

# Equation of State Solver for Smoothed Particle Hydrodynamics

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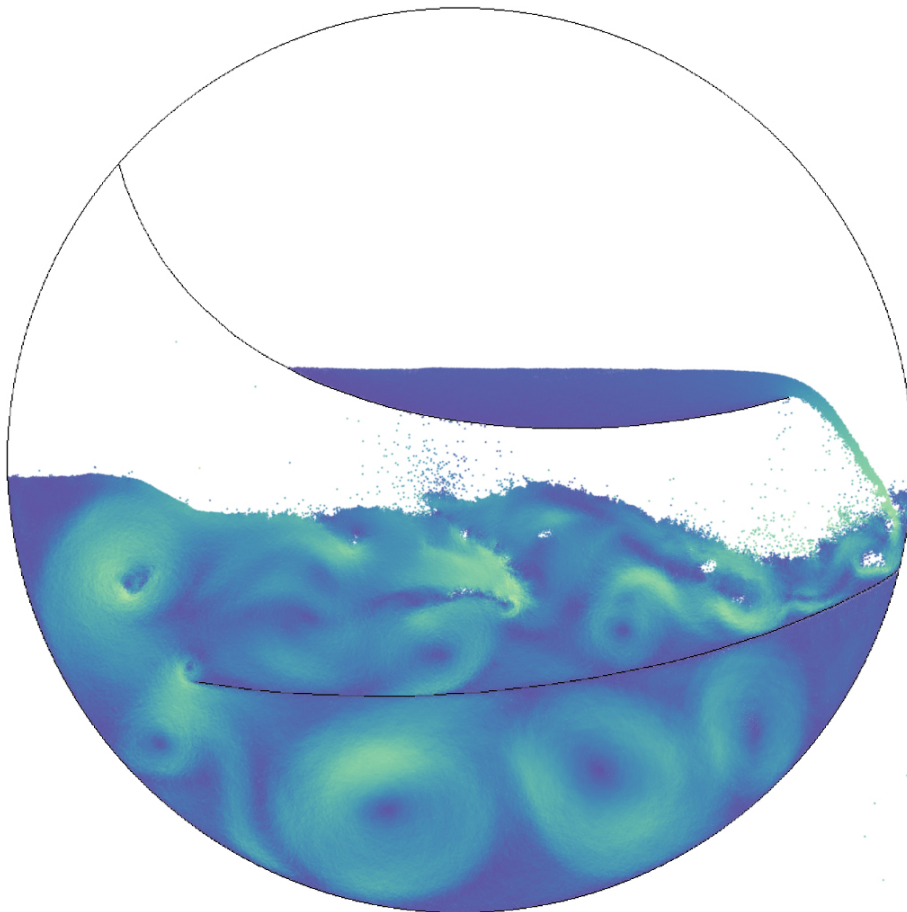


Figure 1: Colour coded velocity field of a simulation with 250000 particles, where the solver handles complex boundary conditions, turbulent flow at low viscosity and a pronounced free surface with less than 0.1% compression. [1]

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# INTRODUCTION

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# GOVERNING EQUATIONS OF FLUID FLOW

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- 2.1 Continuity and Conservation Laws
- 2.2 The Lagrangian Navier-Stokes equations
- 2.3 Equations of State

# **SMOOTHED PARTICLE HYDRODYNAMICS DISCRETIZATION**

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## **3.1 Kernel Functions**

## **3.2 Discretizations and Properties**

## **SOLVING FOR INCOMPRESSIBILITY**

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### **4.1 Weakly Compressible SPH**

### **4.2 Operator Splitting and Iterative Solver**

# **BOUNDARY AND INITIAL CONDITIONS**

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- 5.1 Non-Uniform Single Layer Boundaries**
- 5.2 Jittered Initialization and Lattices**
- 5.3 Solving for Equilibrated Density**

# ANALYSIS

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- 6.1 Oscillation Frequency and Error as a Function of Speed of Sound**
- 6.2 Stability as a Function of Viscosity, Stiffness and Timestep**
- 6.3 Stability over Viscosity and Stiffness**



# CONCLUSION

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# Bibliography

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<sup>1</sup>A. Lahiri, “Chapter 3 - Ray Optics: Optical Systems and Optical Imaging”, in *Basic optics*, edited by A. Lahiri (Elsevier, Amsterdam, 2016), pp. 203–307.