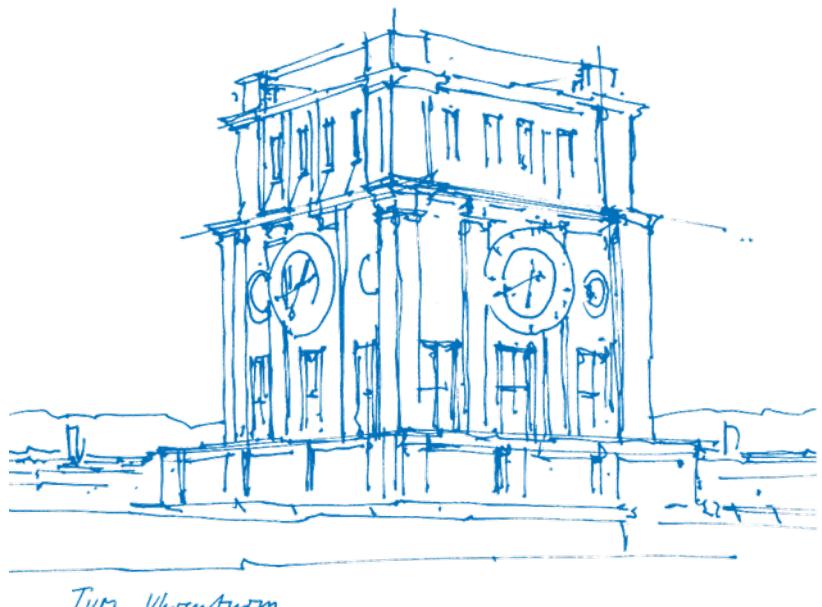
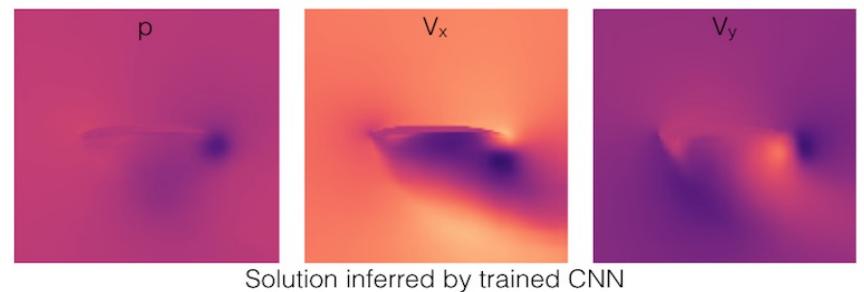
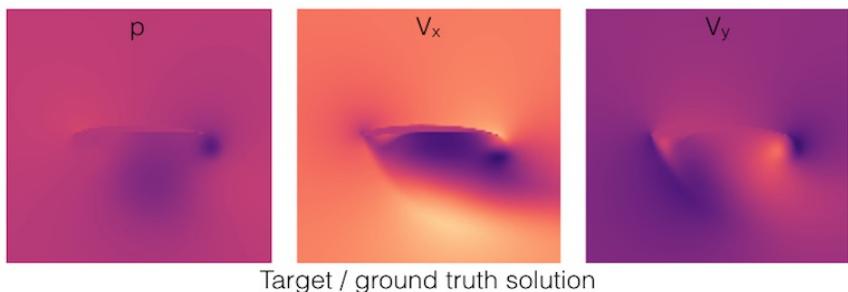


Deep Learning Methods for Reynolds-Averaged Navier-Stokes Simulations of Airfoil Flows

Julian Hohenadel
Technical University of Munich
Chair of Computer Graphics and Visualization
Munich, 11. May 2020



What this paper is about

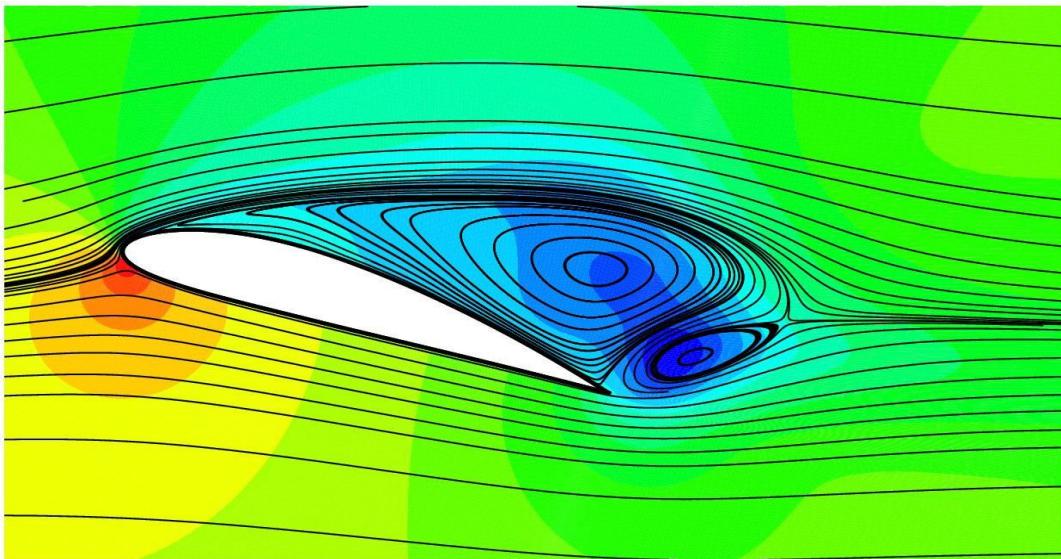


Taken from <https://github.com/thunil/Deep-Flow-Prediction>

What this paper is about

- Reynolds Averaged Navier-Stokes solutions
- Airfoil shapes
- Deep learning – CNN
- U-Net architecture derivative
- supervised training environment
- Inference of pressure and velocity fields
- Generalization
- Evaluation

Background – RANS



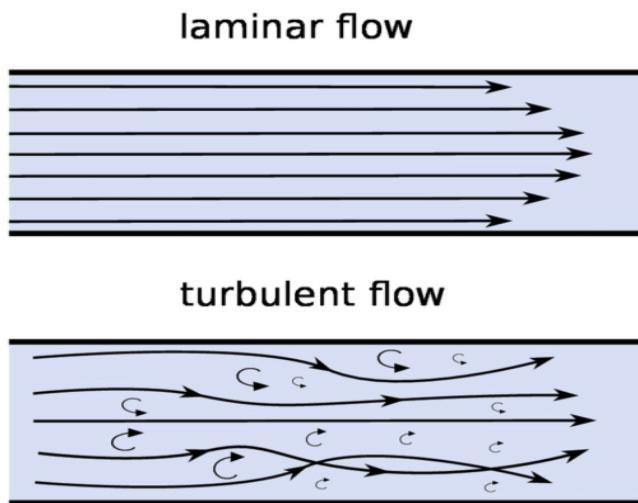
Taken from <https://www.pinterest.ch/pin/615163630322034457/>

Background – RANS

Nonlinear partial differential equation (PDE) system
based on Navier-Stokes equations

Used for the modeling of turbulent incompressible flows

Averages over time component

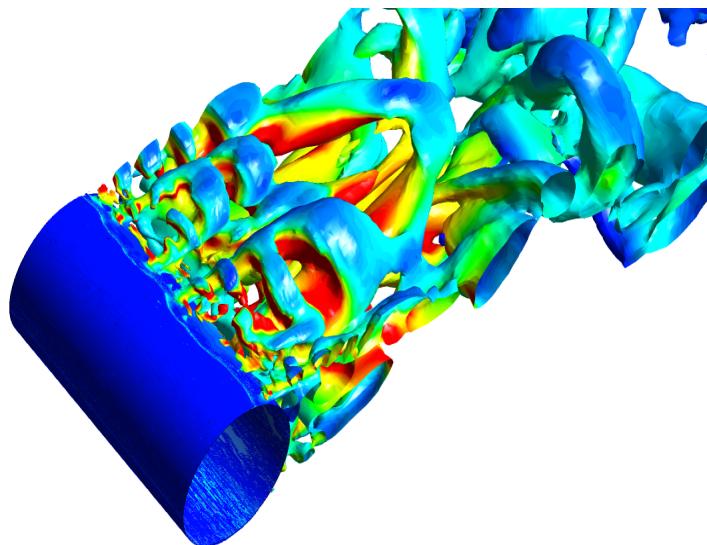


Taken from <https://diffzi.com/laminar-flow-vs-turbulent-flow/>

Background – RANS

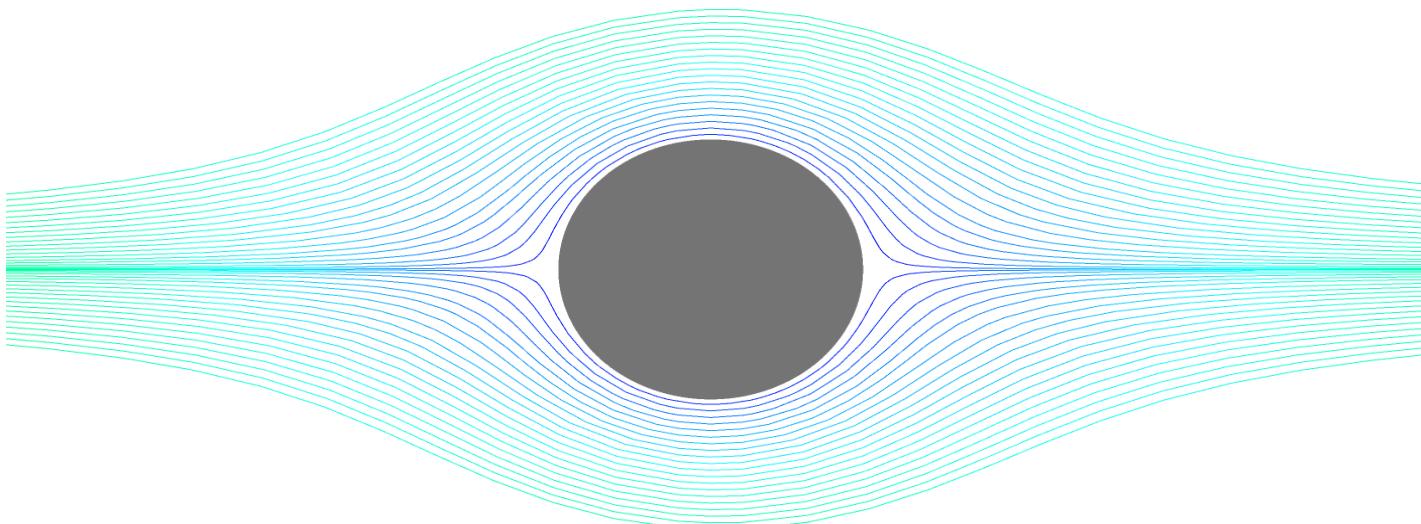
Reynolds number Re :

- dimensionless constant
- needed for calculation of turbulence models
- magnitude decides flow (laminar, turbulent)
- affects lift and drag coefficients



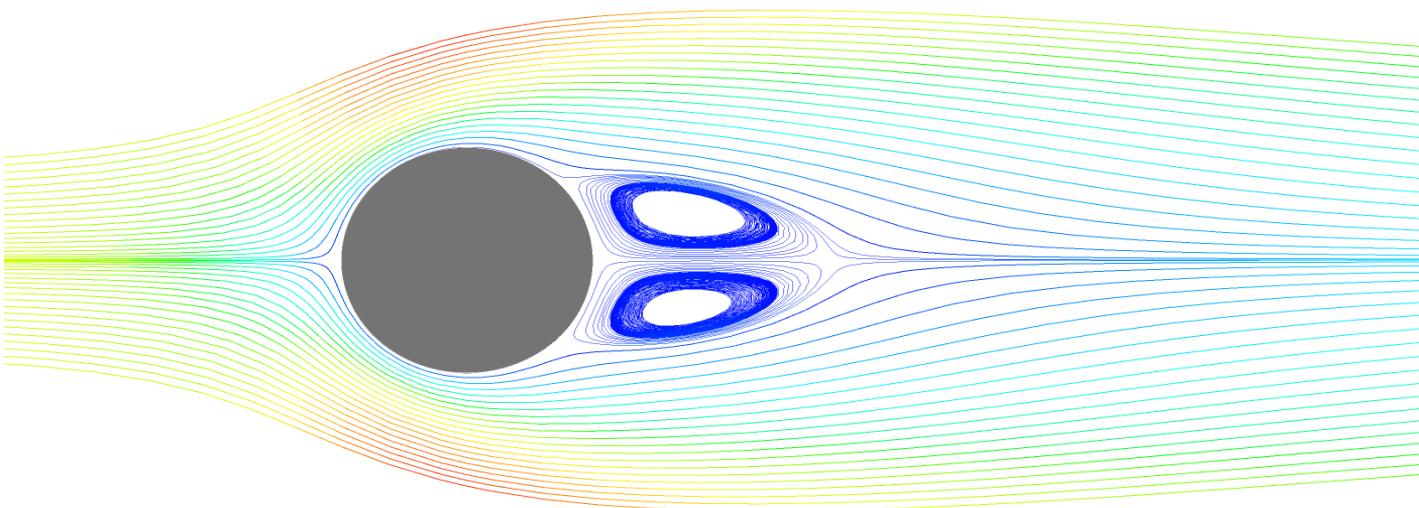
Taken from <https://www.computationalfluidynamics.com.au/reynolds-number-cfd/>

