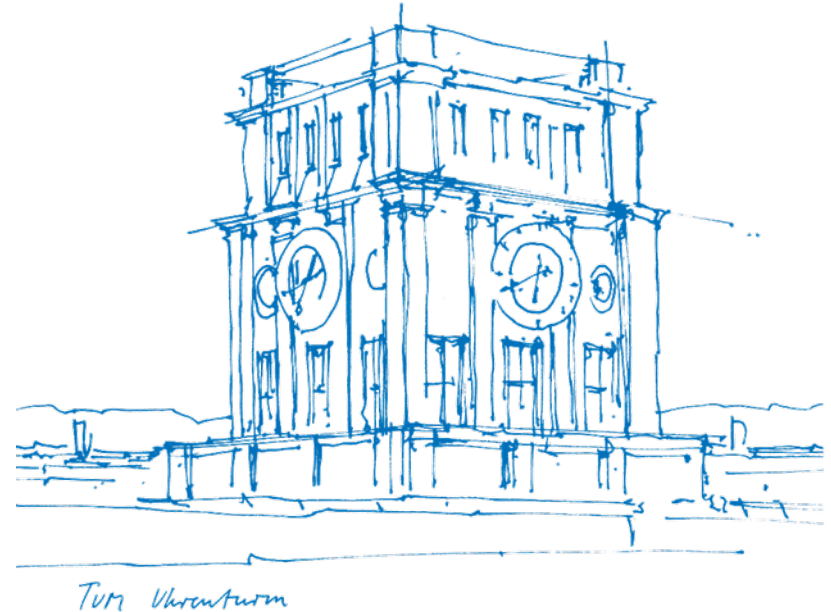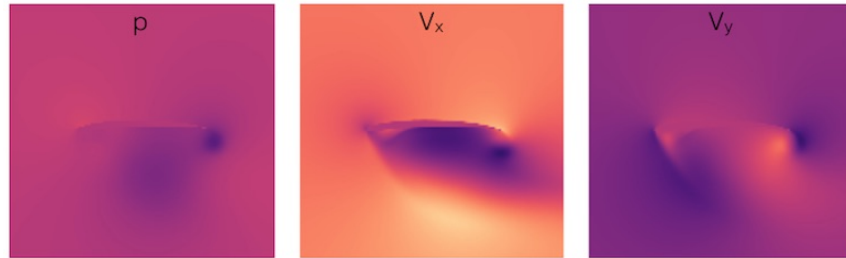# Deep Learning Methods for Reynolds-Averaged Navier-Stokes Simulations of Airfoil Flows
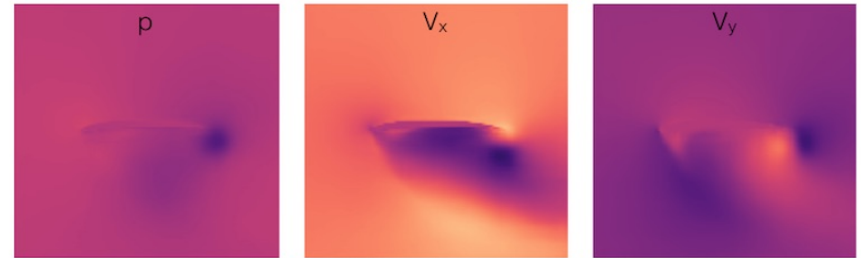
Julian Hohenadel

Technical University of Munich

Chair of Computer Graphics and Visualization

Munich, 11. May 2020

# What this paper is about

Target / ground truth solution

Solution inferred by trained CNN

Taken from `https://github.com/thunil/Deep-Flow-Prediction`

What this paper is about

- Reynolds Averaged Navier-Stokes solutions
- Airfoil shapes
- Deep learning – CNN
- U-Net architecture derivative
- supervised training environment
- Inference of pressure and velocity fields
- Generalization
- Evaluation

Background – RANS

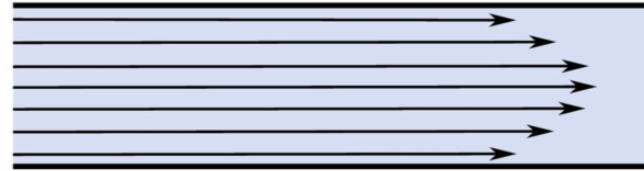Taken from `https://www.pinterest.ch/pin/615163630322034457/`
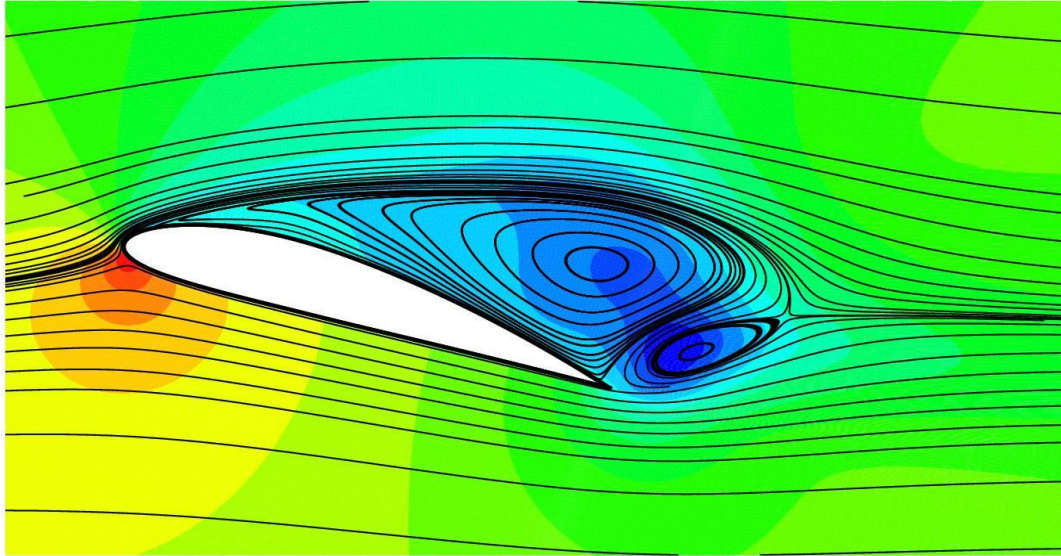
Background – RANS

Nonlinear partial differential equation (PDE) system
based on Navier-Stokes equations
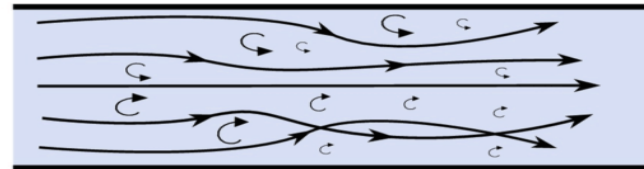
Used for the modeling of turbulent incompressible flows
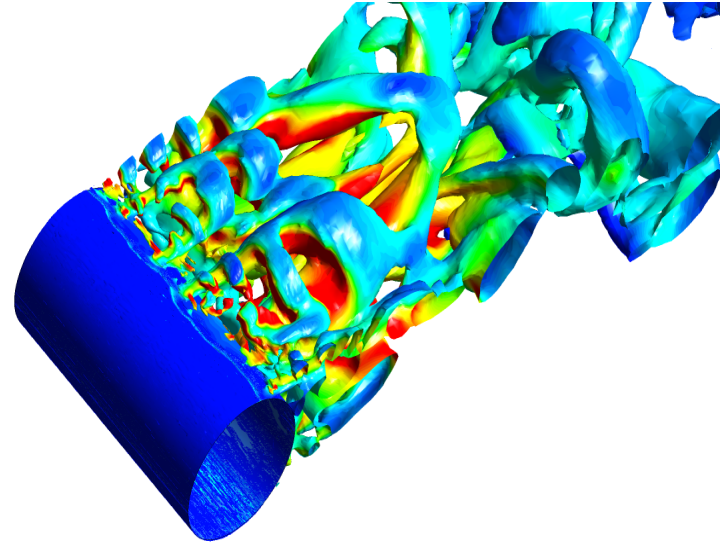
Averages over time component

