klarTeXt Test File

Julian Karrer

August 27, 2025

Arithmetic

Test

$$\int_0^{2\pi} \frac{\partial}{\partial x} \sin(x) + \sin(x) + 5 \, dx = 10\pi - (-\cos(0) + \sin(0)) - \cos(2\pi) + \sin(2\pi)$$

$$\int_{-e}^{2\pi} (\sum_{i=2}^{5} \frac{x}{i}) \, dx = \sum_{i=2}^{5} \frac{1}{i} \frac{(2\pi)^2}{2} - \left(\sum_{i=2}^{5} \frac{1}{i} \frac{-e^2}{2} \right)$$

$$\int_0^{\pi} \Phi(x) \, dx = \exp\left(\frac{-\left(\pi^2\right)}{2} \right) \frac{1}{\sqrt[2]{(2\pi)}} + \pi\Phi(\pi) - \left(\exp(0) \frac{1}{\sqrt[2]{(2\pi)}} \right)$$

Symbolic integration

Integrals are first attempted to be symbolically integrated - if symbolic integration fails, definite integrals are numerically evaluated using Gauss-Kronrod G7K15 quadrature. The following set of rules is used repeatedly and recursively:

$$
\begin{aligned}
\int n\,dx &= xn + C && \text{constants}\\
\int -f(x)\,dx &= -\int f(x)\,dx && \text{negatives}\\
\int f(x)+g(x)\,dx &= \int f(x)\,dx + \int g(x)\,dx && \text{sums}\\
\int \sum_{i=a}^{b} f(x,i)\,dx &= \sum_{i=a}^{b} \int f(x,i)\,dx && \Sigma\text{-sums}\\
\int \sum_{x=a}^{b} f(x)\,dx &= x\left(\sum_{i=a}^{b} \int f(x)\right) + C && \text{shadowing } \Sigma\text{-sums}\\
\int \prod_{x=a}^{b} f(x)\,dx &= x\left(\prod_{i=a}^{b} \int f(x)\right) + C && \text{shadowing } \prod\text{-products}\\
\int \frac{f(x)}{g(y)}\,dx &= \frac{1}{g(y)} \int f(x)\,dx && \text{fractions}\\
\int \frac{n}{x}\,dx &= n\ln(x) + C\\
\int x^n\,dx &= \frac{x^{n+1}}{n+1} && \text{if } n \neq -1\\
\int \sqrt[n]{x}\,dx &= \frac{n}{n+1} x^{\frac{n+1}{n}} && \text{if } n \neq -1\\
\int \sin(x)\,dx &= -\cos(x) + C && \text{trigonometric functions}\\
\int \cos(x)\,dx &= \sin(x) + C\\
\int \tan(x)\,dx &= -\frac{\ln\left(\cos(x)^2\right)}{2} + C\\
\int \arcsin(x)\,dx &= x\arcsin(x) + \sqrt{1-x^2} + C && \text{inverse trigonometric functions}\\
\int \arccos(x)\,dx &= x\arccos(x) - \sqrt{1-x^2} + C\\
\int \arctan(x)\,dx &= x\arctan(x) - \frac{\ln\left(x^2+1\right)}{2} + C\\
\int \sinh(x)\,dx &= \cosh(x) + C && \text{hyperbolic functions}\\
\int \cosh(x)\,dx &= \sinh(x) + C\\
\int \tanh(x)\,dx &= \ln\left(\cosh(x)\right) + C\\
\int \ln(x)\,dx &= x\left(\ln(x) - 1\right) + C && \text{logarithms}\\
\int \lg(x)\,dx &= \frac{x}{\ln(10)}\left(\ln(x) - 1\right) + C\\
\int \exp(x)\,dx &= \exp(x) + C && \text{exponentials}\\
\int \Theta(x)\,dx &= x\Theta(x) + C && \text{Heaviside Theta}\\
\int \Phi(x)\,dx &= x\Phi(x) + \frac{1}{\sqrt{2\pi}}\exp(-\frac{x^2}{2}) + C && \text{CDF of standard normal distribution}
\end{aligned}
$$

where n is constant with respect to x. If n cannot statically be proven to be different from -1, since for example it occurs open in a term of a function body, some rules cannot be applied.

Simplicification

The following rules for simplification of terms, including open terms with free variables, are implemented. Where the symmetric case of a rule applies due to commutativity, this should also be implemented.

$$
\begin{aligned}
x \cdot 0 \cdot y &= 0 && \text{absorbing elements} \\
x + 0 &= x && \text{neutral elements} \\
x \cdot 1 &= x && \\
\frac{x}{1} &= x && \\
\sqrt[1]{x} &= x && \\
x^1 &= x && \\
ax + bx = (a+b)x &= x && \text{combining like terms} \\
x^a + x^b = x^{a+b} &= x && \\
\forall a > b : \sum_a^b x &= 0 && \text{empty sums and products} \\
\forall a > b : \prod_a^b x &= 1 && \\
-(-x) &= x && \text{operations that cancel} \\
\sqrt[y]{x^y} &= x && \textit{roots and powers} \\
\left(\sqrt[y]{x}\right)^y &= x && \\
x + (-x) &= 0 && \textit{plus and minus} \\
(-x) + x &= 0 && \\
x \cdot \frac{a}{x} &= a && \textit{multiplication and division} \\
-5x \cdot 2y \cdot (-6i) &= 5x \cdot 2y \cdot 6i && \text{parity of minus in products} \\
-5x \cdot (-2y) \cdot (-6i) &= -\left(5x \cdot 2y \cdot 6i\right) && \textit{even or odd number of } - \\
x^0 &= 1 && \text{trivial due to argument} \\
\frac{x}{x} &= 1 && \\
\frac{0}{x} &= 0 &&
\end{aligned}
$$

The above rules for analytically solvable integrals and constant folding (if subexpressions are values, evaluate the entire expression, e.g. '5 + 7' becomes '12' while '5x' remains '5x') are also implemented.