Background – Reynolds number: < 1



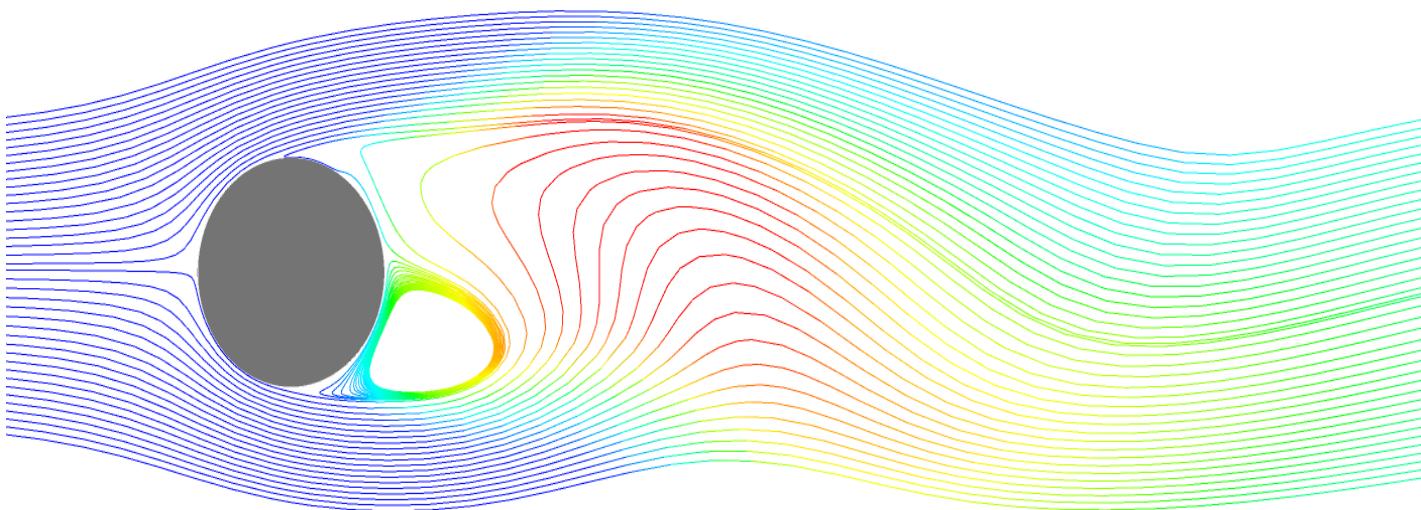
Taken from <https://www.computationalfluidynamics.com.au/reynolds-number-cfd/>

Background – Reynolds number: ≈ 10



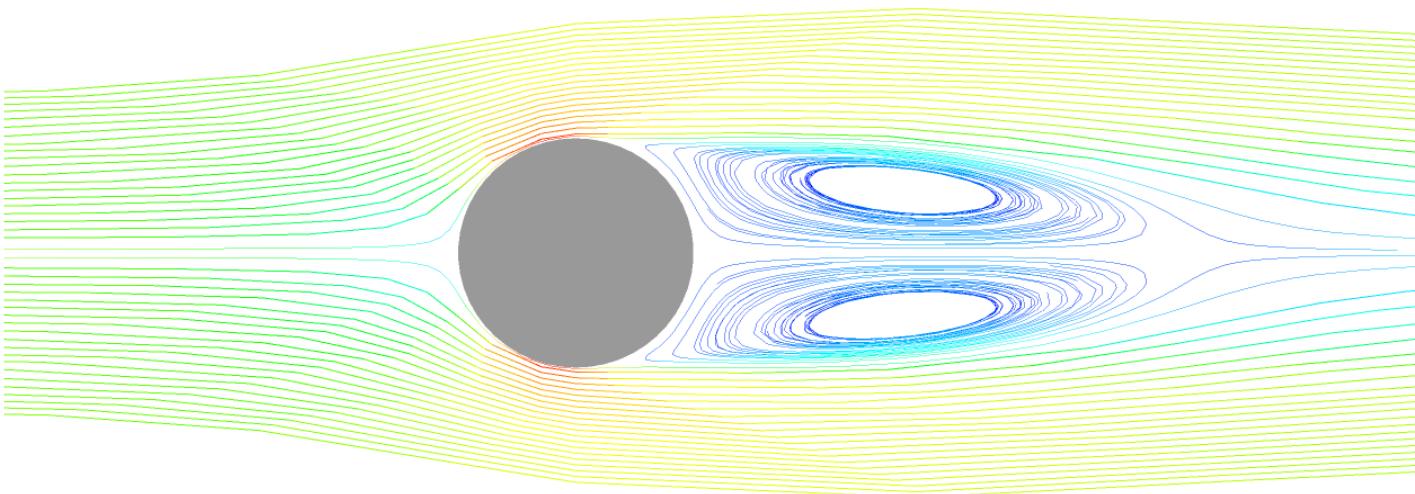
Taken from <https://www.computationalfluidynamics.com.au/reynolds-number-cfd/>

Background – Reynolds number: $\approx 1 \cdot 10^5$



Taken from <https://www.computationalfluidynamics.com.au/reynolds-number-cfd/>

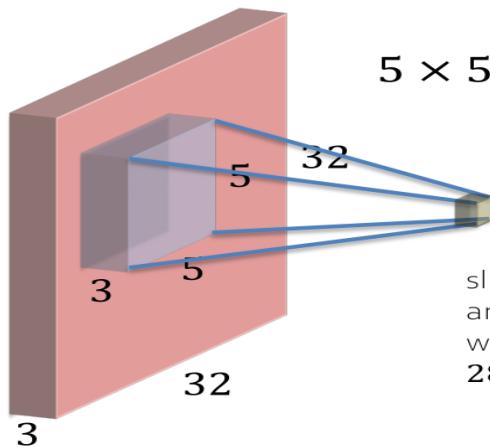
Background – Reynolds number: $\approx 1 \cdot 10^6$



Taken from <https://www.computationalfluidynamics.com.au/reynolds-number-cfd/>

Background – Convolutions

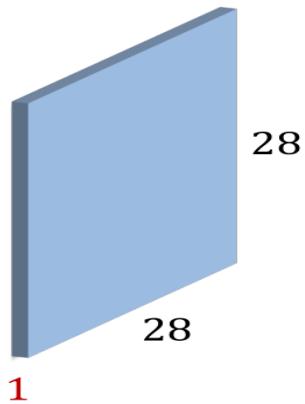
$32 \times 32 \times 3$ image



$5 \times 5 \times 3$ filter

slide over all spatial locations x_i
and compute all output z_i ;
w/o padding, there are
 28×28 locations

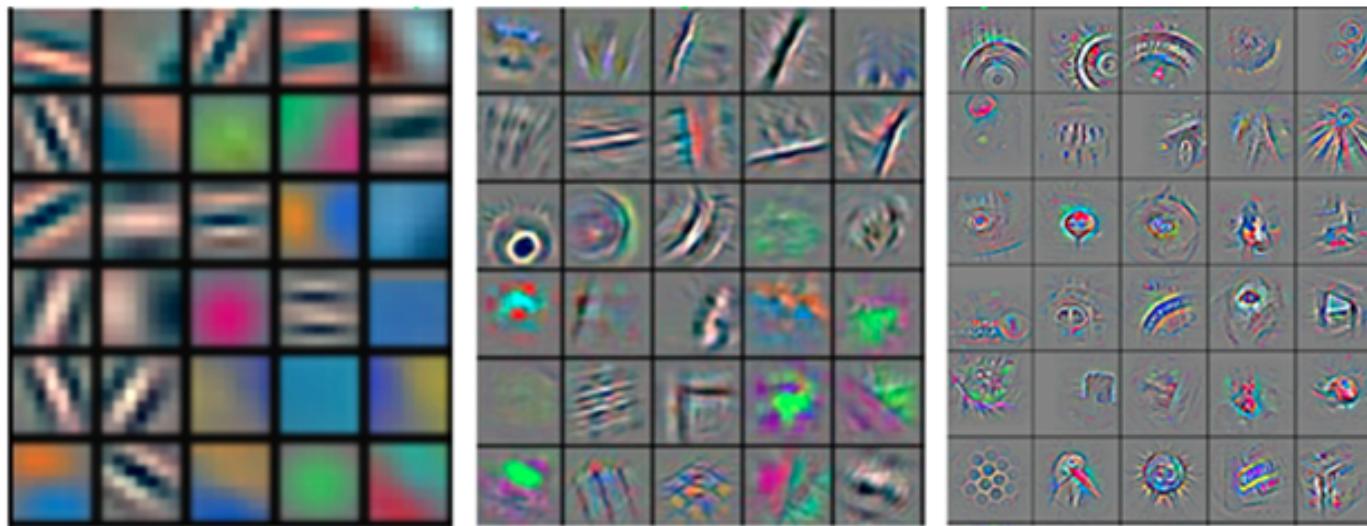
activation map
(also feature map)



Taken from I2DL WS19/20 (TUM)

Background – Convolutions

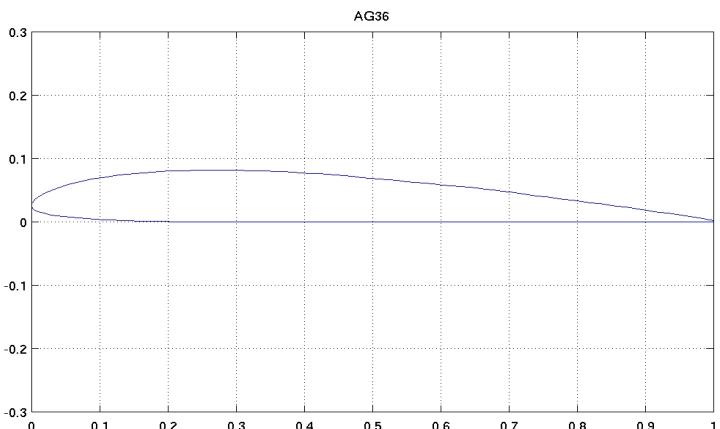
Low-Level Features, Mid-Level Features, High-Level Features: each filter captures different characteristics



Taken from <https://arxiv.org/pdf/1311.2901.pdf>

Data Generation

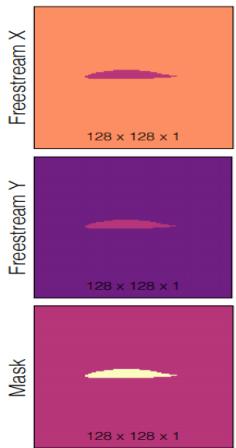
- Airfoil shapes from UIUC database
- Reynolds number: $[0.5, 5] \cdot 10^6$ (highly turbulent)
- Angle of attack: $[-22.5, 22.5]$
- Ground truth generated with OpenFOAM
(pressure, x velocity, y velocity)
- Training data resolution: $3 \times 128 \times 128$
(Inference region < full simulation domain)



Taken from <https://m-selig.ae.illinois.edu/ads/afplots/ag35.gif>

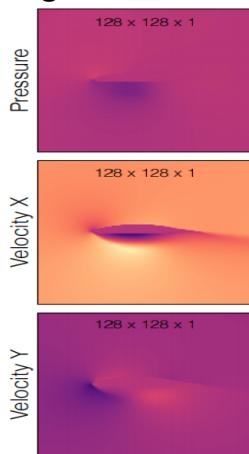
Pre-processing – Data

Input channels



Reynolds number encoded as differently scaled freestream velocity vectors wrt. their magnitude