laminar flow

turbulent flow

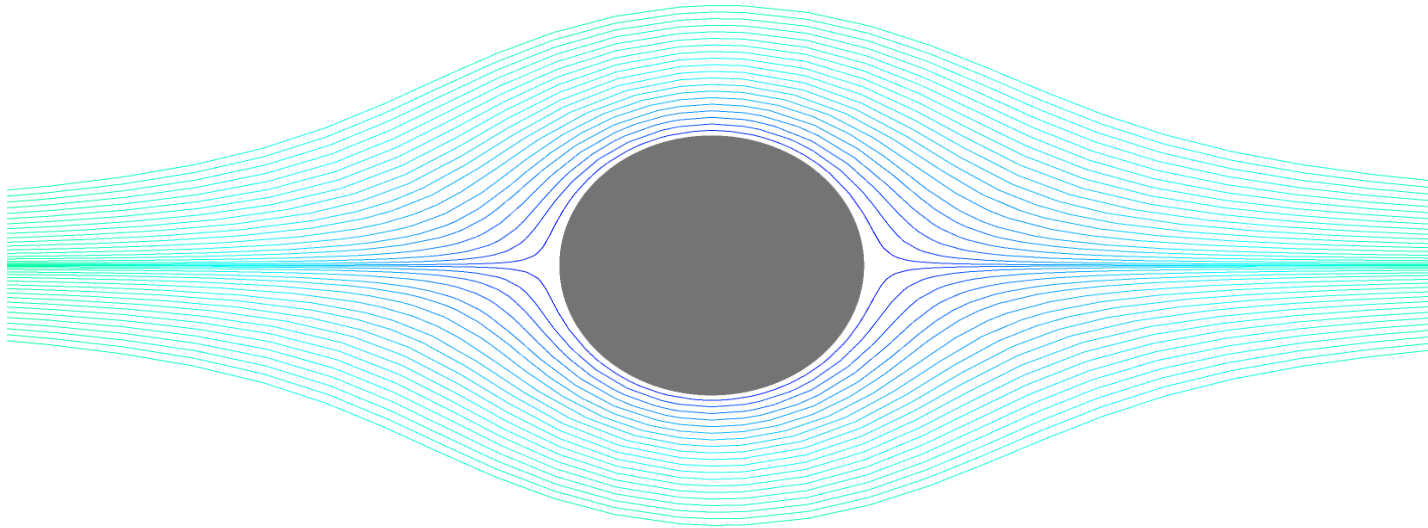Taken from `https://diffzi.com/laminar-flow-vs-turbulent-flow/`

Background – RANS

Reynolds number *Re*:

- dimensionless constant
- needed for calculation of turbulence models
- magnitude decides flow (laminar, turbulent)
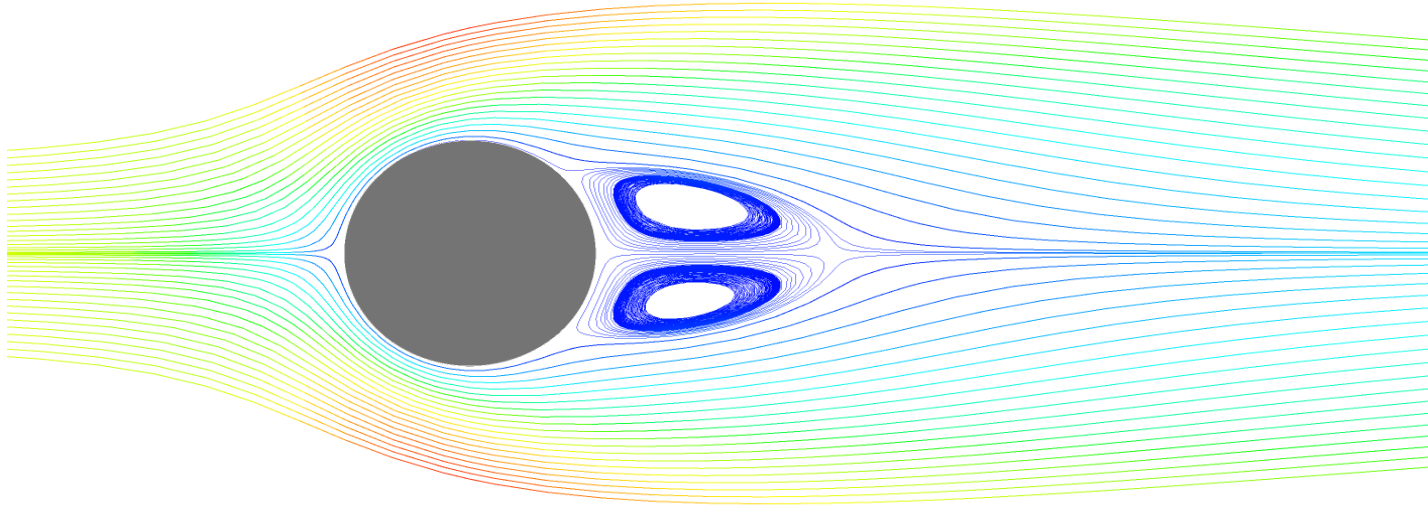- affects lift and drag coeffcients

Taken from `https://www.computationalfluiddynamics.com.au/reynolds-number-cfd/`

Background – Reynolds number: < 1

Taken from `https://www.computationalfluiddynamics.com.au/reynolds-number-cfd/`

Background – Reynolds number: ≈ 10

Taken from `https://www.computationalfluiddynamics.com.au/reynolds-number-cfd/`

Background – Reynolds number: $\approx 1 \cdot 10^5$

Taken from `https://www.computationalfluiddynamics.com.au/reynolds-number-cfd/`
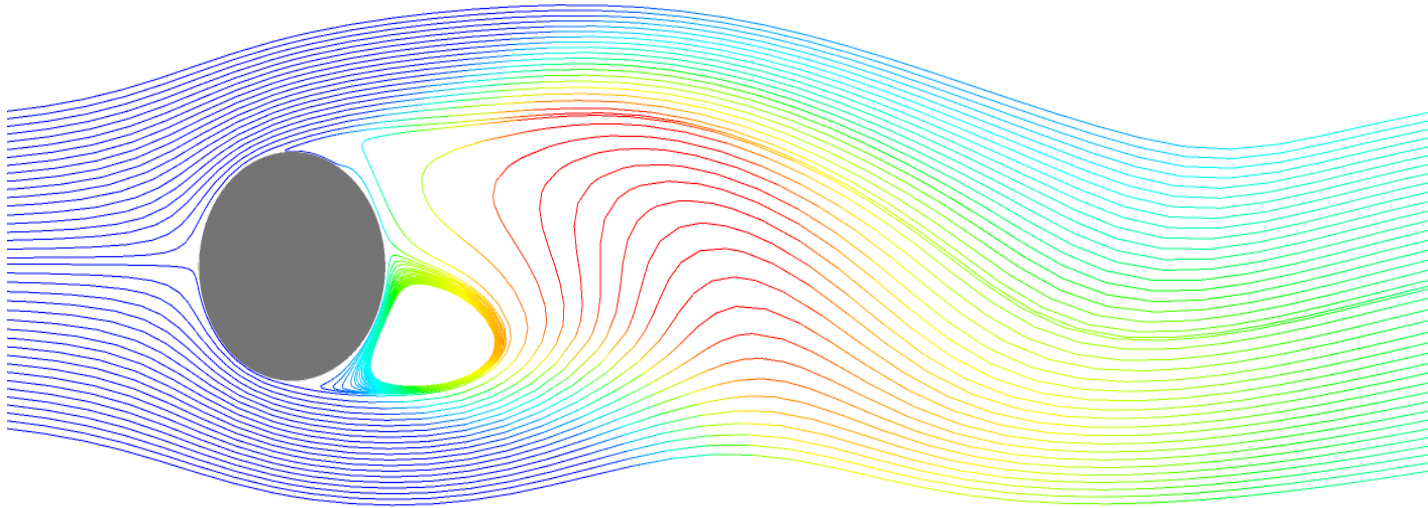
Background – Reynolds number: $\approx 1 \cdot 10^6$

Taken from `https://www.computationalfluiddynamics.com.au/reynolds-number-cfd/`
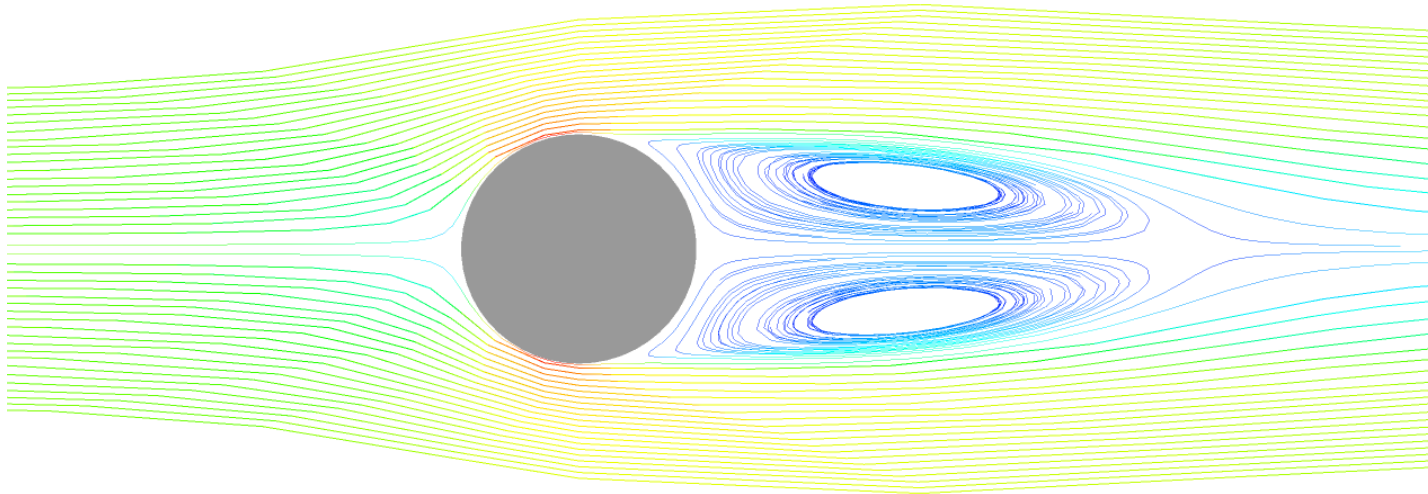
Background – Convolutions

