA) approximation P with 2 regions of the friction force curve $V : [0, v_{\max}] \to \mathbb{R} : v \mapsto v^2$ as follows. We want a perfect match for $v = 0$ and $v = v_{\max}$. This implies that we still have two degrees of freedom, i.e., the coordinates (α, β) of the middle edge point of the PWA curve (see Figure 2). Now determine α and β such that the *squared* area between V and P (i.e., the squared area corresponding the hashed region in Figure 2) is minimized, or equivalently, such that

$$\int_0^{v_{\max}} \left(V(v) - P(v) \right)^2 dv$$

is minimized.
Note: Use analytic computations (by hand or by using MATLAB, Mathematica or Maple, etc.) to determine the optimal α and β (and *not* numerical computations/optimization).

Step 2.3: In this step, assume that the gear is constant and equal to 1 ($r(t) = 1$ for all t). Approximate the friction force using the PWA function P instead of the quadratic function V. Compare the output of the resulting continuous-time PWA model with that of the original model for a sinusoidal throttle input. Can you explain where the differences — if any — come from?
Note: By properly selecting the initial speed you should be able to guarantee that the speed never becomes negative and that both regions of the PWA model are visited.

Step 2.4: From now on again consider the gear ratio ($r(t)$) to be automatically determined based on the vehicle speed at time t. Write a PWA model for the system that considers both the gear ratio and the approximation P of the friction force. Additionally, we assume to know the profile of the road, i.e. the height y as a function of x as

$$y = \min\left\{ \frac{2h}{w}x \, , \, \frac{h}{w}x+h \, , \, 3h \, , \, \frac{3h}{2w}x+9h \right\}.$$

Use this information to compute θ as a function of x. The constants h and w are given in Table 1 as well. Next, add this information to the system model by considering the different variations of θ and assuming $\sin\theta \approx \theta$.
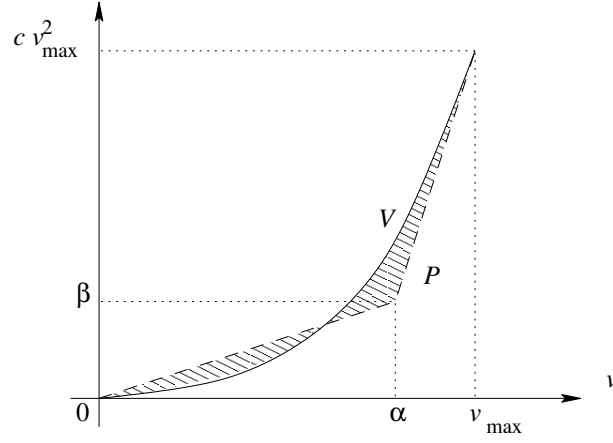
Figure 2: The quadratic function V and its PWA approximation P.

Step 2.5: Now discretize the PWA model of Step 2.4 of the vehicle using the sample step $T = 0.1\,\text{s}$ and a forward Euler rule for the discretization.

Step 2.6: Transform the discrete-time PWA model of Step 2.5 into an MLD model with binary and real variables only.

Hint: Consider to map the term $\dfrac{b}{1 + \gamma r(k)}$ as a whole to a linear expression of one or more binary auxiliary variables $\delta(k)$.

Step 2.7: Now we design an MPC controller for the MLD model using the implicit MPC approach. The performance J should be a trade-off between the tracking J_{track} (i.e., the difference between the speed and the reference speed communicated by the leading vehicle) and the input energy J_{input} (for groups 1, 2, 7, and 8), the smoothness of the throttle signal (for groups 3, 4, 9, and 10), and the smoothness of the derivative of the throttle signal (for groups 5, 6, 11, and 12). More specifically, we have

$$J(k) = J_{\text{track}}(k) + \lambda J_{\text{input}}(k)$$

with $J_{\text{track}}(k)$ and $J_{\text{input}}(k)$ as given in Table 2 where

$$\tilde{v}(k) = [v(k+1)\ldots v(k+N_{\text{p}})]^{\text{T}},$$

$$\tilde{v}_{\text{ref}}(k) = [v_{\text{ref}}(k+1)\ldots v_{\text{ref}}(k+N_{\text{p}})]^{\text{T}},$$

and

$$\tilde{u}(k) = [u(k)\ldots u(k+N_{\text{p}}-1)]^{\text{T}},$$

with N_{p} the prediction horizon. Note that for a signal $s(\cdot)$ we have

$$\Delta s(k) = s(k) - s(k-1),$$

$$\Delta^2 s(k) = \Delta s(k) - \Delta s(k-1) = s(k) - 2s(k-1) + s(k-2).$$

In order to get a well-defined objective function groups 3, 4, 5, 6, 9, 10, 11, and 12 may assume that $u(k_0 - 2) = u(k_0 - 1) = 0$ where k_0 corresponds to the first sample step of the total simulation period $[0, T_{\text{end}}]$ (cf. Step 2.9).