Target channels



Data from the RANS solution

Pre-processing – Normalization

Motivation: Flatten space of solutions, accelerate learning by simplifying the learning task for the NN

Bernoulli equation for incompressible (**laminar**) flow:

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{constant}$$

- v : velocity
- g : acceleration (constant)
- z : elevation (constant)
- p : pressure
- ρ : density (constant)

⇒ $v^2 \sim p$ – e.g. double the speed quadruples the pressure

Pre-processing – Normalization

Normalization of target channels by division with freestream magnitude:

$$\tilde{v}_o = \frac{v_o}{\|v_i\|}, \quad \tilde{p}_o = \frac{p_o}{\|v_i\|^2} - \text{important to remove quadratic scaling of pressure}$$

Pre-processing – Normalization

All units disappear \implies really dimensionless:

- Pressure: $[p]_{SI} = 1 \text{ Pa} = 1 \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$
- Density: $[\rho]_{SI} = 1 \frac{\text{kg}}{\text{m}^3}$ – constant in incompressible flow
- Velocity: $[v]_{SI} = \frac{\text{m}}{\text{s}}$

Pre-processing – Offset removal & value clamping

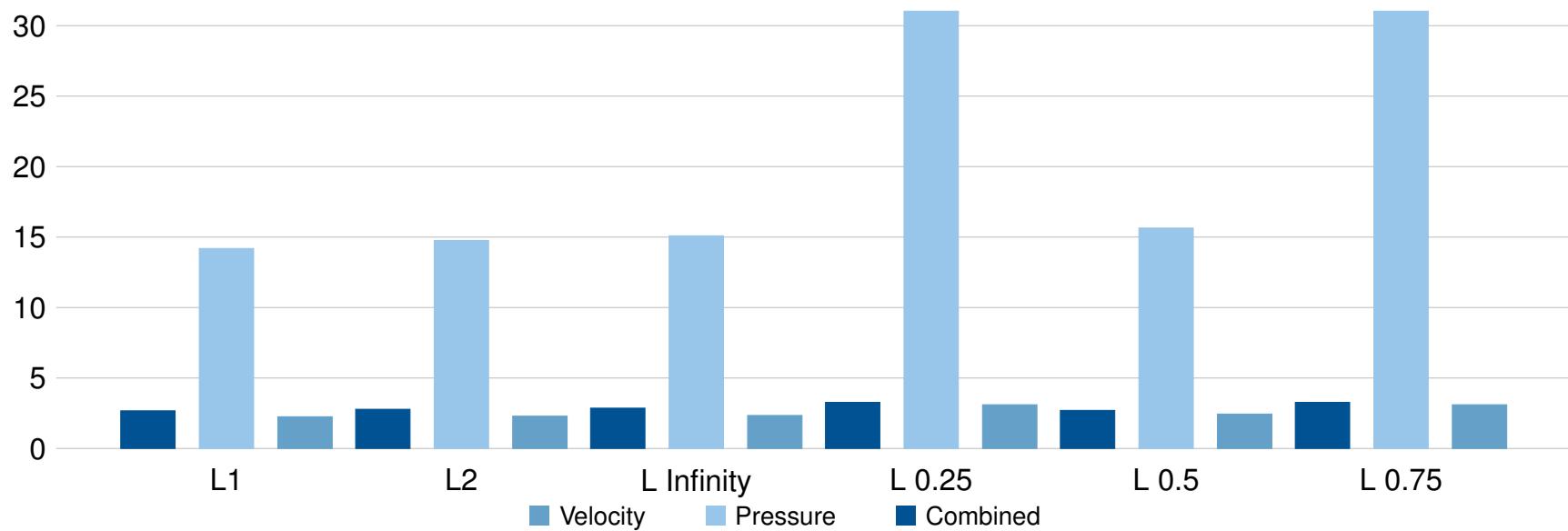
Motivation: eliminate ill-posed learning goal & improve numerical precision

- RANS typically only needs ∇_p for computation
- Spatially move pressure distribution into the origin
- $\hat{p}_o = \tilde{p}_o - p_{mean}$
- Clamp both input and target channels into $[-1, 1]$ range (divide by max abs. value)

Pre-processing – Evaluation

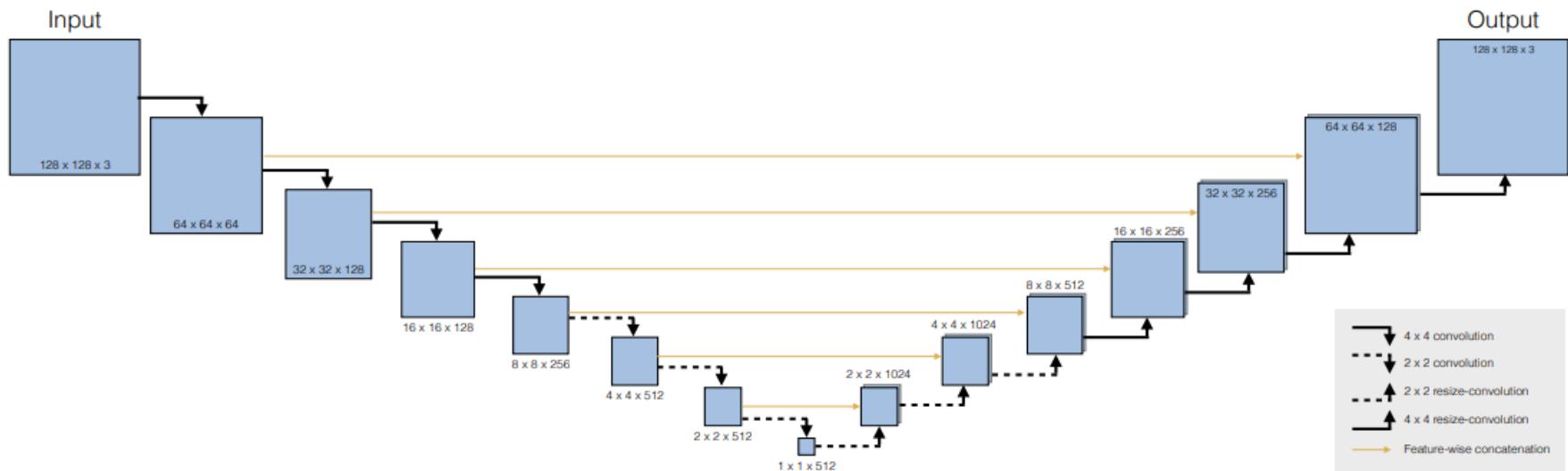
Vector norms used in pre-processing comparision wrt. error, default: L2 (in %)

L1 normalization achieves the best error rates (p, vel, combined: **14.19%**, **2.251%**, **2.646%** – L2: 14.76%, 2.291%, 2.780%)



Architecture

U-Net derivative proposed in the paper:



Taken from <https://arxiv.org/pdf/1810.08217.pdf>

Architecture – Convolutional blocks

Encoder

1. Activation – Leaky ReLu (0.2)
2. Convolution – Width down, Depth up
3. Batch normalization
4. Dropout (1%)

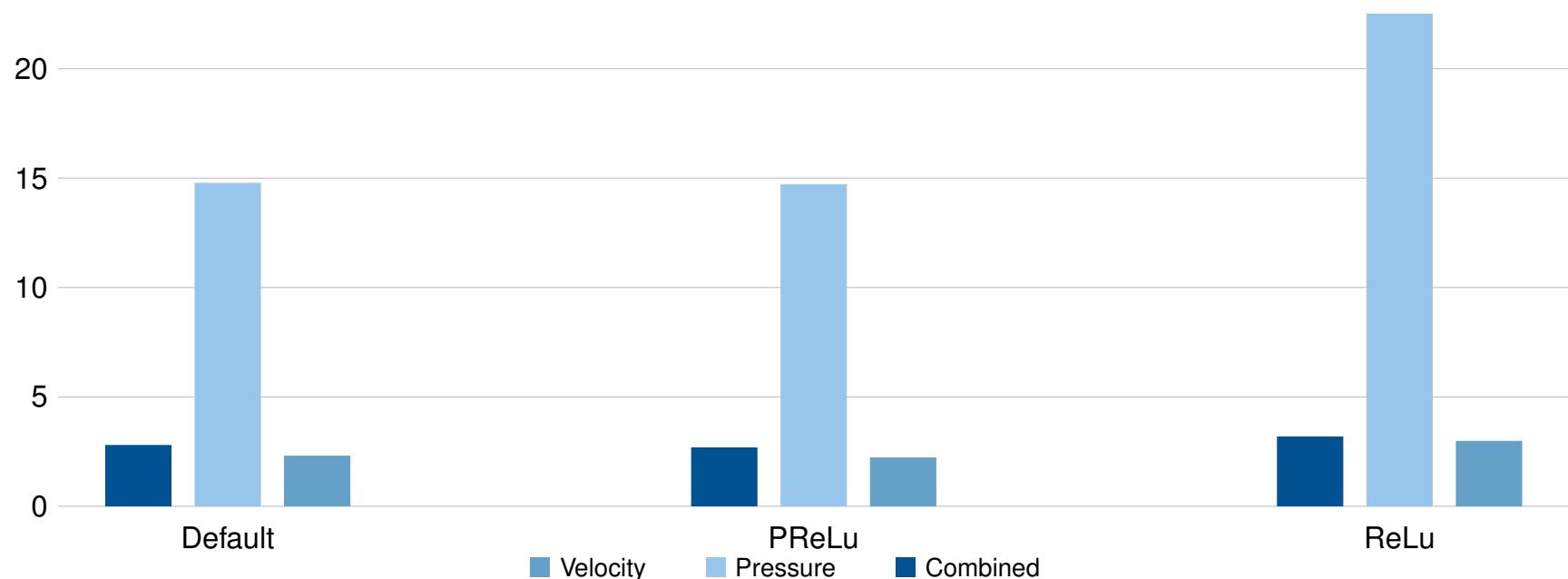
Decoder

1. Activation – ReLu
2. Upsampling – linear (2.0)
3. Convolution – Width up, Depth down
4. Batch normalization
5. Dropout (1%)

Architecture – Evaluation

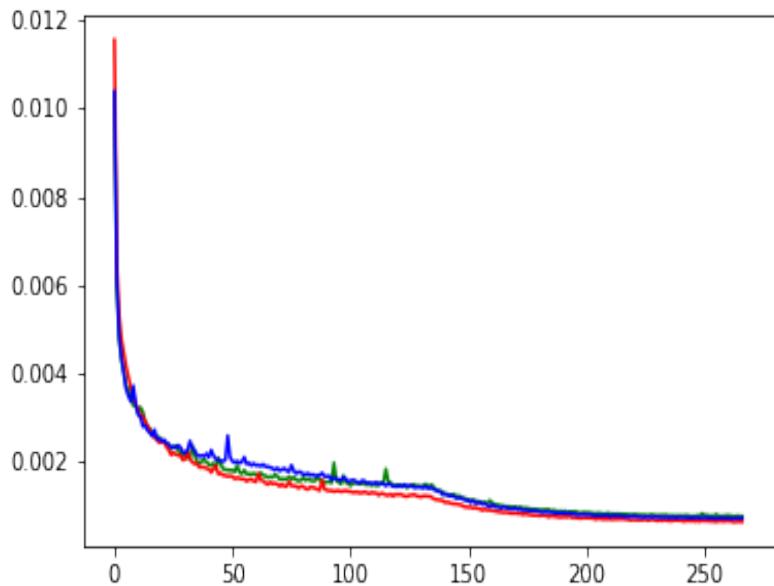
Error percentage of different activation functions after 160k iterations (266 epochs).

