ADAPTIVE SIMULATED ANNEALING (ASA) ©

Lester Ingber

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Adaptive Simulated Annealing (ASA) is a C-language code developed to statistically find the best global fit of a nonlinear constrained non-convex cost-function over a D-dimensional space. This algorithm permits an annealing schedule for "temperature" T decreasing exponentially in annealing-time k, $T = T_0 \exp(-ck^{1/D})$. The introduction of re-annealing also permits adaptation to changing sensitivities in the multi-dimensional parameter-space. This annealing schedule is faster than fast Cauchy annealing, where $T = T_0/k$, and much faster than Boltzmann annealing, where $T = T_0/\ln k$. ASA has over 100 OPTIONS to provide robust tuning over many classes of nonlinear stochastic systems.

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^{*} Adaptive Simulated Annealing (ASA)

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1. ASA-LICENSE

This Adaptive Simulated Annealing (ASA) code is being made available under conditions specified in the ASA-LICENSE file that comes with this code, and is owned by Lester Ingber. Reference is properly given to the internet archive that first published the code[1].

Please read the copy of the public ASA-LICENSE contained in the ASA directory, which is compliant with The Open Source Initiative BSD 3-Clause License at https://opensource.org/licenses/BSD-3-Clause . Its intent is to protect the integrity of the algorithm and promote widespread usage.

2. Physical Studies Institute LLC Terms of Use

Physical Studies Institute LLC (PSI) develops projects in several areas of expertise documented in the www.ingber.com InterNet archive, e.g., this ASA code. Information on terms of use is in the file https://www.ingber.com/ingber_terms.html under WWW.

There is no charge for downloading and using codes or files in the www.ingber.com archive. In general, I have retained all rights such as copyrights to these codes and files, but they may be freely used by any person or group independent of affiliations, e.g., independent of academic or commercial affiliation.

Limited help assisting people with queries on my codes and papers is available only by electronic mail correspondence. Sorry, I cannot mail out hardcopies of code or papers.

3. Documentation

3.1. Table of Contents/Index

A compilation of the three levels of headers with their page numbers may be used as a Table of Contents placed after the first title page (as is done for ASA-README.ps, ASA-README.pdf and ASA-README.html below), or left at the end for quick reference (as is done for ASA-README.txt below).

3.2. ASA-README.ms and ASA-README

The ASA-README.ms file is used to prepare other documentation files using UNIX® MS macros.

3.2.1. ASA-README.txt and ASA-README+.txt

ASA-README.txt is an ASCII file that can be previewed on your screen or sent to an ASCII lineprinter.

ASA-README+.txt is ASA-README.txt without any filters to strip off underlining and bold enhancements.

3.2.2. asa.[13nl] Manpage

The ASA-README.txt or ASA-README+.txt file can be copied to a file named asa.[13], and asa.[13] can be installed as MANPATH/cat1/asa.1 or MANPATH/cat3/asa.3, where MANPATH is the place your man directory is located. If you do not have any cat[13] directories on your system, then installing a copy of ASA-README.txt or ASA-README+.txt as MANPATH/man[13nl]/asa.[13nl], choosing one of the suffixes in [13nl] for your choice of directory and asa file name, should work fine on most machines. However, passing this asa.[13nl] through man may strip out additional "back-slash" characters, leading to missing words or unintended formatting. If such a file looks strange, compare it to the raw ASA-README.ms file to determine the true intended content. You likely can avoid some further undesirable formatting by man by placing '.nf' on the first line of this file.

3.2.3. ASA-README.ps and ASA-README.pdf

ASA-README.ps is a PostScript® formatted file which may be previewed on your screen if you have the proper software, or it may be sent to a PostScript® printer to produce hardcopy.

A PDF version ASA-README.pdf is prepared from ASA-README.ps.

3.2.4. ASA-README.html

ASA-README.html is an HTML version which enables easier access to subsections of this file. Cross-references have been kept local to this file, so you may view it under a local browser if you download the HTML source file.

The background image file asa_back.jpg referenced in ASA-README.html can be downloaded as https://www.ingber.com/asa_back.jpg from the ASA archive.