Write a MATLAB file that computes the optimal MPC input sequence for a given sample step k for values of N_{p} and N_{c} (i.e., the control horizon) up to 6, and for arbitrary values of λ. Note that a discretized version of the comfort constraint $-a_{\text{comf,max}} \leqslant a(t) \leqslant a_{\text{comf,max}}$ should also be taken into account!

Also note that due to the approximation made in Step 2.2, which is only valid for non-negative speeds, we should also *explicitly* add the constraint $v(t) \geqslant 0$.

Group number[2]	$J_{\text{track}}(k) = \|\tilde{v}(k) - \tilde{v}_{\text{ref}}(k)\|_1$ and $J_{\text{input}}(k) =$	Group number[2]	$J_{\text{track}}(k) = \|\tilde{v}(k) - \tilde{v}_{\text{ref}}(k)\|_\infty$ and $J_{\text{input}}(k) =$
1	$\|\tilde{u}(k)\|_1$	7	$\|\tilde{u}(k)\|_1$
2	$\|\tilde{u}(k)\|_\infty$	8	$\|\tilde{u}(k)\|_\infty$
3	$\|\Delta\tilde{u}(k)\|_1$	9	$\|\Delta\tilde{u}(k)\|_1$
4	$\|\Delta\tilde{u}(k)\|_\infty$	10	$\|\Delta\tilde{u}(k)\|_\infty$
5	$\|\Delta^2\tilde{u}(k)\|_1$	11	$\|\Delta^2\tilde{u}(k)\|_1$
6	$\|\Delta^2\tilde{u}(k)\|_\infty$	12	$\|\Delta^2\tilde{u}(k)\|_\infty$

Table 2: Tracking and input cost functions

Hints & notes:

- By introducing one or more dummy variables optimization problems of the form $\min_{\theta\in\mathbb{R}^n}\|\theta\|_1$ subject to $A\theta \leqslant b$ or $\min_{\theta\in\mathbb{R}^n}\|\theta\|_\infty$ subject to $A\theta \leqslant b$ can be transformed into a linear programming (LP) problem:

  - Recall that for $\theta \in \mathbb{R}^n$, we have $\|\theta\|_1 = \sum_{i=1}^n |\theta_i|$. Now it is easy to verify that any optimal solution (ρ^*, θ^*) of the problem

  $$\min_{\rho,\theta\in\mathbb{R}^n} \rho_1 + \cdots + \rho_n \quad \text{subject to } -\rho \leqslant \theta \leqslant \rho \text{ and } A\theta \leqslant b$$

  is also an optimal solution of $\min_{\theta\in\mathbb{R}^n}\|\theta\|_1$ subject to $A\theta \leqslant b$ (and vice versa if we set $\rho^* = \|\theta^*\|_1$).

  - Similarly, recall that for $\theta \in \mathbb{R}^n$, we have $\|\theta\|_\infty = \max_{i=1,\ldots,n} |\theta_i|$. Now it is easy to verify that any optimal solution (τ^*, θ^*) of the problem

  $$\min_{\tau\in\mathbb{R},\theta\in\mathbb{R}^n} \tau \quad \text{subject to } -\tau \leqslant \theta_i \leqslant \tau \text{ for } i=1,\ldots,n \text{ and } A\theta \leqslant b$$

  is also an optimal solution of $\min_{\theta\in\mathbb{R}^n}\|\theta\|_\infty$ subject to $A\theta \leqslant b$ (and vice versa if we set $\tau^* = \|\theta^*\|_\infty$).

- Using the hint above the MPC optimization problem at step k can be transformed into a mixed-integer linear programming problem (MILP). In order to solve this problem you will need an MILP solver, for which you could use one of the following options:

  - Use the `glpk` function of the MPT toolbox version 3 (for MATLAB R2011a and newer) (see `https://www.mpt3.org/`)
  - Use the Matlab interface of the Gurobi optimizer; see `https://www.gurobi.com/products/gurobi-optimizer/`
  - Use the `cplex` command of the TOMLAB cplex toolbox (note that you need a license, see `https://tomopt.com/tomlab/`). With this command you can solve MILP problems (see also the `milp_solve` tomlab cplex script on the course website).
  - As a last resort, you can use the function `intlinprog` of the Matlab Optimization Toolbox.

- As we want you to get some insight into the hybrid MPC method and the relation with integer programming, you are *not* allowed to solve the entire exercise using the `mpt_control` or related commands of the MPT toolbox or the Hybrid Toolbox (see `http://cse.lab.imtlucca.it/~bemporad/hybrid/toolbox/`). However, feel free to compare the results obtained with your own programs to those obtained with the MPT toolbox or the Hybrid Toolbox, and to discuss the differences, if any.

