ENGR 361 Statistical Analysis of Engineering Systems (Spring, 2012)
Homework 2 **due Friday April 20th at 12pm**

Instructions:

1. This page must be signed and stapled to your assignment. Homework handed in without this signed page will not be graded.
2. Your signature indicates your assertion of the truth of the following statement.

I acknowledge that this homework is solely my effort. I have done this work by myself. I have not consulted with others about this homework beyond the allowed level of verbal (non-written) exchanges of thoughts and opinions with my classmates. I have not received outside aid (outside of my own brain) on this homework. I understand that violation of these rules contradicts the class policy on academic integrity.

Name ______________________________

Student ID ______________________________

Signature ______________________________

Date ______________________________

This homework addresses the material covered in Lecture 2, on §1.3 in the text [1].

Recommended supplemental problems from text. These problems are not to be handed in with your assignment. You are encouraged to work with your classmates on the supplemental problems.

- §1.3. Conditional probability: Problems 14, 15, 16, 17.

Required problems for homework:

1. Deer ticks can be carriers of either Lyme disease or human granulocytic ehrlichiosis (HGE). Based on a recent study, suppose that $p_l = 16\%$ of all ticks in a certain location carry Lyme disease, $p_h = 10\%$ carry HGE, and $p_b = 10\%$ of the ticks that carry at least one of these diseases in fact carry both of them. If a randomly selected tick is found to have carried HGE, what is the probability that the selected tick is also a carrier of Lyme disease? *Hints: i) solve the general problem with* $p_l = \mathbb{P}(L), p_h = \mathbb{P}(H), p_b$*, where* L, H *are the events of having Lyme, HGE, respectively; ii) note* $p_b = \mathbb{P}(L \cap H | L \cup H)$*; iii) express* p_b *in terms of* $\mathbb{P}(L \cap H)$ *and* $\mathbb{P}(L \cup H)$*; iv) use* $\mathbb{P}(L \cup H) = \mathbb{P}(L) + \mathbb{P}(H) - \mathbb{P}(L \cap H)$.
2. A class consisting of m graduate and n undergraduate students is randomly divided into m groups of $(m+n)/m$ each (suppose m, n are such that $(m+n)/m$ is an integer). What is the probability that each group includes a graduate student?
3. Generalize the analysis of Example 1.12 (The Monty Hall Problem) from 3 to $n \geq 3$ doors. The prize is equally likely to be found behind any one of the n doors. After indicating a door, one of the remaining $n-1$ doors not containing the prize is opened. You may either i) stay (with the original door you picked) or ii) switch (to one of the remaining $n-2$ doors). Find the probability of winning under the stay and switch strategies as a function of n.
4. Consider a three by three grid with nine squares. Let m squares be selected uniformly at random, where $m \in \{3, 4, 5, 6\}$. Say a set of m squares is a bingo set if there exists a (length 3) row, column, or diagonal that is all in the set. Let $p(m)$ be the probability of a bingo set. Write a computer program to randomly select an m set and evaluate whether or not it is a bingo set. Use this program to estimate $p(m)$ by running $k = 10,000$ independent trials for each $m \in \{3, 4, 5, 6\}$. Produce **ONE** plot with four data sets: the running estimate of $p(m)$ for each of the four values of m versus the number of trials k, for $k = 1, 2, \ldots, 10000$.

References

[1] *Introduction to Probability, 2nd Edition* by Dimitri P. Bertsekas and John N. Tsitsiklis, Athina Scientific Press, 2008.