Eshelby polarization tensor from Integrals

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Mura 1987 page 77

$a_1 \hat{=}$ principle half axes of ellipsoid

$S_{ijkl} \hat{=}$ Eshelby polarization tensor

$I_i \hat{=}$ Integral with
$i \in [1, 2, 3, 11, 22, 33, 12, 13, 23]$

Abbreviations:

$$A_1 = 8\pi\,(1-\nu)$$

$$A_2 = 1 - 2\nu$$

$$A_3 = \frac{A_2}{A_1} = \frac{1-2\nu}{8\pi\,(1-\nu)}$$

set all S_{ijkl} to zero

Set

$$\left.\begin{aligned}
S_{1111} &= \frac{3}{A_1}\, a_1^2\, I_{11} + A_3 I_1 \\
S_{1122} &= \frac{1}{A_1}\, a_2^2\, I_{12} - A_3 I_1 \\
S_{1133} &= \frac{1}{A_1}\, a_3^2\, I_{13} - A_3 I_1 \\
S_{1212} &= \frac{a_1^2 + a_2^2}{2 A_1}\, I_{12} + \frac{A_3}{2}\,(I_1 + I_2)
\end{aligned}\right\}\quad (11.16)$$

$$\left.\begin{aligned}
S_{2112} = S_{1221} &= S_{1212} \\
&= \underset{\substack{\uparrow \\ \text{because } S_{ijkl} = S_{1212} \overset{!}{=} S_{2112} = S_{1221}\ \text{already handled}}}{S_{2112}}
\end{aligned}\right\}\quad \begin{aligned}&\text{Because } S_{ijkl} = S_{jikl} = S_{ijlk} \\ &\text{see } (11.15)\end{aligned}$$

Notes:

$I_{ij} = I_{ji}$ as $I_{12} = 2\pi\, a_1 a_2 a_3 \int_0^{\infty} \frac{ds}{(a_1^2 + s)(a_2^2 + s)\,\Delta(s)}$ (11.14)

Expressions in (11.16) contain parameters a_i of dimension length.
Parameters I_i with $i \in [11, 22, 33, 12, 13, 31]$ have the dimension length.