3.3. Additional Documentation

ASA-CHANGES is a terse record of major changes made in the ASA code. It has three sections, CHANGES, CONTRIBUTORS, and VERSION DATES.

ASA-NOTES is a collection of recommended enhancements, modifications, comments, caveats, etc., that might be of interest. There is a CONTENTS of sections headers that can be used to search on topics in your browser or editor.

There are three files in the ASA archive that should be considered as appendices to the ASA-NOTES file: https://www.ingber.com/asa_contrib.txt, https://www.ingber.com/asa_examples.txt, and https://www.ingber.com/asa_papers.html under WWW.

The file https://www.ingber.com/asa_contrib.txt in the ASA archive contains some code contributed by users. For example, references are giving to MATLAB and RLaB gateway routines to ASA (which include many important parts of the ASA user module), and to function support for ASA_PARALLEL. There is a CONTENTS of sections headers that can be used to search on topics in your browser or editor. In this file I have included the first 1987 VFSR code, the precursor to the ASA code, as used on a specific project, including the RATFOR vfsr.r and vfsr_com.r code, subsequently compiled into FORTRAN to run on a Lawrence Livermore supercomputer. I do not support this old RATFOR code.

The file https://www.ingber.com/asa_examples.txt in the ASA archive contains some example problems using ASA and some code to use in ASA. There is a CONTENTS of sections headers that can be used to search on topics in your browser or editor. This file contains some problems optimized using ASA, which can provide immediate examples on how you can optimize your own problem.

The file https://www.ingber.com/asa_papers.html is an addendum to the ASA-NOTES file in the ASA code, containing references to some patents and papers using ASA or its precursor VFSR.

The file asa_new.txt in the ASA archive is a list of major changes in ASA. The files ASA-README.txt, ASA-README.ps and ASA-README.pdf included with the code also are available independently as https://www.ingber.com/ASA-README.txt , https://www.ingber.com/ASA-README.ps.gz , https://www.ingber.com/ASA-README.html , https://www.ingber.com/ASA-README.pdf .

There is a set of ASA_TEMPLATE's available in the ASA-Makefile and in the user module (some also in the asa module) to illustrate use of particular OPTIONS, as listed under ASA_TEMPLATE below. You can search on these ASA_TEMPLATE's in your browser or editor to see how these are implemented. Note that some OPTIONS require your input, as described below, and code may fail until you add your own code. Once you have determined the most common set of DEFINE_OPTIONS you are likely to use, you might place these in your own TEMPLATE at the top of asa_usr_asa.h at the location specified, e.g.,

```
#if MY_TEMPLATE /* MY_TEMPLATE_asa_user */
/* you can add your own set of #define here */
#define ... TRUE
#define ... 100
#endif
```

See https://www.ingber.com/utils_file_formats.txt for some links to information on gzip, PostScript, PDF, tar, and shar utilities. The file utils_code.txt in that directory gives short statements describing these files, which may be accessed as https://www.ingber.com/utils_code.html under WWW.

3.4. Use of Documentation for Tuning

I am often asked how I can help someone tune their system, and they send me their cost function or a list of the ASA OPTIONS they are using. Most often, the best help I can provide is based on my own experience that nonlinear systems typically are non-typical. In practice, that means that trying to figure out the nature of the cost function under sampling in order to tune ASA (or likely to similarly tune a hard problem under any sampling algorithm), by examining just the cost function, likely will not be as productive as generating more intermediate printout, e.g., setting ASA_PRINT_MORE to TRUE, and looking at this output as a "grey box" of insight into your optimization problem. Larger files with more information is provided by setting ASA_PIPE_FILE to TRUE. Treat the output of ASA as a simulation in the ASA parameter space, which usually is quite a different space than the variable space of your system.

For example, you should be able to see where and how your solution might be getting stuck in a local minima for a very long time, or where the last saved state is still fluctuating across a wide portion of your state space. These observations should suggest how you might try speeding up or slowing down annealing/quenching (see more below on modifying ASA with QUENCH OPTIONS) of the parameter space and/or tightening or loosening the acceptance criteria at different stages by modifying the OPTIONS, e.g., starting with the OPTIONS that can be easily adjusted using the asa_opt file.