PReLU achieves the best error rates (p, vel, combined: **14.69%**, **2.216%**, **2.676%** – Default: 14.76%, 2.296%, 2.787%)

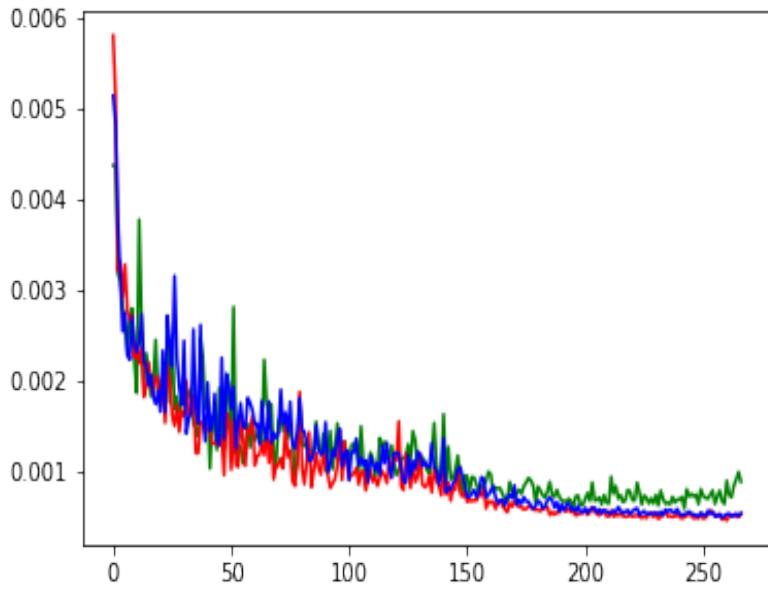


Architecture – Evaluation

Training loss



Validation loss



Transfer

Motivation: Can the network architecture adapt to other PDE systems & how will it perform?

Another use case for PDE systems: predicting wave propagation on shallow water

Governed by Saint-Venant equations (related with Navier-Stokes equations)

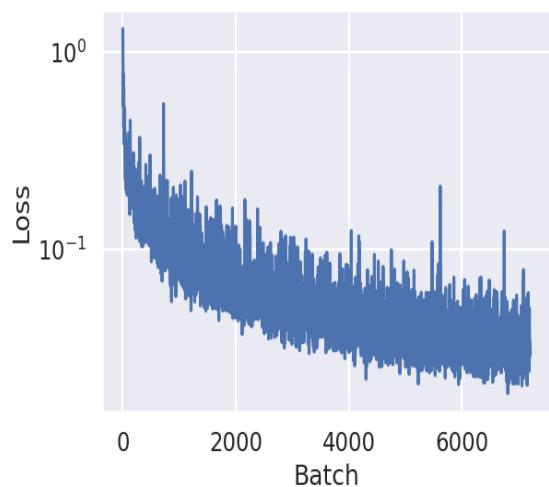
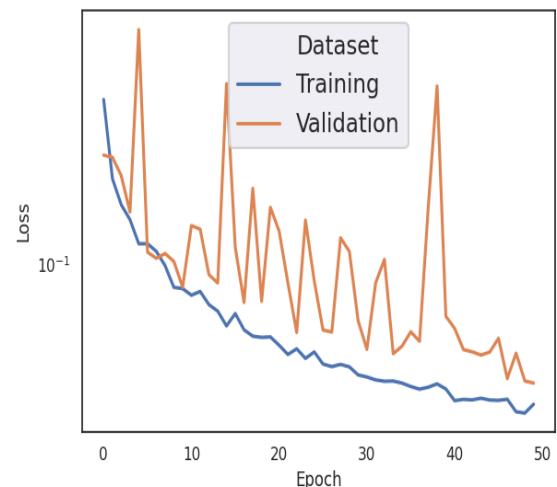
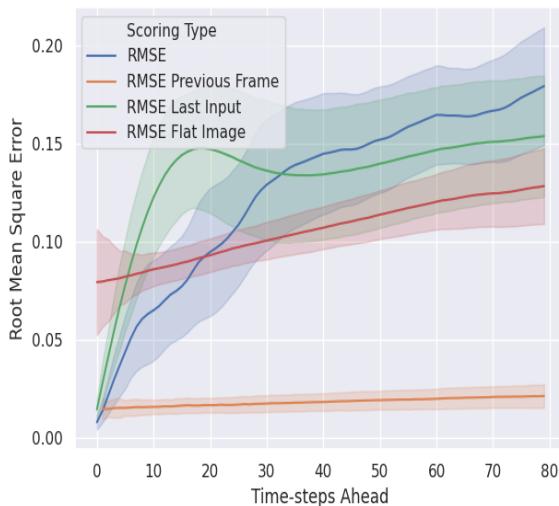
Transfer

U-Net architecture changes:

- Input channels – contain the last n time steps
- Output channels – predict the next m time steps
- Output is refeeded as input to predict time series

Transfer – Evaluation

RMSE with variance, validation loss and batch loss on Bigger Tub environment:

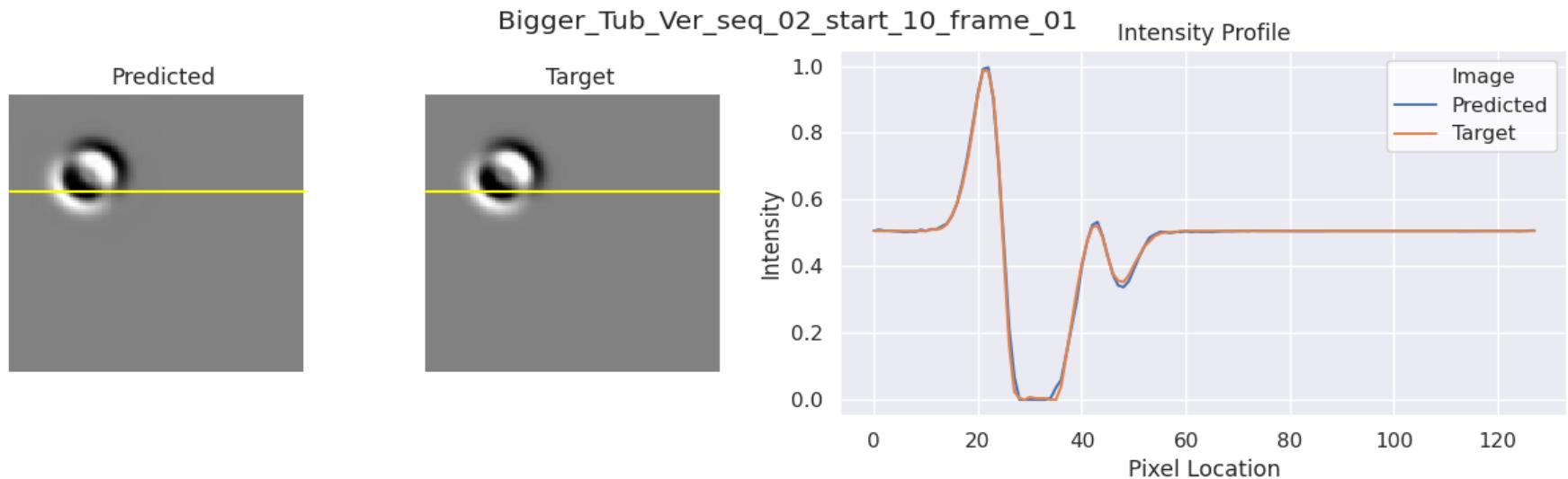


All plots and training in Transfer were made with https://github.com/stathius/wave_propagation

Transfer – Evaluation

Wave propagation prediction

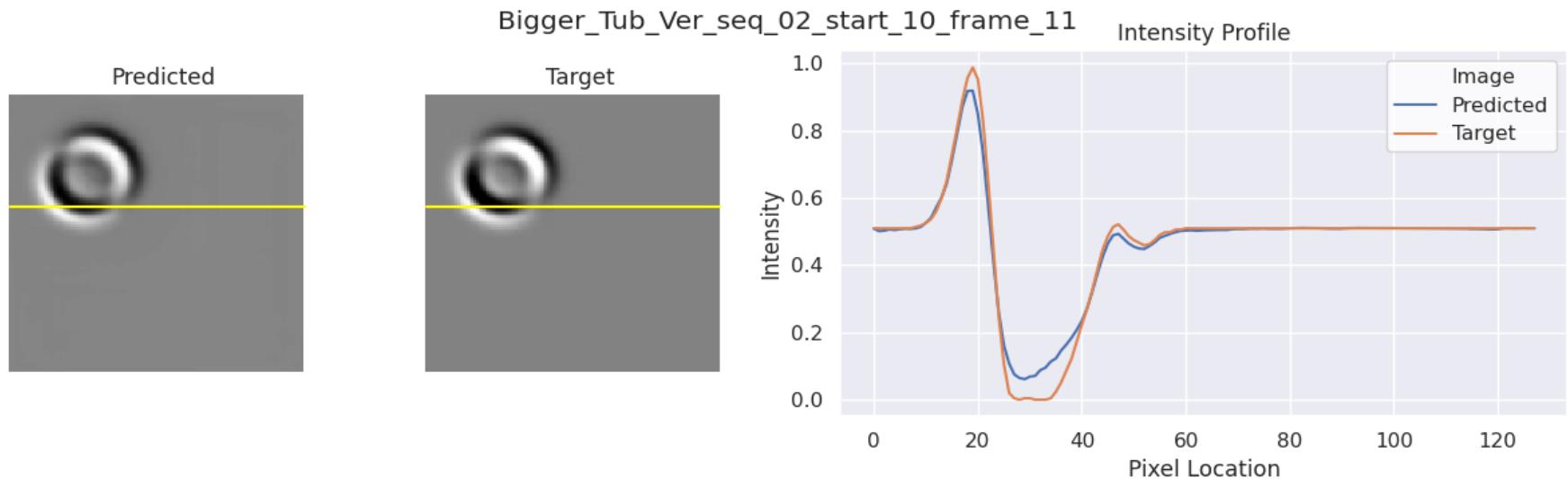
Intensity profile on scanline – Frame 1



Transfer – Evaluation

Wave propagation prediction

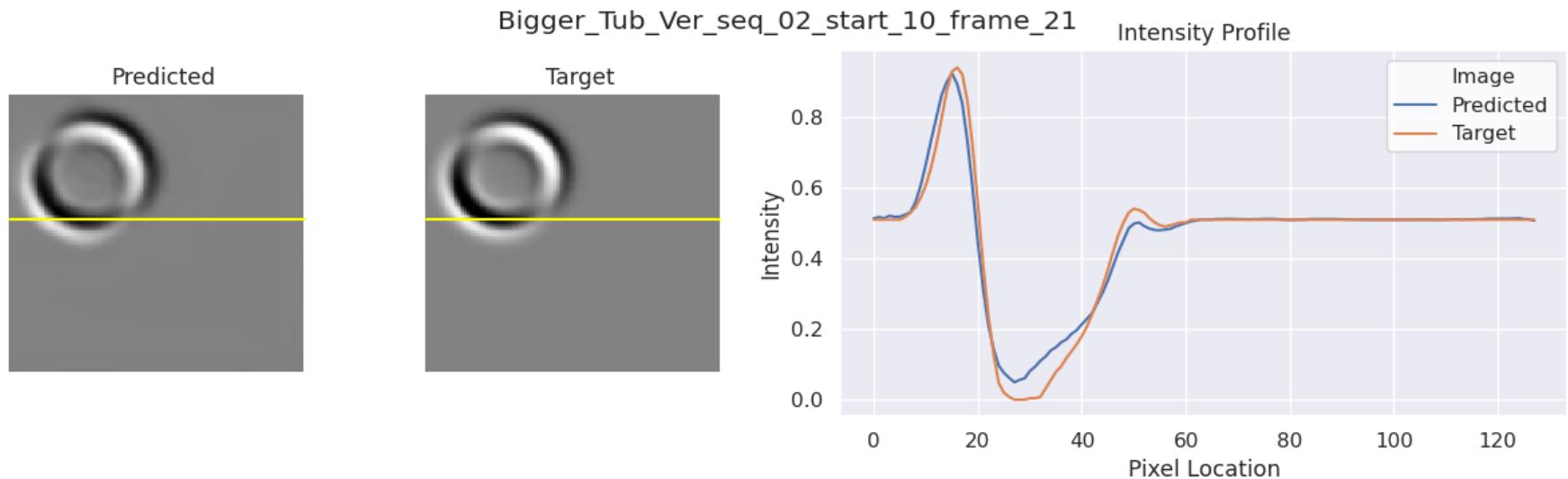
Intensity profile on scanline – Frame 11