$32 \times 32 \times 3$ image
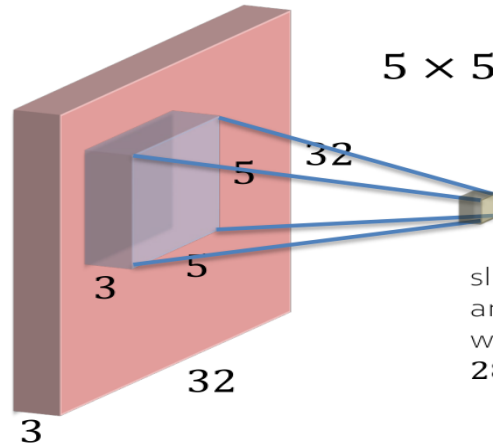
$5 \times 5 \times 3$ filter

Convolve

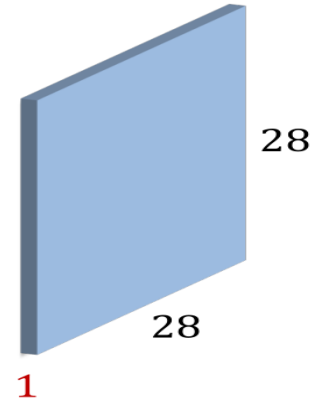slide over all spatial locations x_i and compute all output z_i; w/o padding, there are 28×28 locations
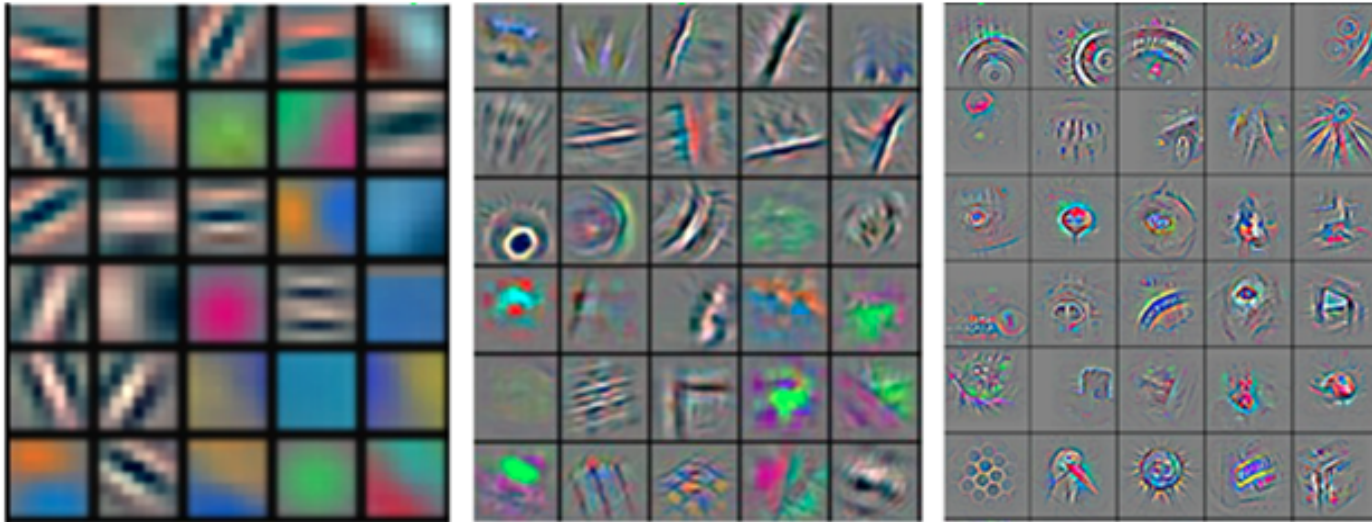
activation map (also feature map)

32

5

3

5

3

32

28

28

1

Taken from I2DL WS19/20 (TUM)

Background – Convolutions

Low-Level Features, Mid-Level Features, High-Level Features: each filter captures different characteristics

Taken from `https://arxiv.org/pdf/1311.2901.pdf`
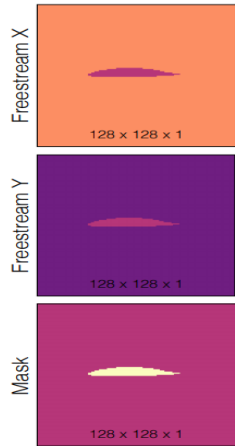
Data Generation

- Airfoil shapes from UIUC database
- Reynolds number: $[0.5, 5] \cdot 10^6$ (highly turbulent)
- Angle of attack: $[-22.5, 22.5]$
- Ground truth generated with OpenFOAM
  (pressure, x velocity, y velocity)
- Training data resolution: $3 \times 128 \times 128$
  (Inference region $<$ full simulation domain)

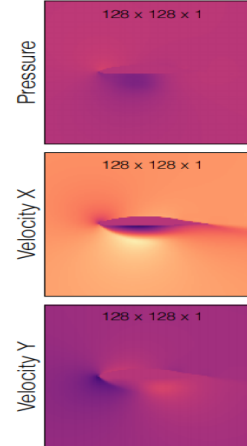Taken from https://m-selig.ae.illinois.edu/ads/afplots/ag35.gif
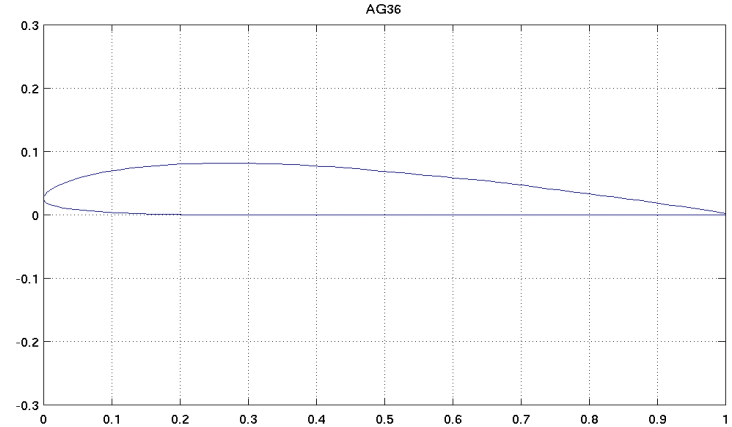
Pre-processing – Data

Input channels

Reynolds number encoded as differently scaled
freestream velocity vectors wrt. their magnitude

Target channels

Data from the RANS solution

Pre-processing – Normalization

Motivation: Flatten space of solutions, accelerate learning by simplifing the learning task for the NN

Normalization of target channels by division with freestream magnitude (vector norm, default: L2):
This makes pressure and velocity dimensionless

$$\tilde{v}_o = \frac{v_o}{\|v_i\|}, \quad \tilde{p}_o = \frac{p_o}{\|v_i\|^2} \text{ – important to remove quadratic scaling of pressure}$$

For a better understanding:
Pressure: $[p]_{SI} = 1\,Pa = 1\frac{kg}{m \cdot s^2}$
Density: $[\rho]_{SI} = 1\frac{kg}{m^3}$ – constant in incompressible flow
Velocity: $[v]_{SI} = \frac{m}{s}$

Pre-processing – Offset removal & value clamping

Motivation: eliminate ill-posed learning goal & improve numerical precision

Spatially move pressure distribution into the origin – RANS typically only needs ∇_p for computation