The ASA-NOTES file that comes with the ASA code provides some guidelines for tuning that may provide some insights, especially the section Some Tuning Guidelines. An especially important guide is to examine the output of ASA at several stages of sampling, to see if changes in parameter and temperatures are reasonably correlated to changes in the cost function. Examples of useful OPTIONS and code that often give quick changes in tuning in some problems are in the file https://www.ingber.com/asa_examples.txt under WWW. Some of the reprint files of published papers in www.ingber.com provide other examples in harder systems, and perhaps you might find some examples of harder systems using ASA similar to your own in https://www.ingber.com/asa_papers.html under WWW. This is the best way to add some Art to the Science of annealing.

While the upside of using ASA is that it has many OPTIONS available for tuning, derived in large part from feedback from many users over many years, making it extremely robust across many systems, the downside is that the learning curve can be steep especially if the default settings or simple tweaking in asa_opt do not work very well for your particular system, and you then must turn to using more ASA OPTIONS. Most of these OPTIONS have useful guides in the ASA_TEMPLATEs in asa_usr.c, as well as being documented here. If you really get stuck, you may consider working with someone else who already has climbed this learning curve and whose experience might offer quick help.

Tuning is an essential aspect of any sampling algorithm if it is to be applied to many classes of systems. It just doesn't make sense to compare sampling algorithms unless you are prepared to properly tune each algorithm to each system being optimized or sampled.

4. Availability of ASA Code

4.1. ingber.com

The latest Adaptive Simulated Annealing (ASA) code and some related papers can be accessed from the home page https://www.ingber.com/ under WWW.

The home page https://www.ingber.com/ (https://www.ingber.com/) under WWW (SSL WWW), and the ASCII version https://www.ingber.com/00index.txt (https://www.ingber.com/00index_s.txt), contain an index of the other files.

The latest version of ASA, ASA-x.y (x and y are version numbers), can be obtained in two formats: https://www.ingber.com/ASA.tar.gz and https://www.ingber.com/ASA.zip. The tar'd versions is compressed in gzip format, and ASA.tar.gz. In the zip'd version, ASA.zip, all files have been processed

for DOS format.

Patches ASA-diff-x1.y1-x2.y2 up to the present version can be prepared if a good case for doing so is presented, e.g. to facilitate updating your own modified codes. These may be concatenated as required before applying. If you require a specific patch, contact lester@ingber.com.

4.2. Electronic Mail

If you do not have WWW, get the Guide to Offline Internet Access, returned by sending an e-mail to mail-server@rtfm.mit.edu with only the words "send usenet/news.answers/internet-services/access-via-email" in the body of the message. The guide gives information on using e-mail to access just about all InterNet information and documents. You will receive the information in utils_access-via-email.txt in the ASA archive.

5. Background

5.1. Context

Too often the management of complex systems is ill-served by not utilizing the best tools available. For example, requirements set by decision-makers often are not formulated in the same language as constructs formulated by powerful mathematical formalisms, and so the products of analyses are not properly or maximally utilized, even if and when they come close to faithfully representing the powerful intuitions they are supposed to model. In turn, even powerful mathematical constructs are ill-served, especially when dealing with multivariate nonlinear complex systems, when these formalisms are butchered into quasi-linear approximations to satisfy constraints of numerical algorithms familiar to particular analysts, but which tend to destroy the power of the intuitive constructs developed by decision-makers.

In order to deal with fitting parameters or exploring sensitivities of variables, as models of systems have become more sophisticated in describing complex behavior, it has become increasingly important to retain and respect the nonlinearities inherent in these models, as they are indeed present in the complex systems they model. ASA can help to handle these fits of nonlinear models of real–world data.

It helps to visualize the problems presented by such complex systems as a geographical terrain. For example, consider a mountain range, with two "parameters," e.g., along the North–South and East–West directions. We wish to find the lowest valley in this terrain. ASA approaches this problem similar to using a bouncing ball that can bounce over mountains from valley to valley. We start at a high "temperature," where the temperature is an ASA parameter that mimics the effect of a fast moving particle in a hot object like a hot molten metal, thereby permitting the ball to make very high bounces and being able to bounce over any mountain to access any valley, given enough bounces. As the temperature is made relatively colder, the ball cannot bounce so high, and it also can settle to become trapped in relatively smaller ranges of valleys.