Transfer – Evaluation

Wave propagation prediction

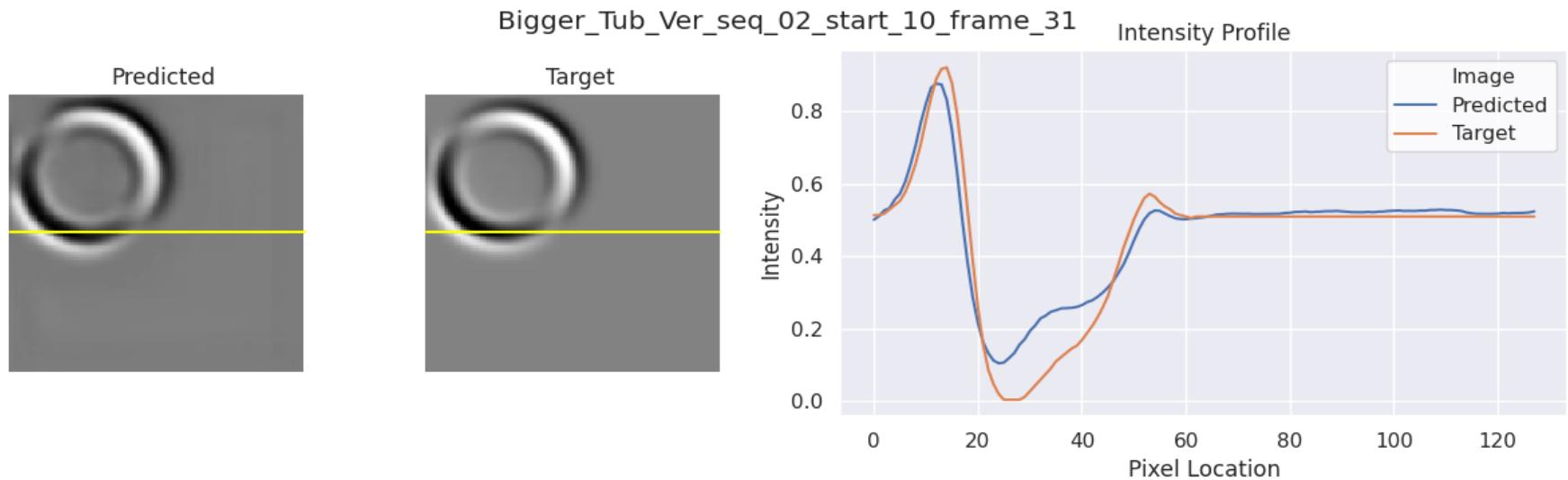
Intensity profile on scanline – Frame 21



Transfer – Evaluation

Wave propagation prediction

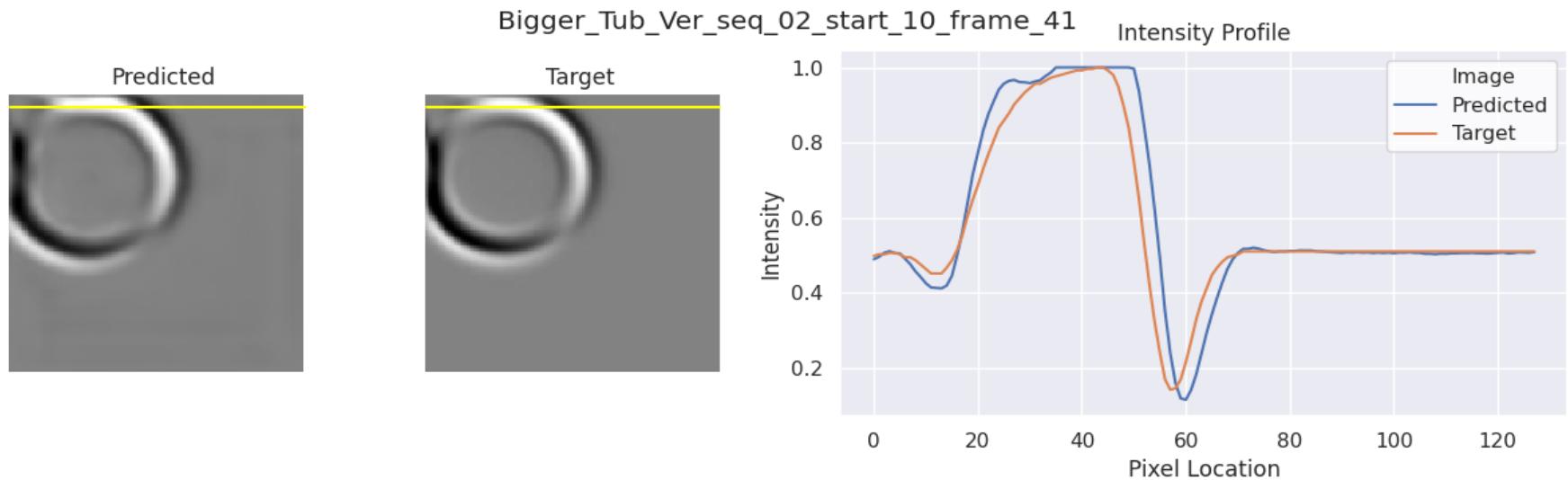
Intensity profile on scanline – Frame 31



Transfer – Evaluation

Wave propagation prediction

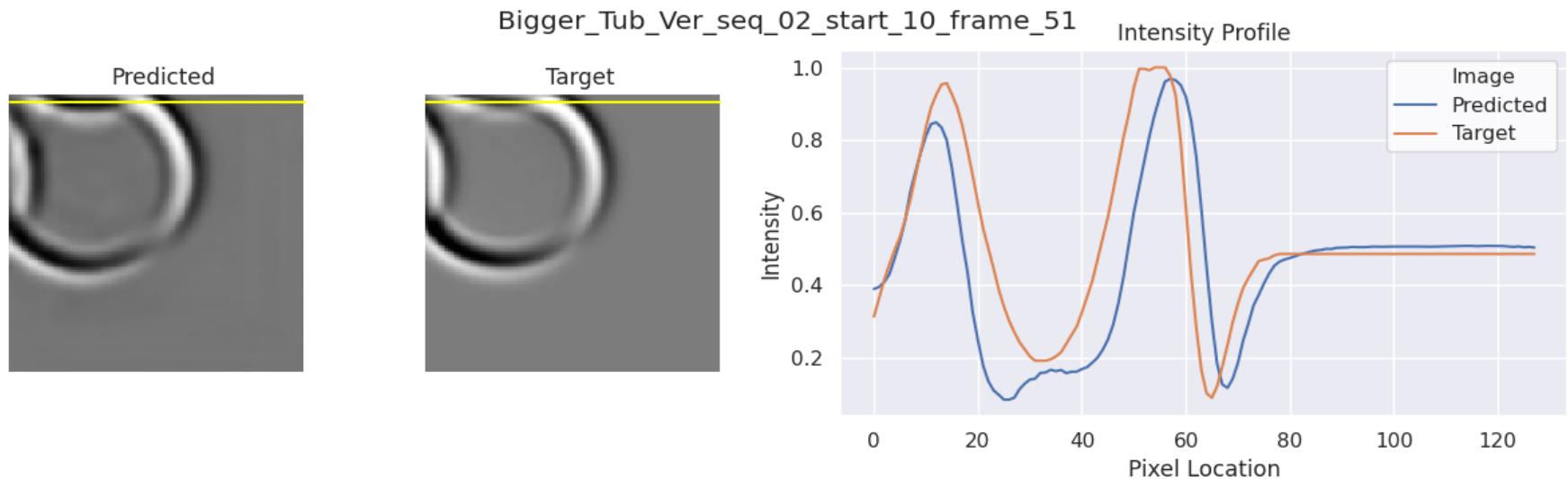
Intensity profile on scanline – Frame 41



Transfer – Evaluation

Wave propagation prediction

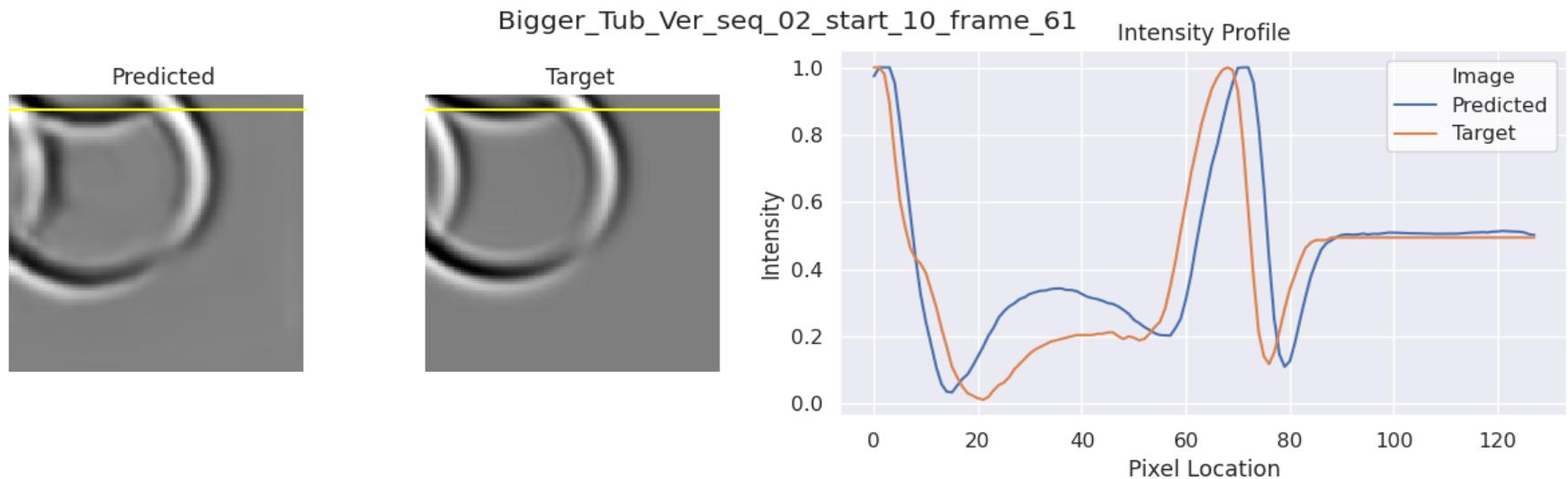
Intensity profile on scanline – Frame 51



Transfer – Evaluation

Wave propagation prediction

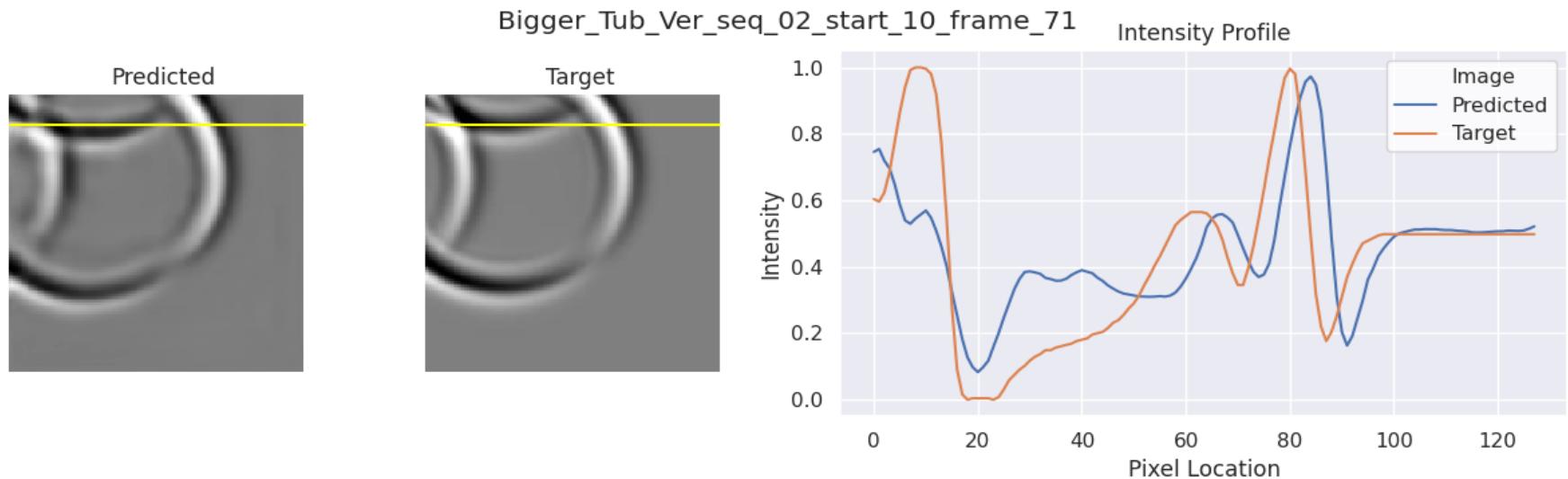
Intensity profile on scanline – Frame 61



