klarTeXt Test File

Julian Karrer

August 28, 2025

Arithmetic

Test

$$\int_{0}^{2\pi} \frac{\partial}{\partial x} \sin(x) + \sin(x) + 5 \, dx = 10\pi - (-\cos(0) + \sin(0)) - \cos(2\pi) + \sin(2\pi)$$

$$\int_{-e}^{2\pi} (\sum_{i=2}^{5} \frac{x}{i}) \, dx = \sum_{i=2}^{5} \frac{1}{i} \frac{(2\pi)^2}{2} - \left(\sum_{i=2}^{5} \frac{1}{i} \frac{-e^2}{2} \right)$$

$$\int_{0}^{\pi} \Phi(x) \, dx = \exp\left(\frac{-\left(\pi^2\right)}{2} \right) \frac{1}{\sqrt[2]{(2\pi)}} + \pi\Phi(\pi) - \left(\exp(0) \frac{1}{\sqrt[2]{(2\pi)}} \right)$$

$$\int_{-1}^{2} \frac{1}{x} \, dx = -\ln(1) + \ln(2)$$

Symbolic integration

Integrals are first attempted to be symbolically integrated - if symbolic integration fails, definite integrals are numerically evaluated using Gauss-Kronrod G7K15 quadrature. The following set of rules is used repeatedly and recursively:

$$\int n\,dx = xn + C \qquad \text{constants}$$

$$\int -f(x)\,dx = -\int f(x)\,dx \qquad \text{negatives}$$

$$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx \qquad \text{sums}$$

$$\int \sum_{i=a}^{b} f(x,i)\,dx = \sum_{i=a}^{b} \int f(x,i)\,dx \qquad \Sigma\text{-sums}$$

$$\int \sum_{x=a}^{b} f(x)\,dx = x\left(\sum_{i=a}^{b} \int f(x)\right) + C \qquad \text{shadowing } \Sigma\text{-sums}$$

$$\int \prod_{x=a}^{b} f(x)\,dx = x\left(\prod_{x=a}^{b} \int f(x)\right) + C \qquad \text{shadowing } \prod\text{-products}$$

$$\int \frac{f(x)}{g(y)}\,dx = \frac{1}{g(y)} \int f(x)\,dx \qquad \text{fractions}$$

$$\int \frac{n}{x}\,dx = n\ln(x) + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} \qquad \text{if } n \neq -1$$

$$\int \sqrt[n]{x}\,dx = \frac{n}{n+1} x^{\frac{n+1}{n}} \qquad \text{if } n \neq -1$$

$$\int \sin(x)\,dx = -\cos(x) + C \qquad \text{trigonometric functions}$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int \tan(x)\,dx = -\frac{\ln\left(\cos(x)^2\right)}{2} + C$$

$$\int \arcsin(x)\,dx = x\arcsin(x) + \sqrt{1-x^2} + C \qquad \text{inverse trigonometric functions}$$

$$\int \arccos(x)\,dx = x\arccos(x) - \sqrt{1-x^2} + C$$

$$\int \arctan(x)\,dx = x\arctan(x) - \frac{\ln\left(x^2+1\right)}{2} + C$$

$$\int \sinh(x)\,dx = \cosh(x) + C \qquad \text{hyperbolic functions}$$

$$\int \cosh(x)\,dx = \sinh(x) + C$$

$$\int \tanh(x)\,dx = \ln\left(\cosh(x)\right) + C$$

$$\int \ln(x)\,dx = x\left(\ln(x) - 1\right) + C \qquad \text{logarithms}$$

$$\int \lg(x)\,dx = \frac{x}{\ln(10)}\left(\ln(x) - 1\right) + C$$

$$\int \exp(x)\,dx = \exp(x) + C \qquad \text{exponentials}$$

$$\int \Theta(x)\,dx = x\Theta(x) + C \qquad \text{Heaviside Theta}$$

$$\int \Phi(x)\,dx = x\Phi(x) + \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2}) + C \qquad \text{CDF of standard normal distribution}$$

where n is constant with respect to x. If n cannot statically be proven to be different from -1, since for example it occurs open in a term of a function body, some rules cannot be applied.

Simplicification

The following rules for simplification of terms, including open terms with free variables, are implemented. Where the symmetric case of a rule applies due to commutativity, this should also be implemented.

$x \cdot 0 \cdot y = 0$	absorbing elements
$x + 0 = x$	neutral elements
$x \cdot 1 = x$	
$\frac{x}{1} = x$	
$\sqrt[1]{x} = x$	
$x^1 = x$	
$ax + bx = (a + b)x = x$	combining like terms
$x^a + x^b = x^{a+b} = x$	
$\forall a > b : \sum_a^b x = 0$	empty sums and products
$\forall a > b : \prod_a^b x = 1$	
$-(-x) = x$	operations that cancel
$\sqrt[y]{x^y} = x$	*roots and powers*
$\left(\sqrt[y]{x}\right)^y = x$	
$x + (-x) = 0$	*plus and minus*
$(-x) + x = 0$	
$x \cdot \frac{a}{x} = a$	*multiplication and division*
$-5x \cdot 2y \cdot (-6i) = 5x \cdot 2y \cdot 6i$	parity of minus in products
$-5x \cdot (-2y) \cdot (-6i) = -(5x \cdot 2y \cdot 6i)$	*even or odd number of* $-$
$x^0 = 1$	trivial due to argument
$\frac{x}{x} = 1$	
$\frac{0}{x} = 0$	

The above rules for analytically solvable integrals and constant folding (if subexpressions are values, evaluate the entire expression, e.g. '5 + 7' becomes '12' while '5x' remains '5x') are also implemented.