We imagine that our mountain range is aptly described by a "cost function." We define probability distributions of the two directional parameters, called generating distributions since they generate possible valleys or states we are to explore. We define another distribution, called the acceptance distribution, which depends on the difference of cost functions of the present generated valley we are to explore and the last saved lowest valley. The acceptance distribution is used to decide whether to stay in a new lower valley or to bounce out of it. All the generating and acceptance distributions depend on temperatures.

The ASA code was first developed in 1987 as Very Fast Simulated Reannealing (VFSR) to deal with the necessity of performing adaptive global optimization on multivariate nonlinear stochastic systems[2]. The first published use of VFSR for a complex systems was in combat analysis, using a model of combat first developed in 1986, and then applied to exercise and simulation data in a series of papers that spanned 1988-1993[3]. The first applications to combat analysis used code written in RATFOR and converted into FORTRAN. Other applications since then have used new code written in C. (The ASA–NOTES file contains some comments on interfacing ASA with FORTRAN codes.)

In November 1992, the VFSR C-code was rewritten, e.g., changing to the use of long descriptive names, and made publicly available as version 6.35 under a "copyleft" GNU General Public License

(GPL)[4], and copies were placed in NETLIB and STATLIB.

Beginning in January 93, many adaptive features were developed, largely in response to users' requests, leading to this ASA code. Until 1996, ASA was located at alumni.caltech.edu. Pointers were placed in NETLIB and STATLIB to this location. ASA versions 1.1 through 5.13 retained the GPL, but subsequent versions through this one have incorporated a simpler ASA-LICENSE, based in part on a University of California license, that protects the integrity of the algorithm, promotes widespread usage, and requires reference to current source code. Beginning with version 29.1 the ASA-LICENSE was replaced with The Open Source Initiative BSD 3-Clause License.

As the archive grew, more room and maintenance was required, and in February 1996 the site was moved to the present www.ingber.com location. Pointers were placed in the Caltech site to this location.

Mirrors of ASA (which may not always be current) are listed at https://sourceforge.net/projects/asa-caltech/ since January 2007, and at https://code.google.com/p/adaptive-simulated-annealing since in March 2001. FTP access to all archives was discontinued in July 2010.

In February 2011, beginning with Version 28.1, the License was simplified and brought into the framework described on https://en.wikipedia.org/wiki/BSD_License as the New BSD License.

ASA has been examined in the context of a review of methods of simulated annealing using annealing versus quenching (faster temperature schedules than permitted by basic heuristic proof of ergodicity -- see more below on modifying ASA with QUENCH OPTIONS)[5]. A paper has indicated how this technique can be enhanced by combining it with some other powerful algorithms, e.g., to produce an algorithm for parallel computation[6]. ASA is now used world–wide across many disciplines[7,8,9,10], including specific disciplines such as finance[11,12,13,14,15,16,17,18], neuroscience[19,20,21,22,23,24,25,26,27,28], and combat analyses[29,30,31,32,33,34,35]. Some papers illustrate the combined use of ASA for optimization and sampling[36,37,38,39,40,41,42]. The https://www.ingber.com/asa_papers.html file in the ASA archive contains references to some patents and papers using ASA and VFSR.

5.2. Outline of ASA Algorithm

Details of the ASA algorithm are best obtained from the published papers. There are three parts to its basic structure: Generating Probability Density Functions, Acceptance Probability Density Functions, and Temperature Schedules.

5.2.1. Simulated Annealing is NOT Simulated Quenching

As an aside, it is very important to understand that Simulated Annealing (SA) is NOT Simulated Quenching (SQ). The issue is that SA is not just one algorithm -- it is a class of approaches to importance sampling.

Different SA algorithms may have different probability distributions they use to generate new states for consideration by the acceptance criteria. The often-cited SA proof of convergence depends on this generating probability distribution, which leads to different temperature schedules for different distributions.

If a schedule-1 derived for one SA-1 is faster than a schedule-2 derived for a different SA-2, but one uses schedule-1 with the SA-2 algorithm, then SQ-2 is being used and not SA-2. The proof for SA-2 will fail using schedule-1 (although the mixed algorithm might work just fine).