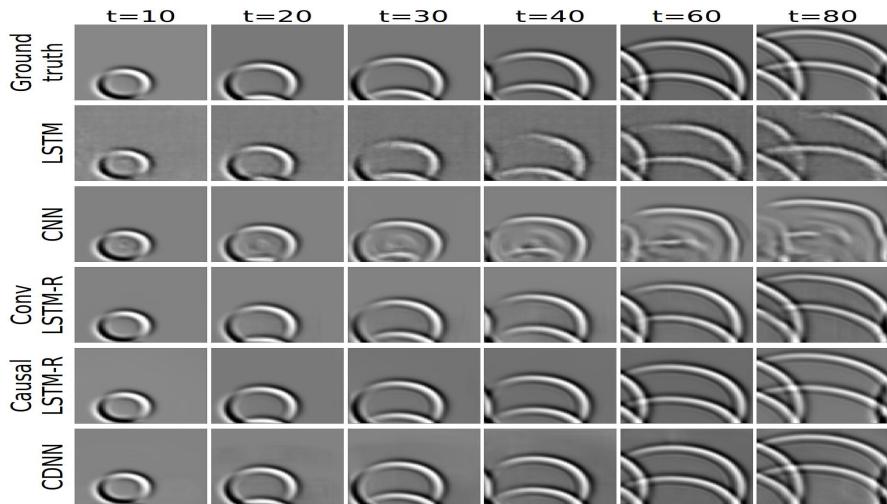
Transfer – Evaluation

Wave propagation prediction

Intensity profile on scanline – Frame 71



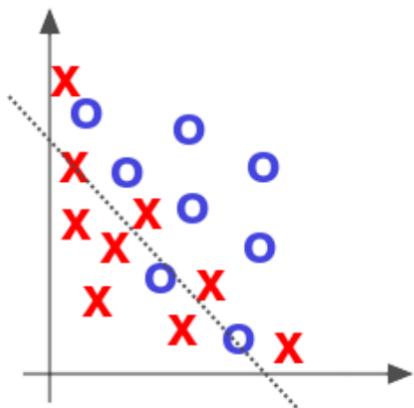
Transfer – Evaluation



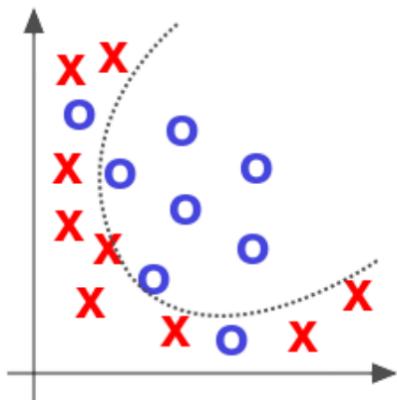
Taken from https://github.com/stathius/wave_propagation

Generalization

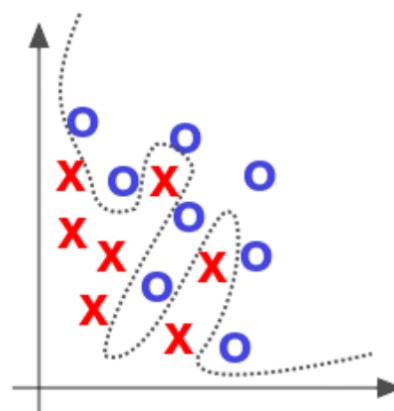
Motivation: Key question in deep learning: How well does my NN perform on unseen data?



Underfitted



Appropriate



Overfitted

Taken from Deep Learning by Adam Gibson, Josh Patterson, O'Reilly Media Inc., 2017

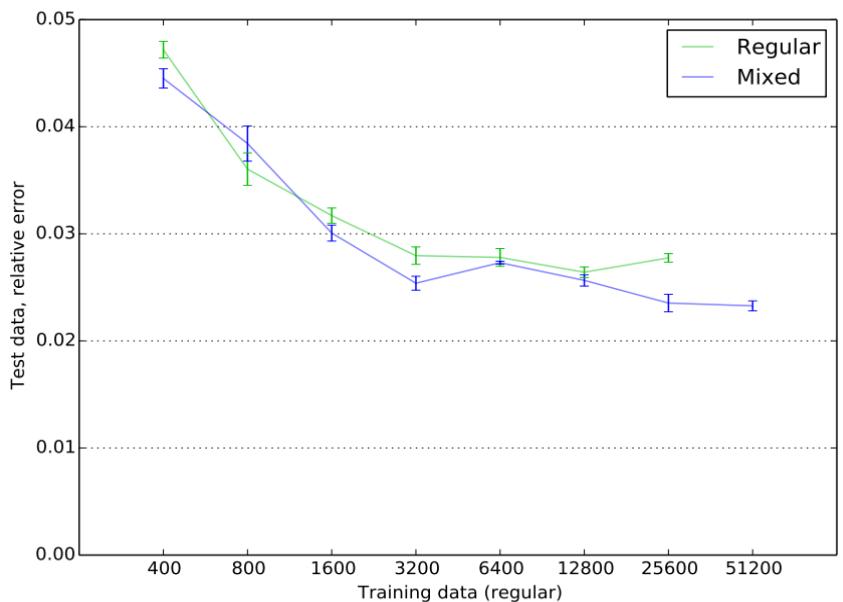
Generalization

Split up training data:

- Regular
- Mixed (50% regular, 50% sheared (± 15 degrees))

The plot shows training with a $30.9 \cdot 10^6$ parameter model

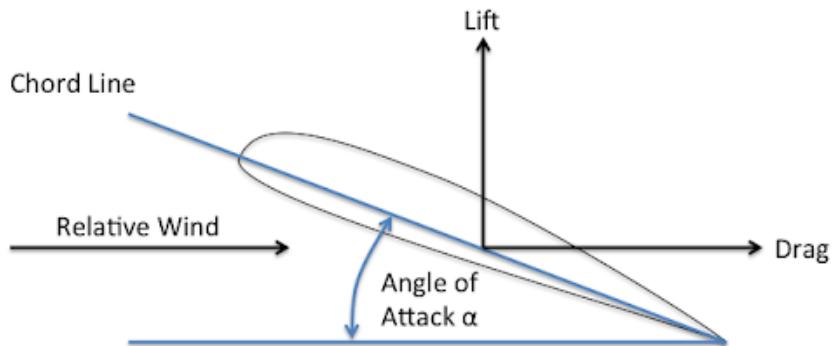
The high capacity supports training with the mixed dataset, achieving a even lower error



Taken from <https://arxiv.org/pdf/1810.08217.pdf>

Generalization – Evaluation

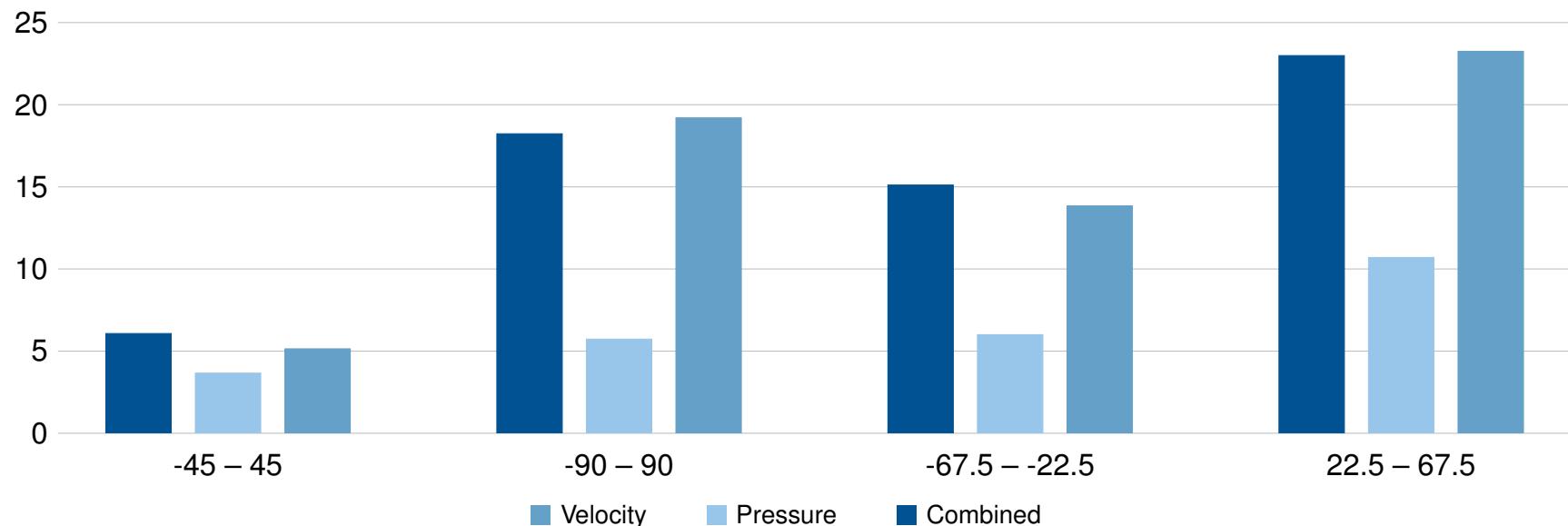
Generalization with different angels of attack:



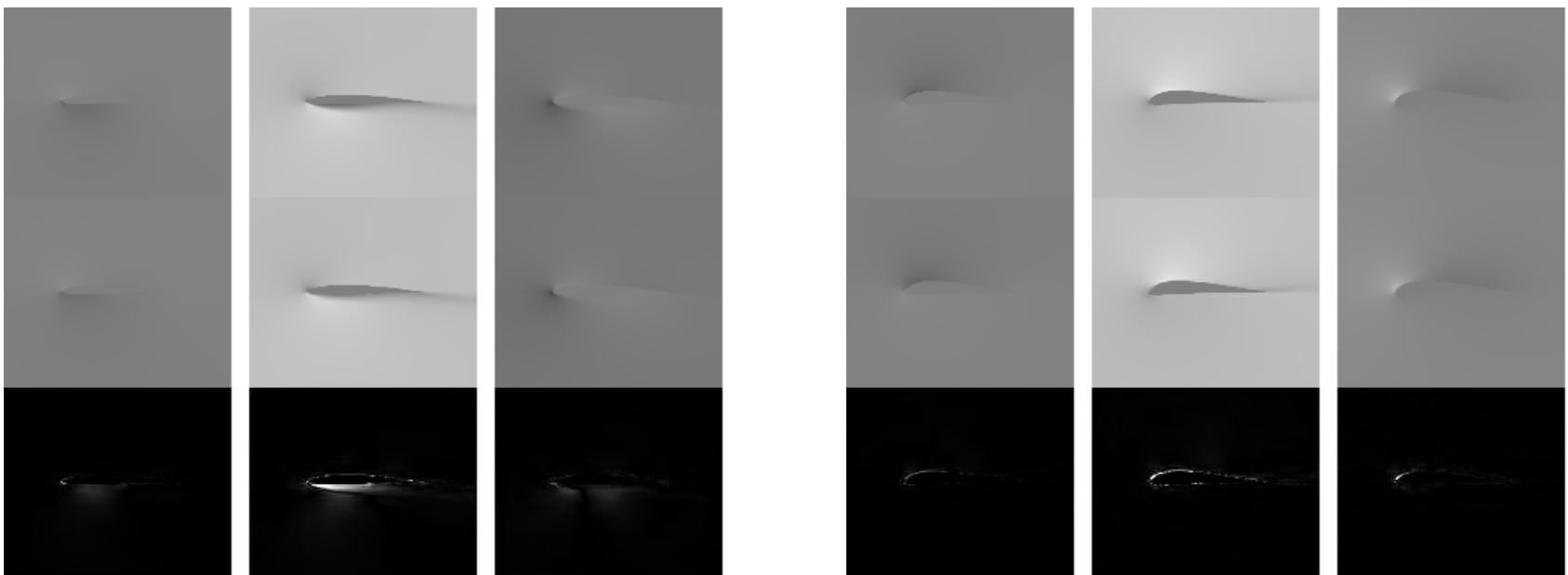
Taken from <http://www.aviationchief.com/angle-of-attack.html>