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[2]If your group number is larger than 12, subtract 12 from it until you get a number in the range 1–12 and select the corresponding tracking and input cost function.

Step 2.8: Write a MATLAB file to simulate the closed-loop behavior of the system (i.e., apply the receding horizon approach in which at each step the optimal MPC control input is recomputed and applied to the system) using the original continuous-time model as simulation model.

Step 2.9: Assume $\lambda = 0.1$ (in general, an appropriate value of λ is determined by taking into account the nominal values of J_{track} and J_{input} and by some tuning and iteration). Consider two combinations of N_{p} and N_{c}: first the combination $(N_{\text{p},1}, N_{\text{c},1}) = (5,4)$ which is the same for all groups, and another combination $(N_{\text{p},2}, N_{\text{c},2})$ that you may select yourself with $N_{\text{p},i} \in \{7,8,9,10\}$ and $1 < N_{\text{c},i} < N_{\text{p},i}$. For each combination run your program for the time interval $[0, T_{\text{end}}]$ with $T_{\text{end}} = 25$, for $v(0) = 0.925\alpha$ where α is the value found in Step 2.2, and for the speed reference signal v_{ref} which defined as follows (see also Figure 3):

$$
v_{\text{ref}}(t) = \begin{cases}
0.85\alpha & \text{for } 0 \leqslant t \leqslant 3 \\
1.2\alpha & \text{for } 3 < t \leqslant 9 \\
1.2\alpha - \dfrac{1}{12}\alpha(t-9) & \text{for } 9 < t \leqslant 15 \\
0.7\alpha & \text{for } 15 < t \leqslant 18 \\
0.7\alpha + \dfrac{4}{15}\alpha(t-18) & \text{for } 18 < t \leqslant 21 \\
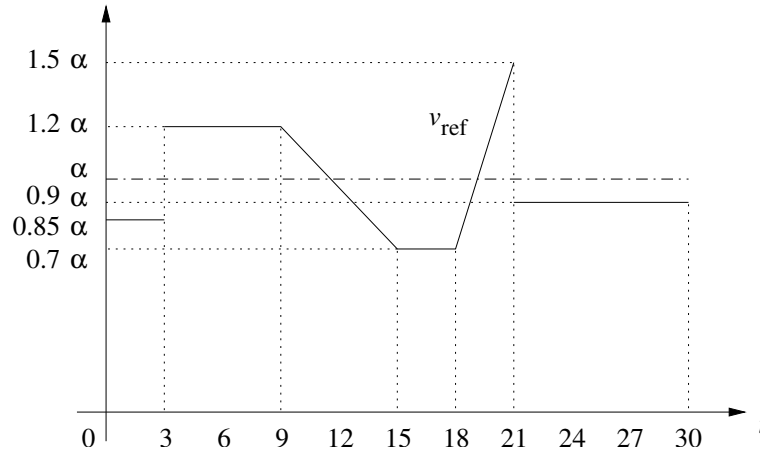0.9\alpha & \text{for } 21 < t \leqslant 30 \ .
\end{cases}
$$

Figure 3: The reference speed signal to be used.

Make a plot of the evolution of the controlled closed-loop system in the (x,v) phase plane and of the evolution of x, v, the acceleration a, v_{ref}, $v - v_{\text{ref}}$, u and Δu over time. Compare the obtained trajectories and discuss the differences, if any.

Step 2.10: You will have noticed that computing the optimal MPC input using the an MILP solver requires quite some computation time, especially for large N_{p} and N_{c}. If this time is larger than the sampling time of the controller (in our case $T = 0.1\,\text{s}$) then the (basic) on-line MPC optimization approach is not feasible. One of the possible solutions is then to use the explicit MPC approach in which for each possible current state $v(k)$ and future reference vector $\tilde{v}_{\text{ref}}(k)$ the optimal MPC input $u^*(k)$ is computed off-line using multi-parametric mixed-integer linear programming, and stored in a look-up table (cf. page 112 of the lecture notes and the references [23, 24, 28, 29, 36] of the lecture notes).
Now you should apply the explicit MPC approach to the ACC example and repeat Steps 7–9 but now with explicit MPC instead of implicit MPC. In order to compute the explicit MPC solution you can use the Multi-Parametric Toolbox.
Compare the off-line and on-line computation times required for each approach with $N_{\text{p}} = N_{\text{c}} \in \{2,3,4\}$, and explain the results.

Note: Do not forget to include into your report an evaluation and conclusion section of 1–1.5 pages, in which you briefly outline the main insights you have obtained while doing Parts 1 and 2 of this assignment.