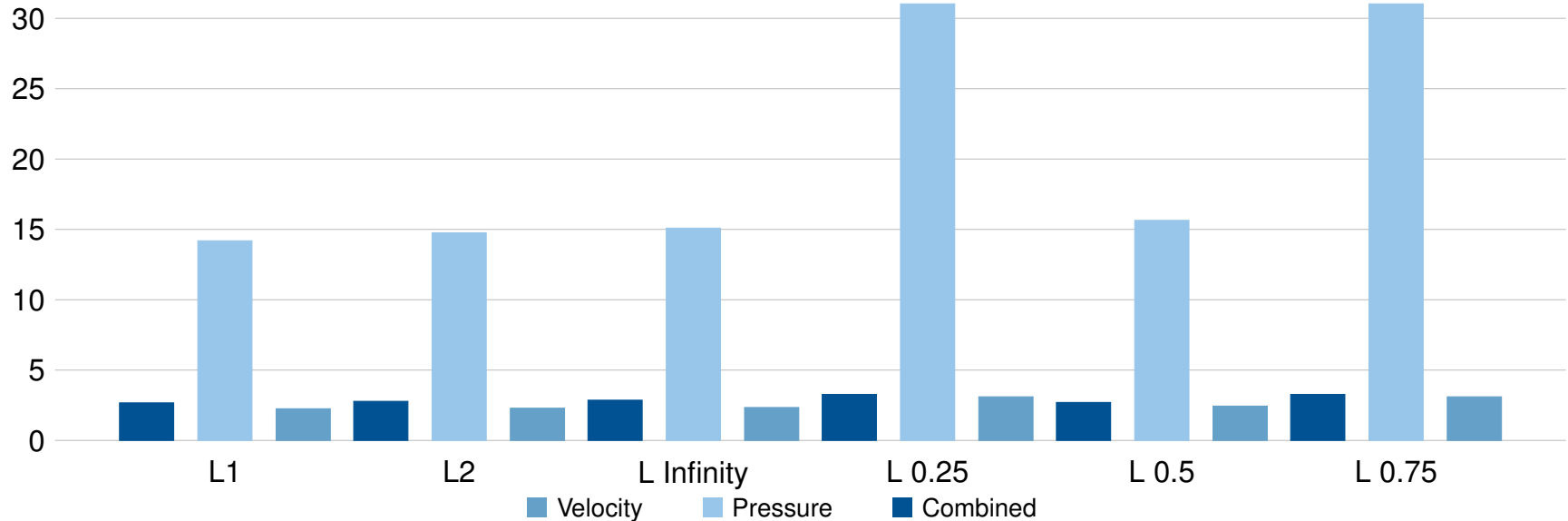$$\hat{p_o} = \tilde{p_o} - p_{mean}$$

Clamp both input and target channels into $[-1, 1]$ range by diving by the maximum absolute value
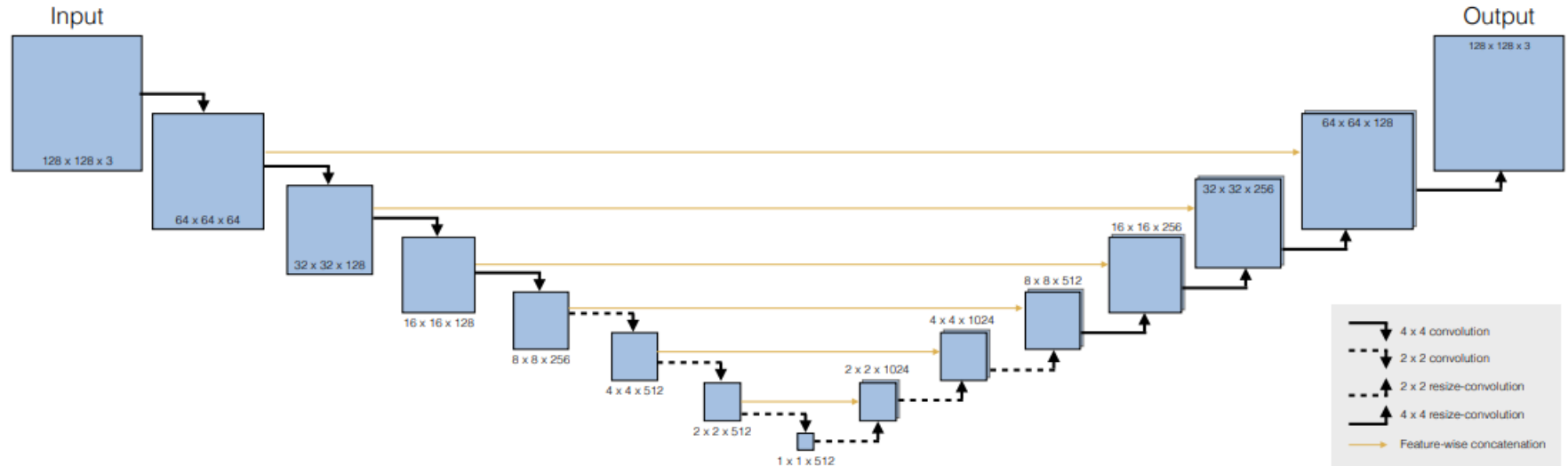
Pre-processing – Evaluation

Vector norms used in pre-processing comparision wrt. error, default: L2 (in %)

L1 normalization achieves the best error rates (p, vel, combined: **14.19**%, **2.251**%, **2.646**% – L2: 14.76%, 2.291%, 2.780%)

Architecture

U-Net derivative proposed in the paper:

Taken from https://arxiv.org/pdf/1810.08217.pdf

Architecture – Convolutional blocks

Encoder

1. Activation – Leaky ReLu (0.2)

2. Convolution – Width down, Depth up
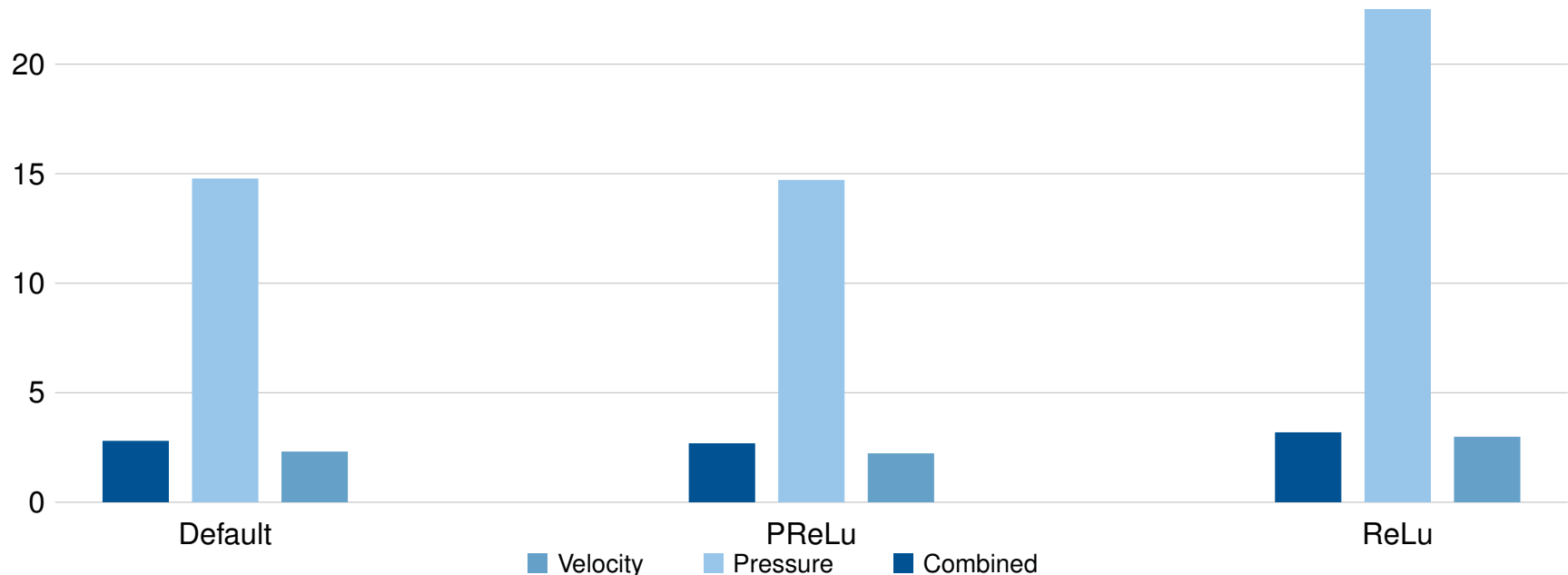
3. Batch normalization

4. Dropout (1%)

Decoder

1. Activation – ReLu

2. Upsampling – linear (2.0)

3. Convolution – Width up, Depth down

4. Batch normalization

5. Dropout (1%)
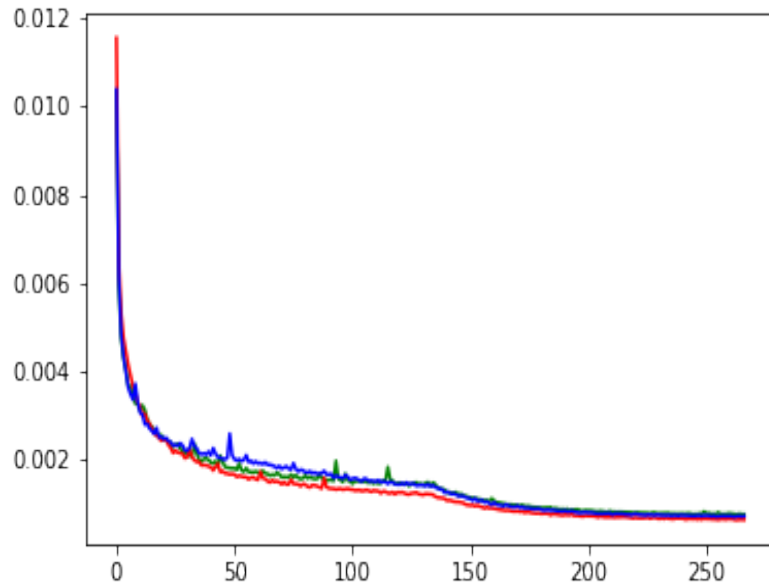
Architecture – Evaluation

Error percentage of different activation functions after 160k iterations (266 epochs).