Generalization – Evaluation

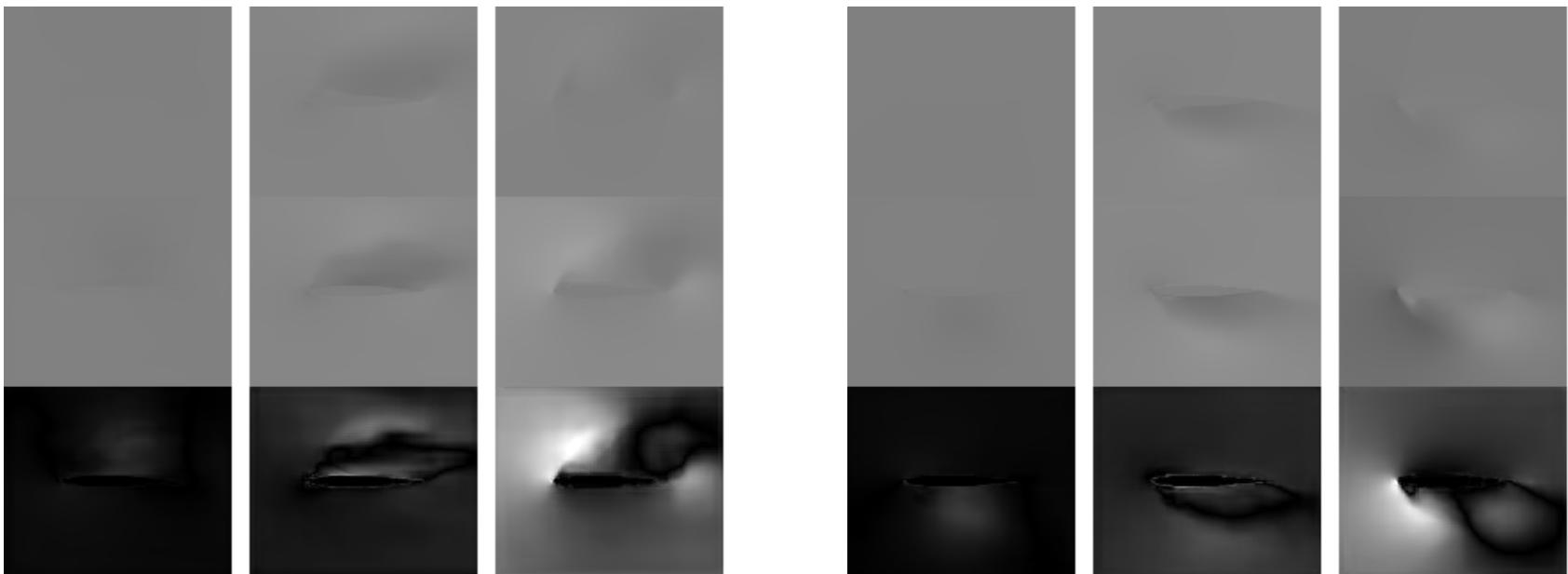
Error increase of different angle of attack intervals wrt. ground truth $[-22.5, 22.5]$



