ENGR 361 Statistical Analysis of Engineering Systems (Spring, 2012)
Homework 4 **due Friday May 4th at 12pm**

Instructions:

1. This page must be signed and stapled to your assignment. Homework handed in without this signed page will not be graded.
2. Your signature indicates your assertion of the truth of the following statement.

I acknowledge that this homework is solely my effort. I have done this work by myself. I have not consulted with others about this homework beyond the allowed level of verbal (non-written) exchanges of thoughts and opinions with my classmates. I have not received outside aid (outside of my own brain) on this homework. I understand that violation of these rules contradicts the class policy on academic integrity.

Name ______________________________

Student ID ______________________________

Signature ______________________________

Date ______________________________

This homework addresses the material covered in Lectures 3 and 4 on Chapter 1 in the text [1].

Recommended supplemental problems from text. These problems are not to be handed in with your assignment. You are encouraged to work with your classmates on the supplemental problems.

- §1.4. Total probability theorem and Bayes' rule: Problems 20, 21, 22, 23, 27.
- §1.5. Independence: Problems 40, 41.
- §1.6. Counting: Problems 51, 53, 59.

Required problems for homework:

1. Alice searches for her term paper in her filing cabinet which has several drawers. She knows that she left her term paper in drawer j with probability $p_j > 0$. The drawers are so messy that even if she correctly guesses that the term paper is in drawer i, the probability that she finds it is only d_i. Alice searches in a particular drawer, say drawer i, but the search is unsuccessful. Conditioned on this event, show that the probability that her paper is in drawer j is given by
$$\frac{p_j}{1-p_i d_i},\ j \neq i, \qquad \frac{p_i(1-d_i)}{1-p_i d_i},\ j = i. \tag{1}$$
Hint: define events I_j (term paper is in drawer j), L_j (Alice looks in drawer j), and F_j (Alice finds paper in drawer j). Then the given probabilities are $\mathbb{P}(I_j) = p_j$ and $\mathbb{P}(F_j | I_j \cap L_j) = d_j$. The probability of interest is $\mathbb{P}(I_j | L_i \cap \bar{F}_i)$ for $j \neq i$ and then for $j = i$. By the conditioned (on L_i) version of Bayes's rule:
$$\mathbb{P}(I_j | \bar{F}_i \cap L_i) = \frac{\mathbb{P}(\bar{F}_i | I_j \cap L_i)\mathbb{P}(I_j | L_i)}{\sum_k \mathbb{P}(\bar{F}_i | I_k \cap L_i)\mathbb{P}(I_k | L_i)}. \tag{2}$$
2. Consider two dependent events A, B. Suppose $\mathbb{P}(A) = p$ and $\mathbb{P}(B|A) = q$ and $\mathbb{P}(B|A^c) = r$. Find an expression in terms of p, q, r for
$$\mathbb{P}(A \cap B | A \cup B). \tag{3}$$
Give a clear derivation of the expression with each step labeled (e.g., definition of conditional expectation, law of total probability, etc.).
3. Using a biased coin to make an unbiased decision. Alice and Bob want to choose between the opera and the movies by tossing a fair coin. Unfortunately, the only available coin is biased (though the bias is not known exactly). How can they use the biased coin to make a decision so that either option (opera or the movies) is equally likely to be chosen?
Hint: map the four outcomes of two coin flips to three decisions: opera, movies, and repeat.
4. De Méré's puzzle. A six-sided die is rolled three times independently. Which is more likely: a sum of 11 or a sum of 12? (This question was posed by the French nobleman de Méré to his friend Pascal in the 17th century.)
5. Memory. The game of memory consists of n pairs of cards where the identical cards in pair i each have the number i on them, for $i = 1, \ldots, n$. The game is played by repeatedly picking two cards, and if the two cards are a pair then they are removed from the game. We will consider the case of perfect memory where if the two cards picked are not a pair they are returned to their places face up. Assume a player uses the following strategy: *i*) the player picks a pair if a pair is known to her, *ii*) else the player picks an as of yet untried card — if this card has as its pair a previously picked card the player picks the pair, else the player picks a second as of yet untried card. The game ends once all pairs have been picked. Let $t(n)$ be the (random) number of turns until the game ends. For each $n \in \mathcal{N} = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$ run $k = 1000$ random realizations of the game. Thus for each n you should generate $t_1(n), \ldots, t_k(n)$ random realizations of the game. Plot $\bar{t}(n) = \frac{1}{k}\sum_{i=1}^{k} t_i(n)$ versus $n \in \mathcal{N}$.

References

[1] *Introduction to Probability, 2nd Edition* by Dimitri P. Bertsekas and John N. Tsitsiklis, Athina Scientific Press, 2008.