PReLu achieves the best error rates (p, vel, combined: **14.69**%, **2.216**%, **2.676**% – Default: 14.76%, 2.296%, 2.787%)
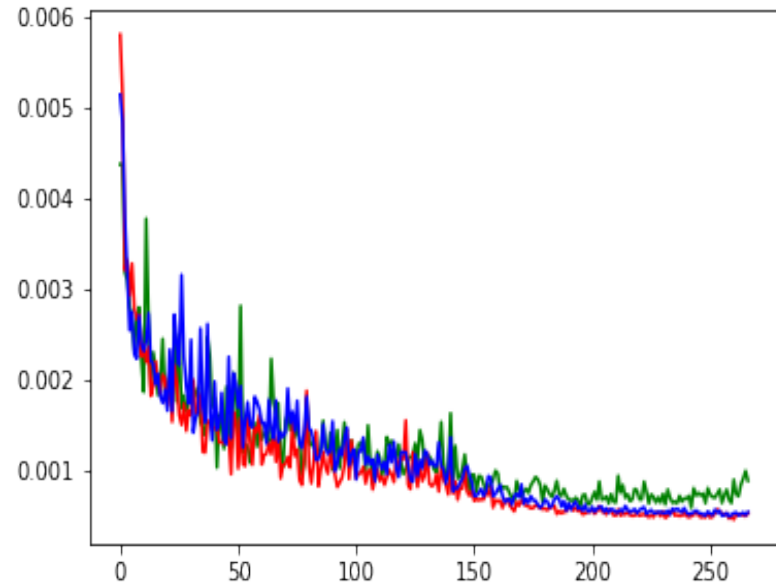
Architecture – Evaluation

Training loss

Validation loss

Transfer

Motivation: Can the network architecture adapt to other PDE systems and how well will it perform?

Another use case for PDE systems like RANS is predicting wave propagation on shallow water

Wave propagation, in this case, is governed by the Saint-Venant equations (related with Navier-Stokes equations)

U-Net architecture changes:

- Input channels – contain the last n time steps
- Output channels – predict the next m time steps
- Output is refeeded as input to predict time series

Transfer – Evaluation

RMSE with variance, validation loss and batch loss on Bigger Tub environment:
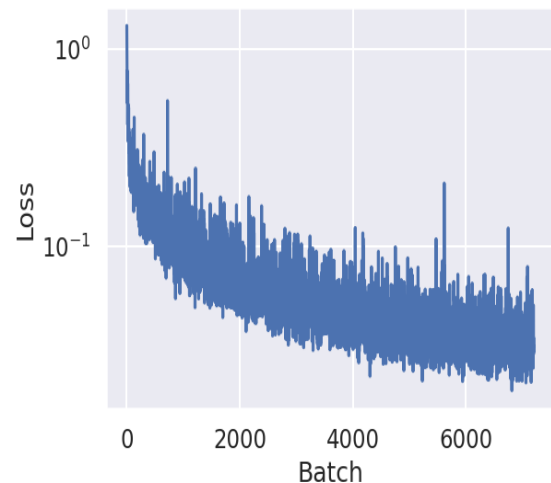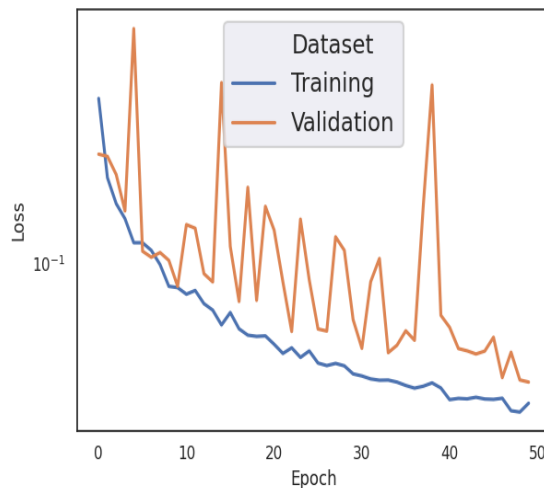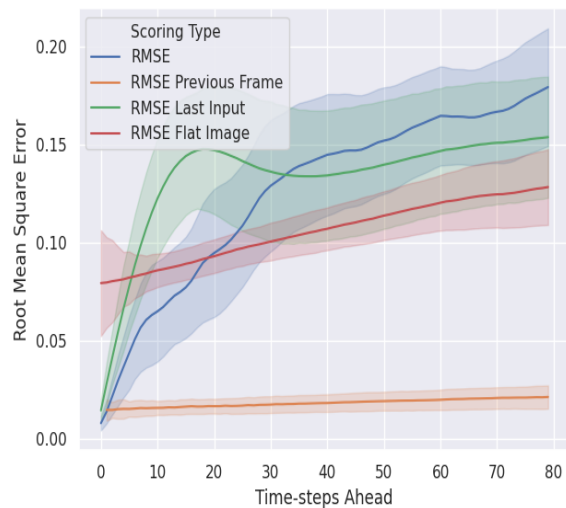
All plots and training in Transfer were made with `https://github.com/stathius/wave_propagation`

Transfer – Evaluation

Wave propagation prediction

Intensity profile on scanline – Frame 1
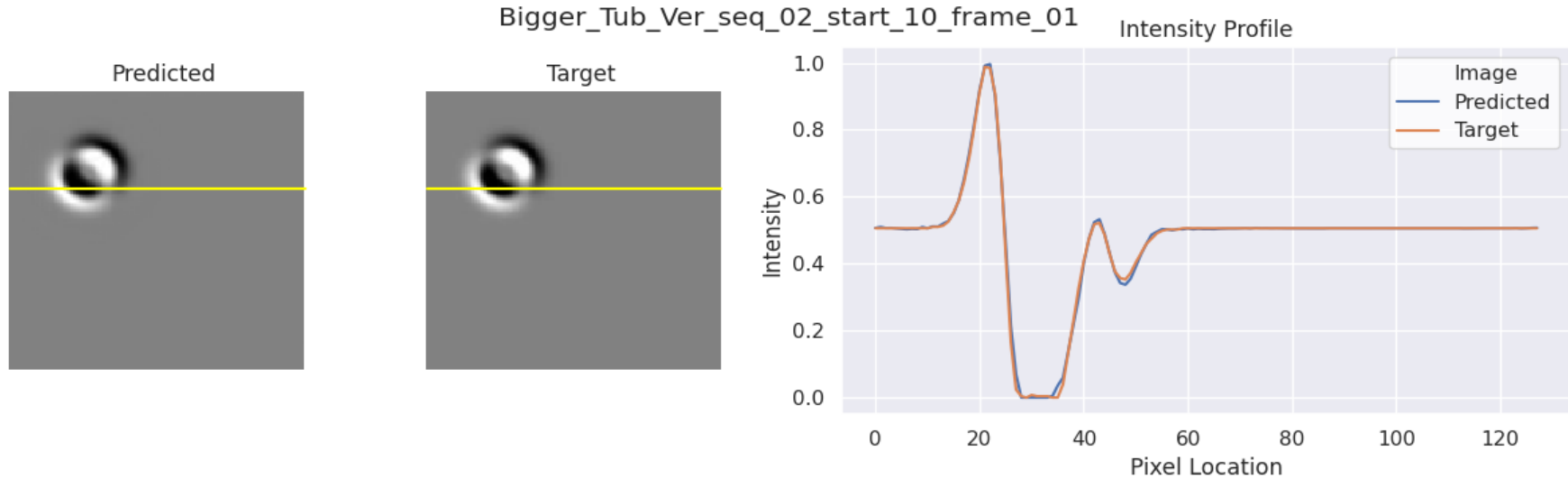
Transfer – Evaluation

