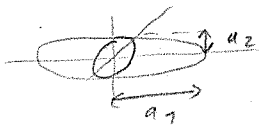
Approximate cylinder by spheroid

Cylinder: 

$a_c = \frac{l}{2r}$ $\quad V_c = l\pi r^2$

Spheroid: 

$a_s = \frac{a_1}{a_2}$ $\quad V_s = \frac{4}{3}\pi a_2^2 a_1$

Input: l, r

Output: a_1, a_2 $\Rightarrow$ DOF = 2

Possible requirements:

1. Equal volume
2. Equal aspect ratio
3. Equal length
4. Equal radius

$\Rightarrow$ 6 Possibilities with 4 diffent results

① + ②

$V_c = V_s$

$l\pi r^2 = \frac{4}{3}\pi a_2^2 a_1$

$r^2 = \frac{4}{3} a_2^2 a_2 \frac{l}{2r}$

$r^3 = a_2^3$

$r = a_2$ //

$a_c = a_s$

$\frac{l}{2r} = \frac{a_1}{a_2}$

$\Rightarrow a_1 = a_2 \frac{l}{2r}$

$\Rightarrow a_1 = \sqrt[3]{\frac{3}{2}} \frac{l}{2}$ //

① + ③

$V_c = V_s$

$l\pi r^2 = \frac{4}{3}\pi a_2^2 a_1$

$l r^2 = \frac{4}{3} a_2^2 \frac{l}{2}$

$r^2 = \frac{2}{3} a_2^2$

$a_2 = \sqrt{\frac{3}{2}}\, r$ //

$2 \cdot a_1 = l$

$\Rightarrow a_1 = \frac{l}{2}$ //

① + ④

$V_c = V_s$

$l\pi r^2 = \frac{4}{3}\pi r^2 a_1$

$a_1 = \frac{3}{4} l$ //

$a_2 = r$

$\Rightarrow a_2 = r$ //

② + ③

$a_c = a_s$

$\frac{l}{2r} = \frac{a_1}{a_2}$

$\frac{l}{2r} = \frac{l}{2 a_2}$

$\Rightarrow a_2 = r$ //

$2 a_1 = l$

$\Rightarrow a_1 = \frac{l}{2}$ //

② + ④

$a_c = a_s$

$\frac{l}{2r} = \frac{a_1}{a_2}$

$\frac{l}{2r} = \frac{a_1}{r}$

$\Rightarrow a_1 = \frac{l}{2}$ //

$a_2 = r$

$\Rightarrow a_2 = r$ //

③ + ④

$2 a_1 = l$

$a_1 = \frac{l}{2}$ //

$a_2 = r$

$a_2 = r$ //