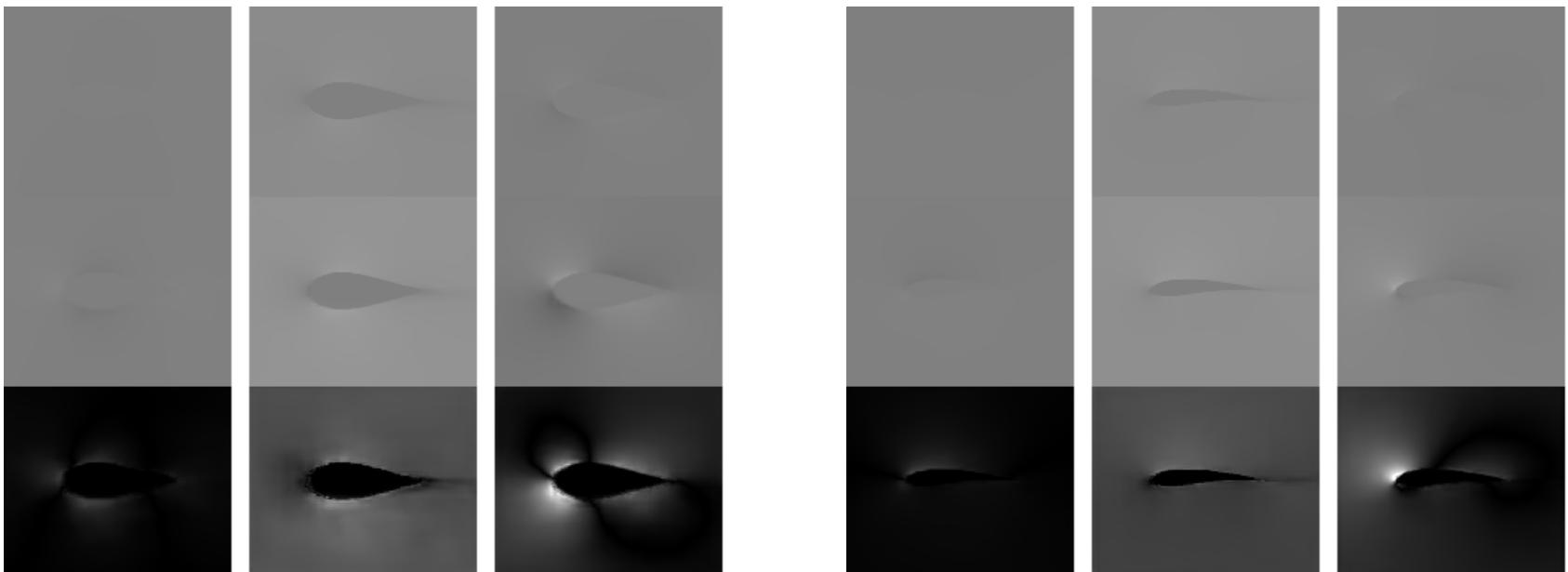
Generalization – $[-22.5, 22.5]$



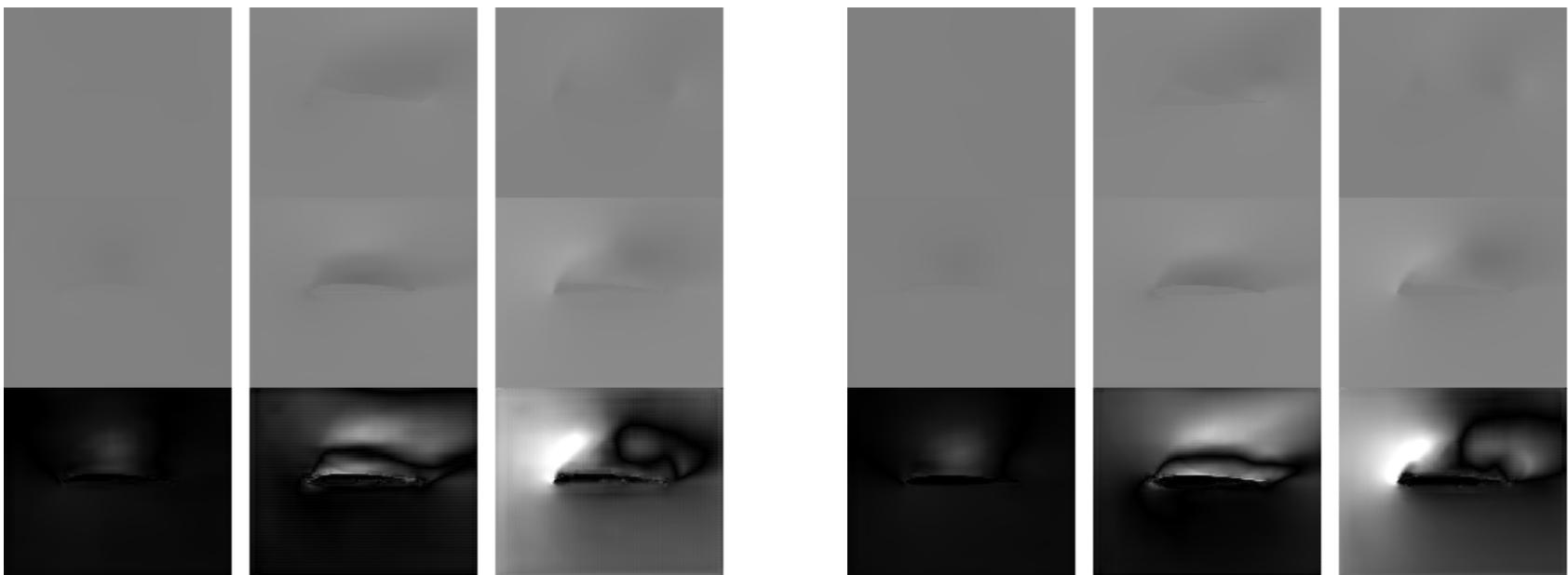
Generalization – $[-45, 45]$



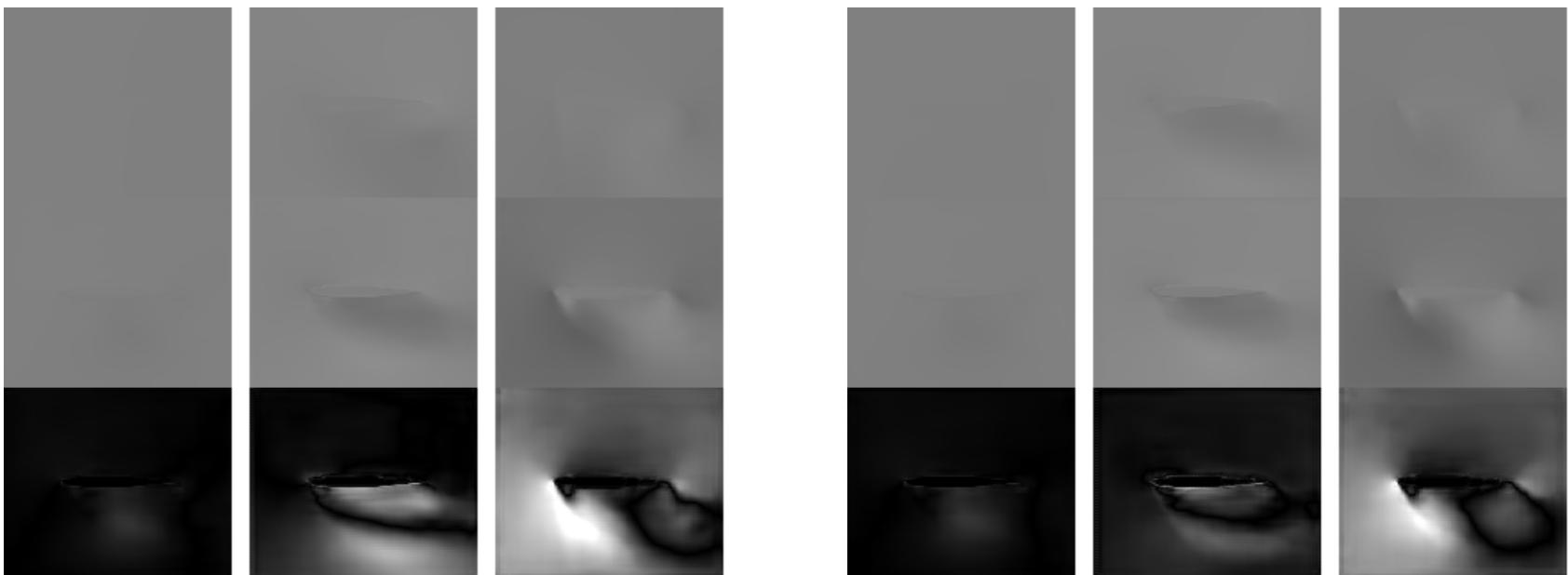
Generalization – $[-90, 90]$



Generalization – $[-67.5, -22.5]$

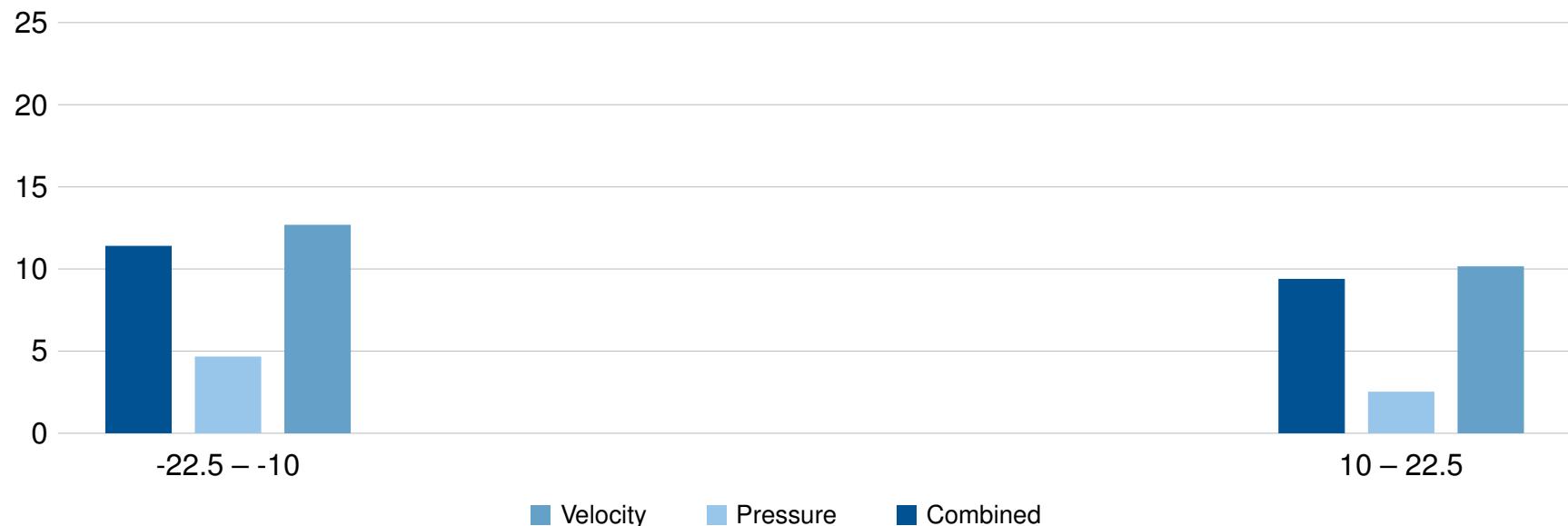


Generalization – [22.5, 67.5]

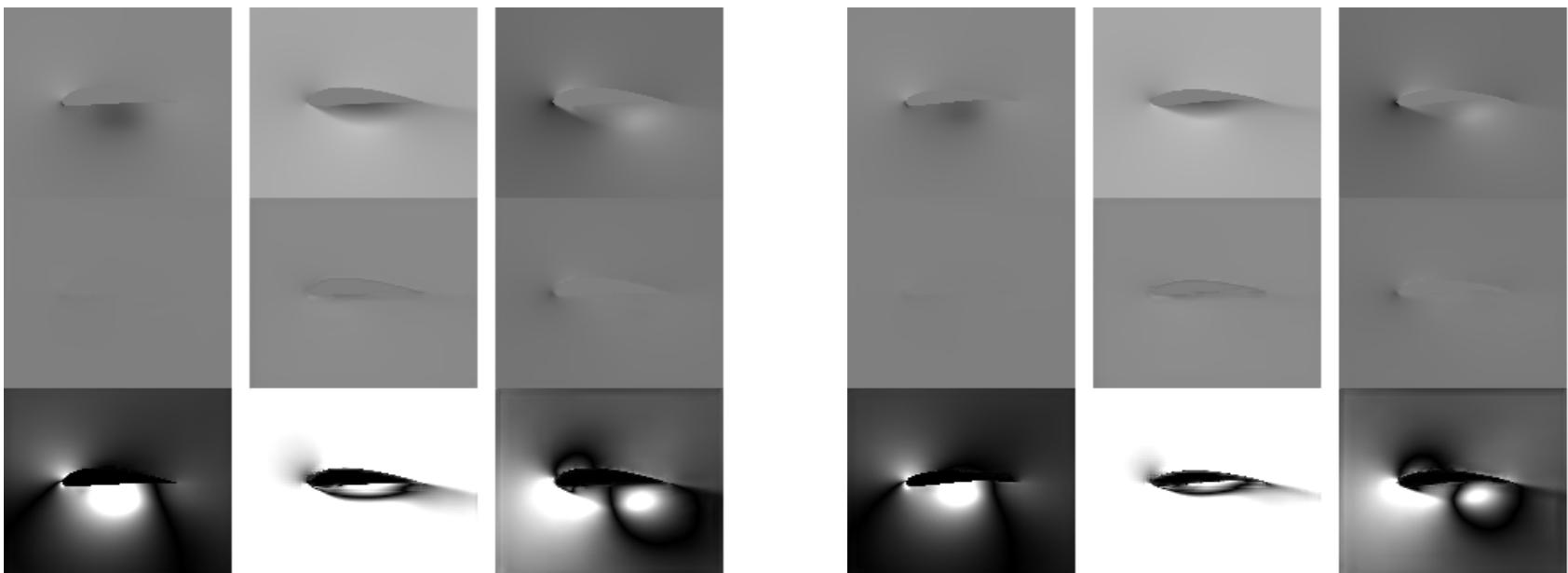


Generalization – Evaluation

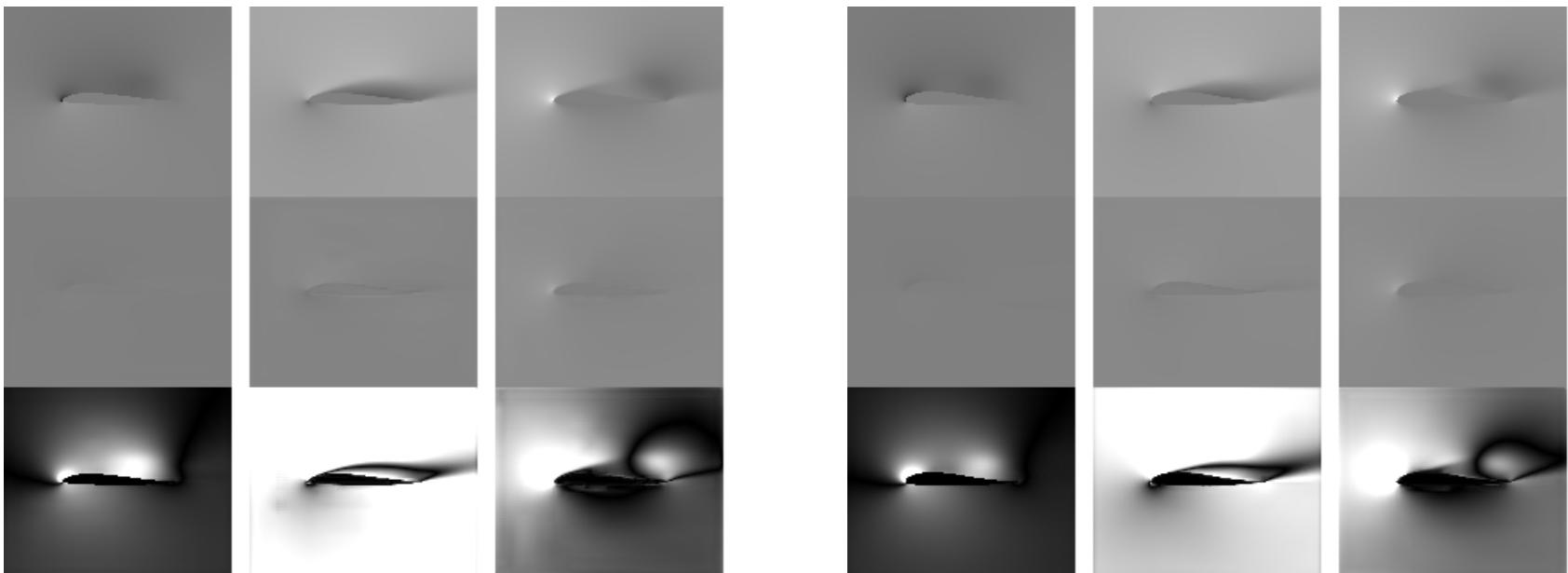
Error increase of different angle of attack interpolations wrt. ground truth $[-22.5, 22.5]$



Generalization – training: $[-10, 22.5]$, testing: $[-22.5, -10]$



Generalization – training: $[-22.5, 10]$, testing: $[10, 22.5]$



Discussion

Positiv

- Relative error < 3%
- Convolutions paired with an encoder decoder structure seem to catch regions of interest fast and reliable
- U-Nets can outperform LSTM's in accuracy as well as in speed with a fraction of capacity (in time-series problems)
- accuracy does not suffer too much from models with a lot less capacity mostly affects sharpness of solutions
- Inference speed is $1000\times$ when compared with OpenFOAM solver
- Accuracy improvements still possible (bigger models, more training data)

Discussion

Negativ

- Prior knowledge needed for proper pre-processing
- Solvers needed for dataset generation
- Extrapolation yields mediocre results
- Fresh Training needed for other shapes (e.g. cars in wind tunnel)
 - transfer learning unlikely
- Trade off: training speed – grid resolution
- Possible data loss from transformation:
 - adaptive grid (solver) \implies cartesian grid (NN)
- no guarantee for correctness
- Accuracy improvements computationally expensive likely requires tailored architectures and loss functions

Summary

Investigate the accuracy of U-Net models for the inference of Reynolds-Averaged Navier-Stokes solutions

Data Generation $6 \times 128 \times 128$

- Input (encodes Reynolds number): Bit Mask, x & y velocity
- Target (RANS solution): Pressure, x & y velocity

Pre-Processing

- Make data dimensionless, flatten space of solutions
- Pressure offset removal, numerical precision

Architecture

- U-Net – Encoder - Bottleneck - Decoder structure
- Activations highly depend on current task

Transfer

- U-Net as time-series prediction NN for wave propagation
- Input: last n frames, Output: next m frames, refeed

Generalization

- NN performance on unseen data: Different angels of attack
- mediocre performance on wider intervals (velocity)

Discussion

- low error (improvable), speed up, even with low capacity
- prior knowledge and solvers needed, poor extrapolation

Backup slides

Backup slides – Training Setup

Adam optimizer ($\beta_1 = 0.5, \beta_2 = 0.999$)

Learning rate: 0.0004

Learning rate decay: On

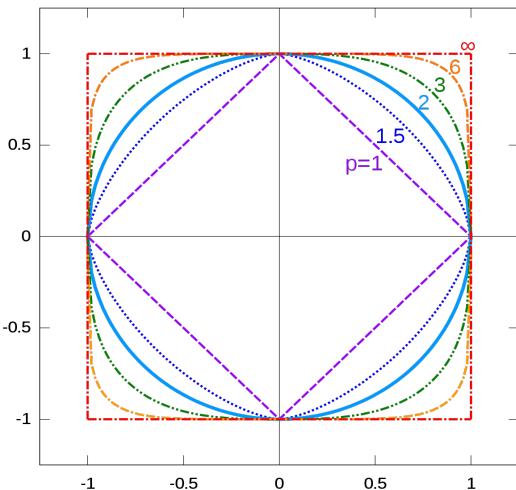
Batch size: 10

Iterations: 80000

Model parameters:

122.979, 487.107, 1.938.819, 7.736.067, 30.905.859

Backup slides – Norms on unit circle



Taken from: [https://de.wikipedia.org/wiki/Norm_\(Mathematik\)#/media/Datei:Vector-p-Norms_qtl1.svg](https://de.wikipedia.org/wiki/Norm_(Mathematik)#/media/Datei:Vector-p-Norms_qtl1.svg)

Backup slides – Navier-Stokes

Navier-Stokes equation for incompressible flow:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u - \nu \nabla^2 u = -\nabla(\frac{p}{\rho_0}) + g$$

- u : velocity
- p : pressure
- ν : kinematic viscosity
- ρ_0 : uniform density
- g : gravitational acceleration

Backup slides – Saint-Venant

Saint-Venant equations for incompressible flow (1D):

$$\begin{aligned}\frac{\partial A}{\partial t} + \frac{\partial(Au)}{\partial x} &= 0 \\ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} &= -\frac{P}{A} \frac{\tau}{\rho}\end{aligned}$$

- x : coordinate
- t : time
- $A(x, t)$: cross-sectional area of the flow at x
- $u(x, t)$: flow velocity
- $\zeta(x, t)$: free surface elevation
- $\tau(x, t)$: wall shear stress along the wetted perimeter $P(x,t)$ of the cross section at x
- ρ : (constant) fluid density
- g : gravitational acceleration