Wave propagation prediction

Intensity profile on scanline – Frame 11

Transfer – Evaluation

Wave propagation prediction

Intensity profile on scanline – Frame 21

Transfer – Evaluation

Wave propagation prediction

Intensity profile on scanline – Frame 31

Transfer – Evaluation

Wave propagation prediction

Intensity profile on scanline – Frame 41
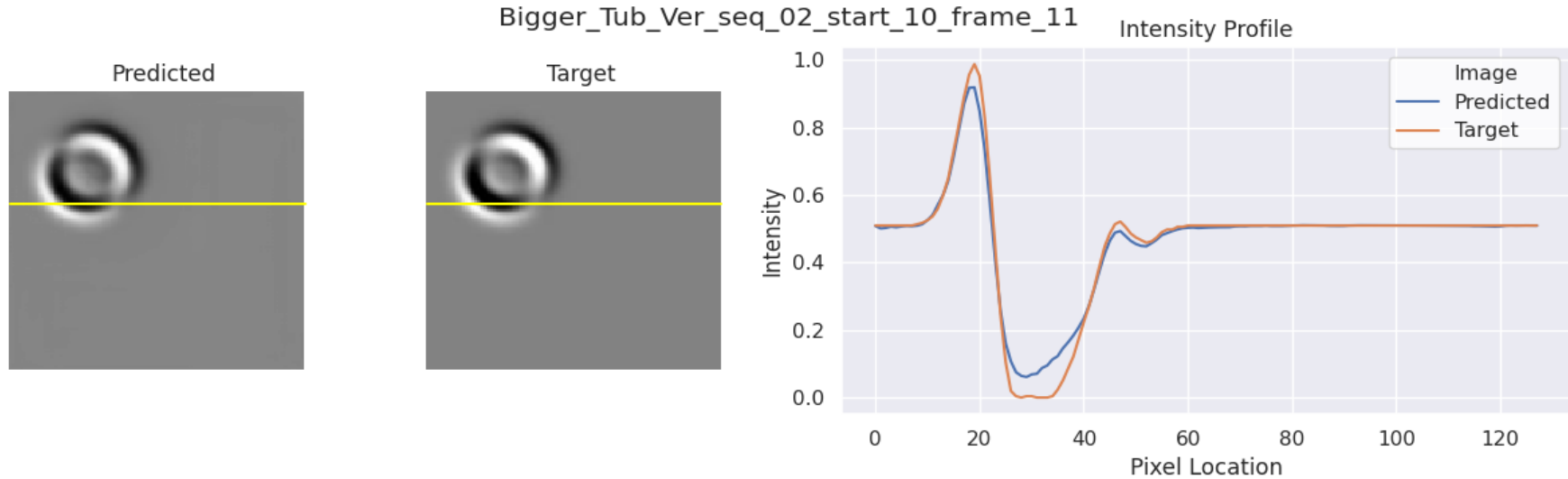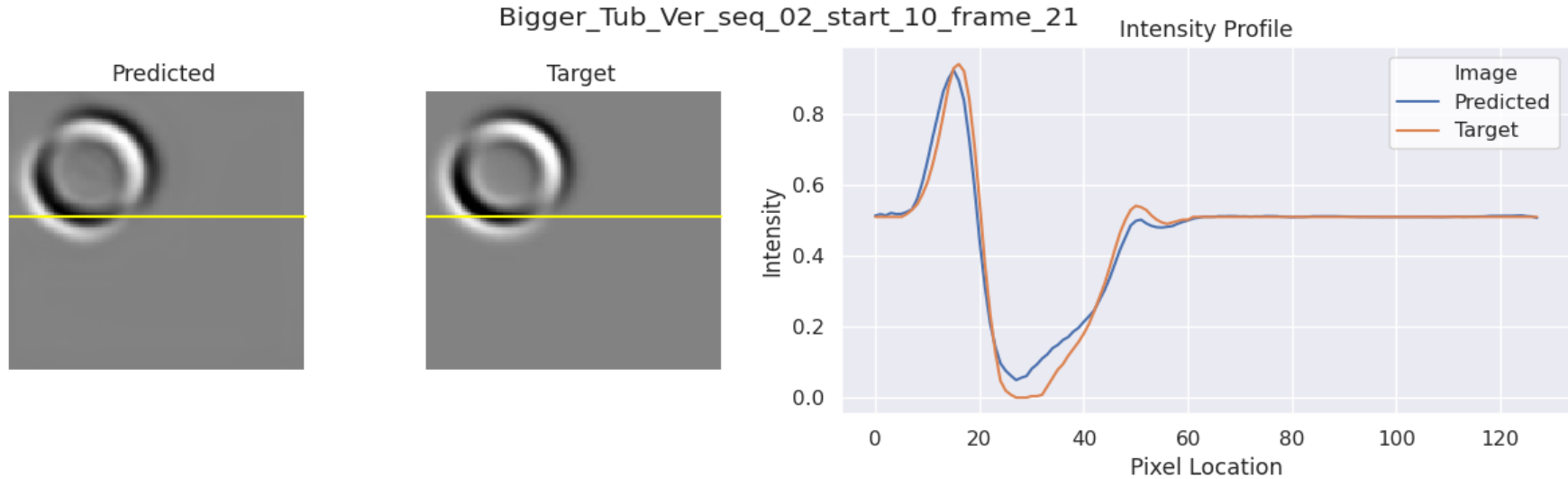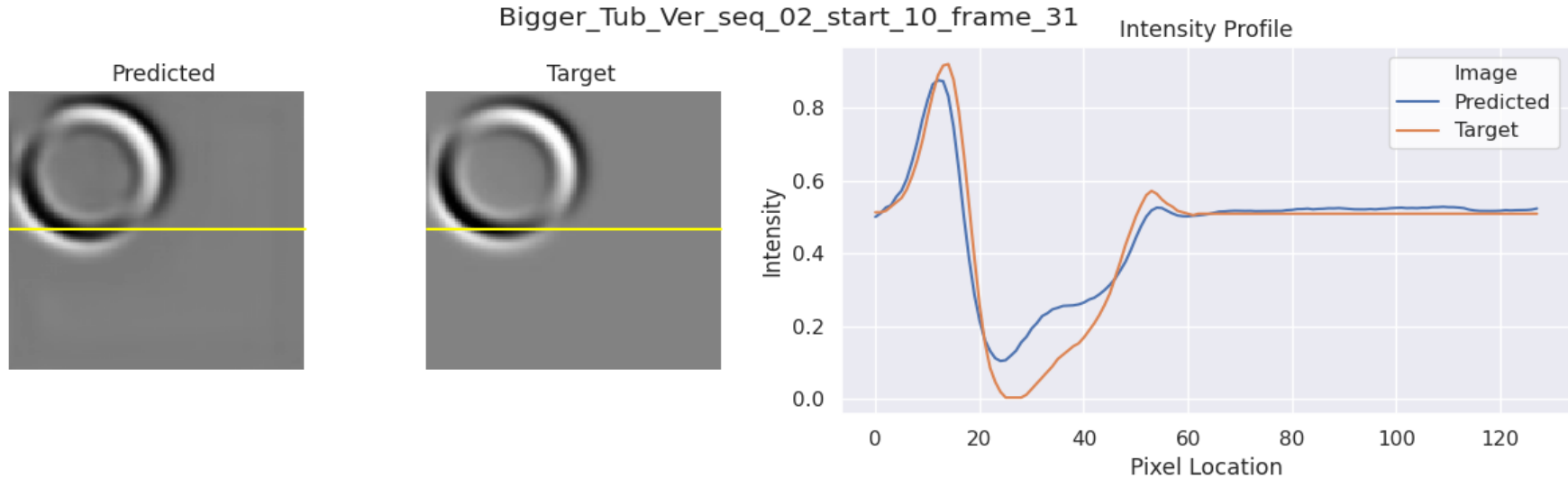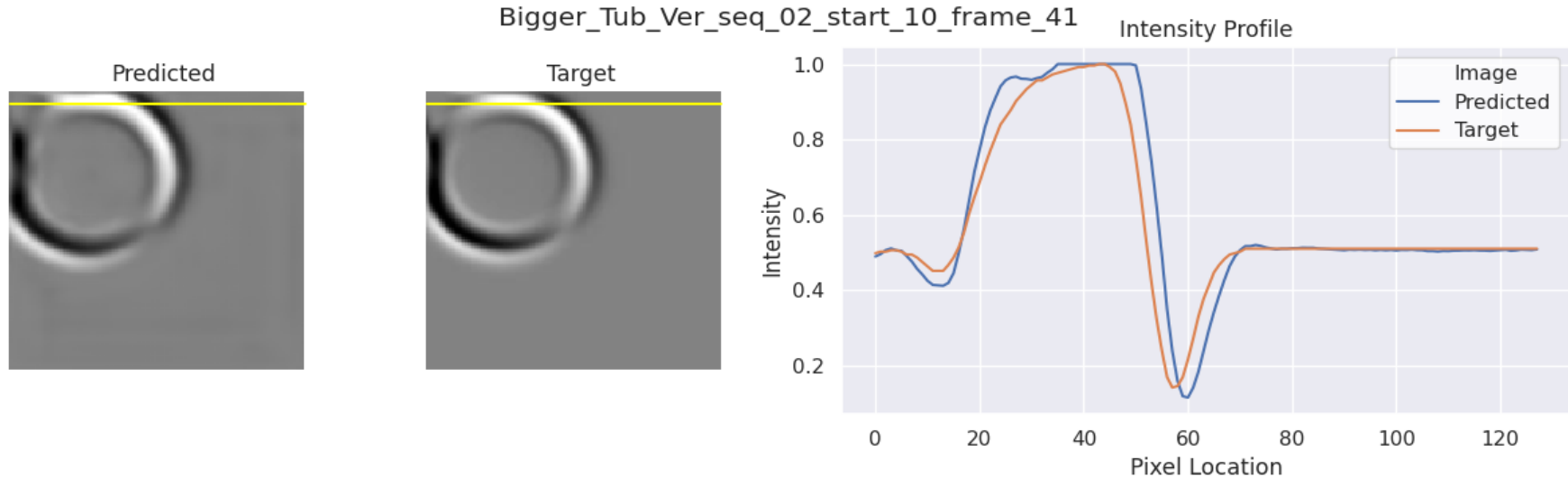
Transfer – Evaluation

