ENGR 361 Statistical Analysis of Engineering Systems (Spring, 2012)
Homework 1 **due Friday April 13th at 12pm**

Instructions:

1. This page must be signed and stapled to your assignment. Homework handed in without this signed page will not be graded.
2. Your signature indicates your assertion of the truth of the following statement.

I acknowledge that this homework is solely my effort. I have done this work by myself. I have not consulted with others about this homework beyond the allowed level of verbal (non-written) exchanges of thoughts and opinions with my classmates. I have not received outside aid (outside of my own brain) on this homework. I understand that violation of these rules contradicts the class policy on academic integrity.

Name ________________________________

Student ID ________________________________

Signature ________________________________

Date ________________________________

This homework addresses the material covered in Lecture 1, on §1.1–§1.3 in the text [1].

Recommended supplemental problems from text. These problems are not to be handed in with your assignment. You are encouraged to work with your classmates on the supplemental problems.

- §1.2. Probabilistic models: Problems 6, 7, 8, 9, 10.
- §1.3. Conditional probability: Problems 15, 16.

Required problems for homework:

1. We roll a four-sided die once and then we roll it as many times as is necessary to obtain a different face than the one obtained in the first roll. Let the outcome of the experiment be (r_1, r_2) where r_1 and r_2 are the results of the first and the last rolls, respectively. Assume that all possible outcomes have equal probability. Find the probability that:

   (a) r_1 is even
   (b) Both r_1 and r_2 are even.
   (c) $r_1 + r_2 < 5$.

2. A magical four-sided die is rolled twice. Let S be the sum of the results of the two rolls. We are told that the probability that $S = k$ is proportional to k, for $k = 2, 3, \cdots, 8$, and that all possible ways that a given sum k can arise are equally likely. Construct an appropriate probabilistic model and find the probability of getting doubles.

3. Alice and Bob each choose at random a number between zero and two. We assume a uniform probability law under which the probability of an event is proportional to its area. Consider the following events:

   (a) A: The magnitude of the difference of the two numbers is greater than 1/3.
   (b) B: At least one of the numbers is greater than 1/3.
   (c) C: The two numbers are equal.
   (d) D: Alice's number is greater than 1/3.

   Find the probabilities $\mathbb{P}(A), \mathbb{P}(B), \mathbb{P}(A \cap B), \mathbb{P}(C), \mathbb{P}(D), \mathbb{P}(A \cap D)$.

4. Consider a game where two fair (unbiased) coins are tossed, repeatedly as necessary as described below. Each toss results in *heads* (two heads), *tails* (two tails), or *odds* (one of each). A bet can be made that the spinner (coin tosser) will obtain three heads before a single tails and before five consecutive odds. Notice that at most 15 tosses are required to resolve this bet. (If it is unresolved after 14 tosses, then the 14 tosses must consist of *oooohoooohoooo*, and the next toss is decisive.) Write a computer program to simulate this game, and run your program 100,000 times keeping track of the fraction of n plays that result in a win. Plot this fraction of winning plays versus n, for $n = 100, 200, \ldots, 99900, 100000$. That is, every 100 plays you should compute the *cumulative fraction of plays that have been wins*, and plot that fraction vs. n. What is your final estimate of the probability of winning this bet. Make sure your plot has labeled axes and a title.

References

[1] *Introduction to Probability, 2nd Edition* by Dimitri P. Bertsekas and John N. Tsitsiklis, Athina Scientific Press, 2008.