Wave propagation prediction

Intensity profile on scanline – Frame 51

Transfer – Evaluation

Wave propagation prediction
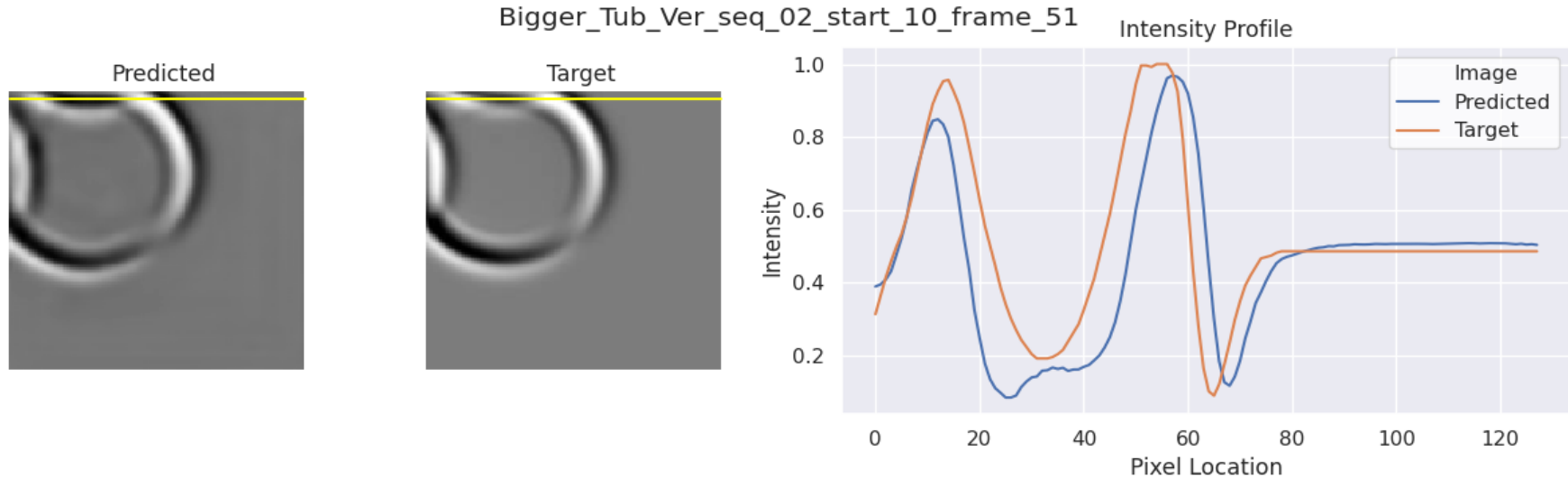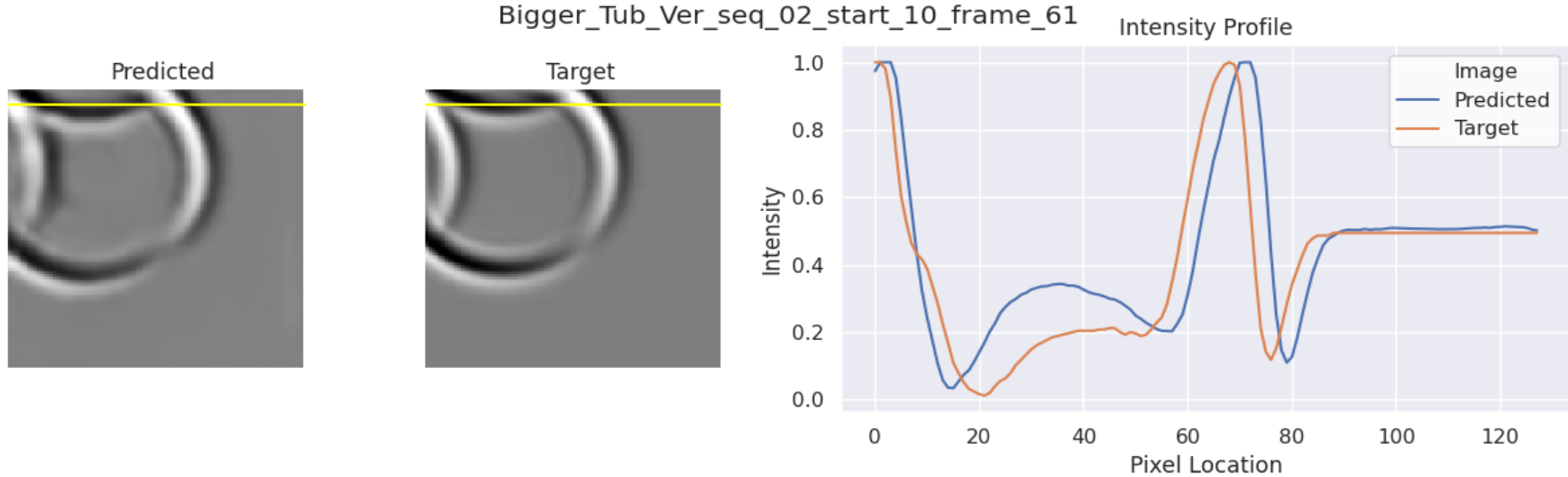
Intensity profile on scanline – Frame 61

Transfer – Evaluation

Wave propagation prediction

Intensity profile on scanline – Frame 71
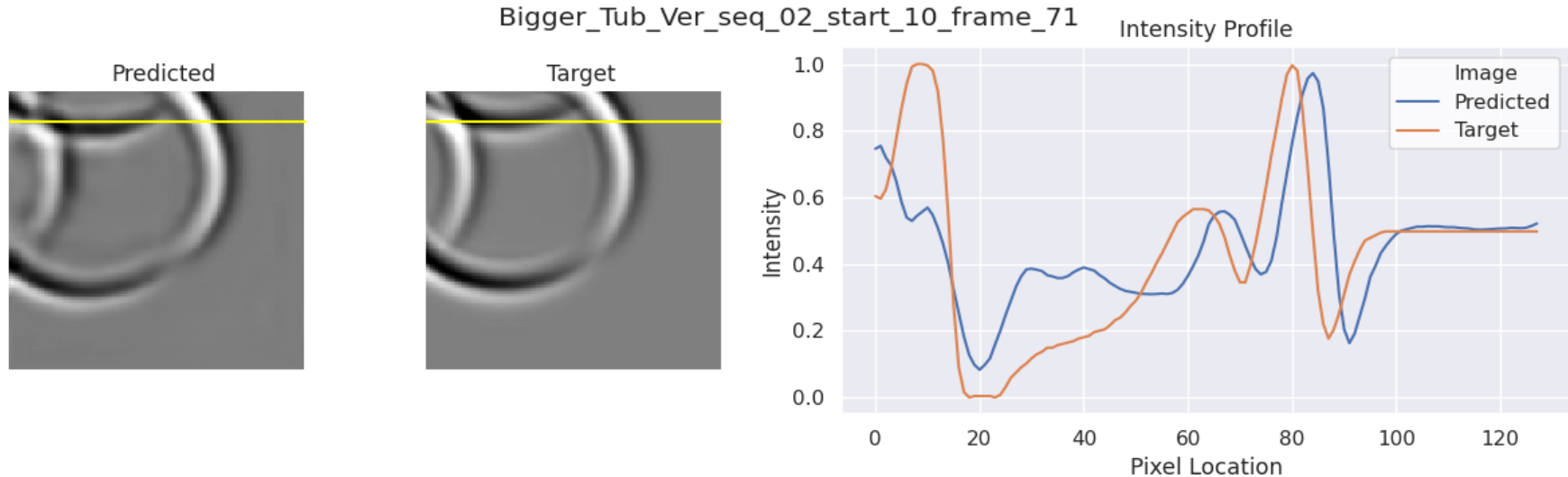
Generalization

Motivation: Key question in deep learning: How well does my NN perform on unseen data?
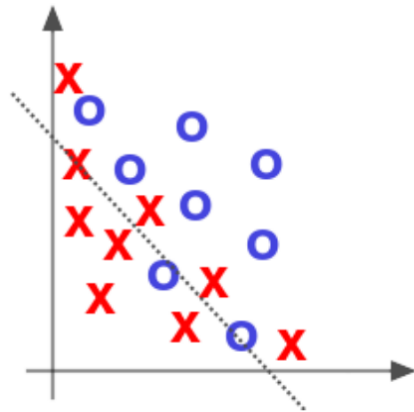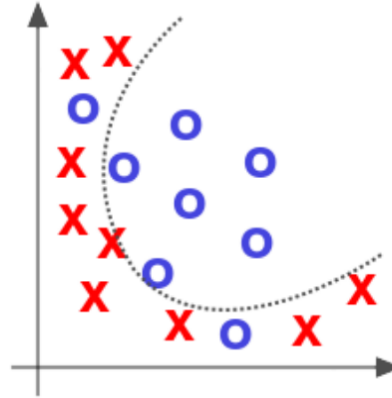
Underfitted   Appropriate   Overfitted
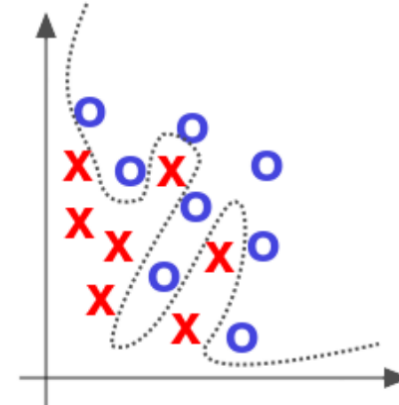
Taken from Deep Learning by Adam Gibson, Josh Patterson, O'Reily Media Inc., 2017

Generalization

Splitted up training data:

- Regular
- Mixed (50% regular, 50% sheared ($\pm$15 degrees))

The plot shows training with a $30.9 \cdot 10^6$ parameter model

The high capacity supports training with the mixed dataset, achieving a even lower error

Taken from `https://arxiv.org/pdf/1810.08217.pdf`
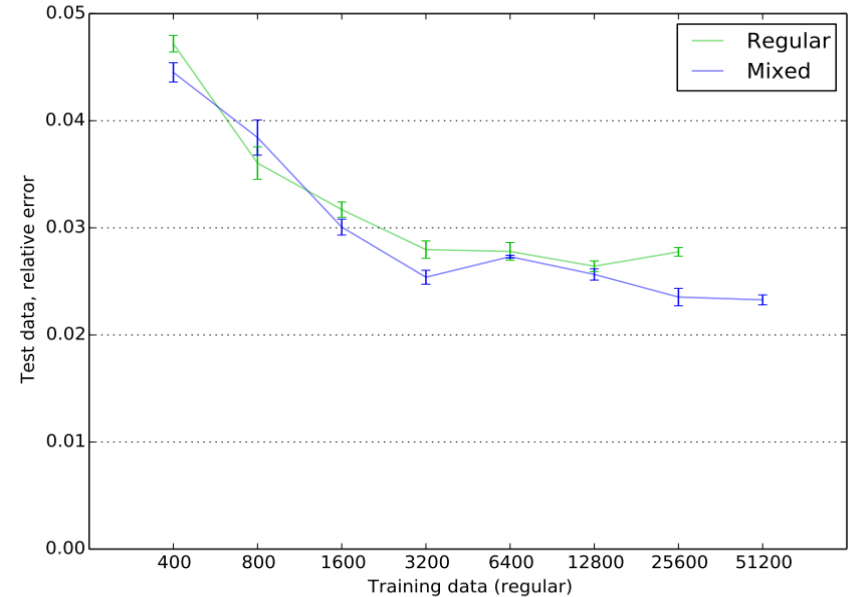
Generalization – Evaluation

To test generalization new datasets need to be created

Generation of 1 training sample: ≈ 70 seconds (using Google Colab)

Generation of $12.8k$ training samples: > 10 days – not feasible

$\implies$ Generate only new test sets containing 90 samples: < 2 hours – feasible

Extrapolation with different angle of attack intervals, default: $[-22.5, 22.5]$:

- $[-45, 45]$
- $[-90, 90]$
- $[-67.5, -22.5]$
- $[22.5, 67.5]$

Generalization – Evaluation

Error increase of different angle of attack intervals wrt. ground truth $[-22.5, 22.5]$

Discussion

Positiv

- Relative error $< 3\%$
- Convolutions paired with an encoder decoder structure seem to
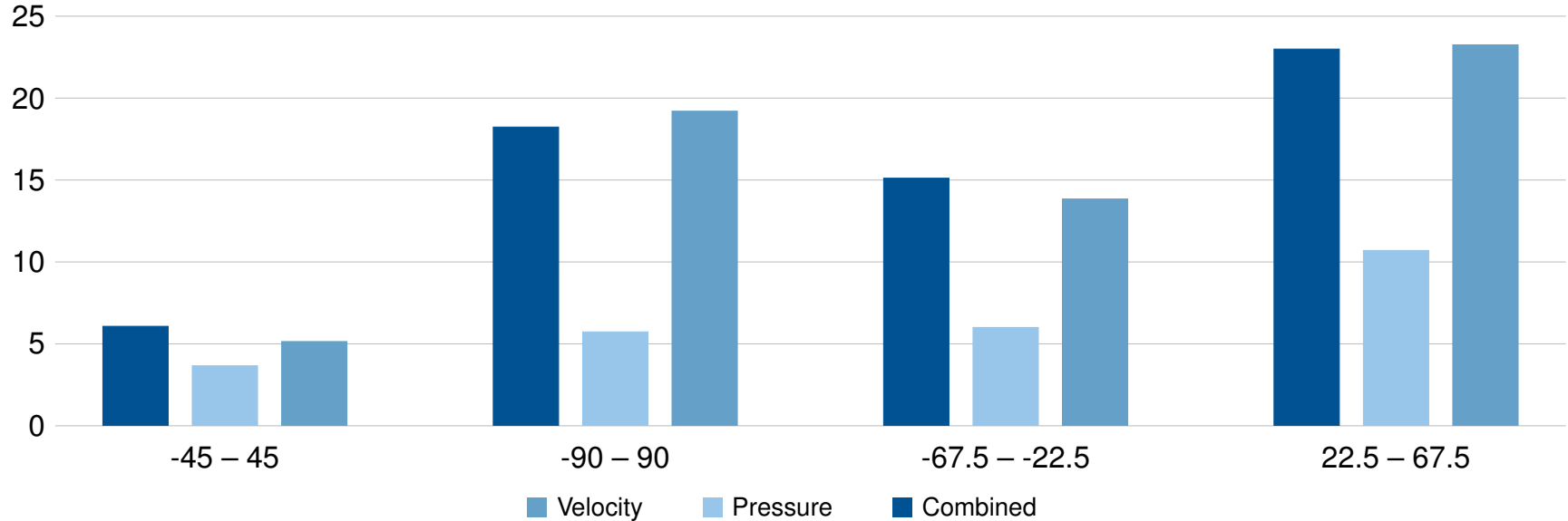   catch regions of interest fast and reliable
- U-Nets can outperform LSTM's in accuracy as well as in speed
    with a fraction of capacity (in time-series problems)
- accuracy does not suffer too much from models with a lot less capacity
    mostly affects sharpness of solutions
- Inference speed is $1000\times$ when compared with OpenFOAM solver
- Accuracy improvements still possible (bigger models, more training data)

Discussion

Negativ

- Proir knowledge needed for proper pre-processing
- Solvers needed for dataset generation
- Extrapolation yields mediocre results
- Fresh Training needed for other shapes (e.g. cars in wind tunnel)
    - transfer learning unlikely
- Trade off: training speed – grid resolution
- Possible data loss from transformation:

    adaptive grid (solver) $\implies$ cartesian grid (NN)
- no guarantee for correctness
- Accuracy improvements computationally expensive likely requires
    tailored architectures and loss functions

Summary

Investigate the accuracy of U-Net models for the inference of Reynolds-Averaged Navier-Stokes solutions

Data Generation $6 \times 128 \times 128$

- Input (encodes Reynolds number): Bit Mask, x & y velocity
- Target (RANS solution): Pressure, x & y velocity

Pre-Processing

- Make data dimensionless, flatten space of solutions
- Pressure offset removal, numerical precision

Architecture

- U-Net – Encoder - Bottleneck - Decoder structure
- Activations highly depend on current task

Transfer

- U-Net as time-series prediction NN for wave propagation
- Input: last n frames, Output: next m frames, refeed

Generalization

- NN performance on unseen data: Different angels of attack
- mediocre performance on wider intervals (velocity)

Discussion

- low error (improvable), speed up, even with low capacity
- prior knowledge and solvers needed, poor extrapolation

Backup slides

Backup slides – Training Setup

Adam optimizer ($\beta_1 = 0.5, \beta_2 = 0.999$)

Learning rate: 0.0004

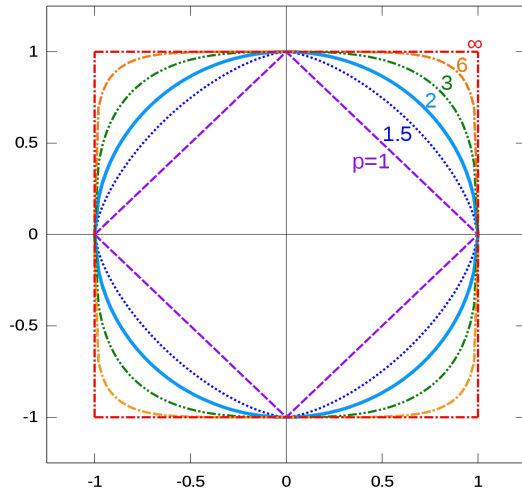Learning rate decay: On

Batch size: 10

Iterations: 80000

Model parameters:
122.979,   487.107,   1.938.819,   7.736.067,   30.905.859

Backup slides – Norms on unit circle

Taken from: `https://de.wikipedia.org/wiki/Norm_(Mathematik)#/media/Datei:Vector-p-Norms_qtl1.svg`