SQ is not a "bad" thing. It's just that one cannot cite the SA proof of convergence, although the SQ being used might be really among the best algorithms being used for a given system. For example, there is no doubt that Newton's method is superior to SA for parabolic objective functions. The point is that there are proofs that SA likely can do more for more complex problems, although it might not be the most efficient algorithm for a specific system.

See more below on how to modify ASA with QUENCH OPTIONS, easily permitting various degrees of quenching. This is especially useful if you determine that your optimal state(s) can be reached using SQ.

5.2.2. Generating Probability Density Function

In a *D*-dimensional parameter space with parameters p^i having ranges $[A_i, B_i]$, about the k'th last saved point (e.g., a local optima), p_k^i , a new point is generated using a distribution defined by the product of distributions for each parameter, $g^i(y^i; T_i)$ in terms of random variables $y^i \in [-1, 1]$, where $p_{k+1}^i = p_k^i + y^i(B_i - A_i)$, and "temperatures" T_i ,

$$g^{i}(y^{i};T_{i}) = \frac{1}{2(|y^{i}| + T_{i})\ln(1 + 1/T_{i})} \ .$$

The DEFINE_OPTIONS USER_GENERATING_FUNCTION permits using an alternative to this ASA distribution function.

5.2.3. Acceptance Probability Density Function

The cost functions, $C(p_{k+1}) - C(p_k)$, are compared using a uniform random generator, $U \in [0, 1)$, in a "Boltzmann" test: If

$$\exp[-(C(p_{k+1}) - C(p_k))/T_{\text{cost}}] > U$$
,

where $T_{\rm cost}$ is the "temperature" used for this test, then the new point is accepted as the new saved point for the next iteration. Otherwise, the last saved point is retained. The DEFINE_OPTIONS USER_ACCEPT_ASYMP_EXP or USER_ACCEPT_THRESHOLD permit using alternatives to this Boltzmann distribution function.

5.2.4. Reannealing Temperature Schedule

The annealing schedule for each parameter temperature, T_i from a starting temperature T_{i0} , is

$$T_i(k_i) = T_{0i} \exp(-c_i k_i^{1/D})$$
.

This is discussed further below.

The annealing schedule for the cost temperature is developed similarly to the parameter temperatures. However, the index for reannealing the cost function, $k_{\rm cost}$, is determined by the number of accepted points, instead of the number of generated points as used for the parameters. This choice was made because the Boltzmann acceptance criteria uses an exponential distribution which is not as fat-tailed as the ASA distribution used for the parameters. This schedule can be modified using several OPTIONS. In particular, the Pre-Compile DEFINE_OPTIONS USER_COST_SCHEDULE permits quite arbitrary functional modifications for this annealing schedule, and the Pre-Compile DEFINE_OPTIONS

As determined by the Program Options selected, the parameter "temperatures" may be periodically adaptively reannealed, or increased relative to their previous values, using their relative first derivatives with respect to the cost function, to guide the search "fairly" among the parameters.

As determined by the Program Options selected, the reannealing of the cost temperature resets the scale of the annealing of the cost acceptance criteria as

$$T_{\rm cost}(k_{\rm cost}) = T_{0\;{\rm cost}} \exp(-c_{\rm cost} k_{\rm cost}^{1/D}) \; . \label{eq:tost}$$

The new $T_{0 \text{ cost}}$ is taken to be the minimum of the current initial cost temperature and the maximum of the absolute values of the best and last cost functions and their difference. The new k_{cost} is calculated taking T_{cost} as the maximum of the current value and the absolute value of the difference between the last and best saved minima of the cost function, constrained not to exceed the current initial cost temperature. This procedure essentially resets the scale of the annealing of the cost temperature within the scale of the current best or last saved minimum.

This default algorithm for reannealing the cost temperature, taking advantage of the ASA importance sampling that relates most specifically to the parameter temperatures, while often is quite efficient for some systems, may lead to problems in dwelling too long in local minima for other systems. In such case, the user may also experiment with alternative algorithms effected using the Reanneal_Cost OPTIONS, discussed below. For example, ASA provides an alternative calculation for the cost

temperature, when Reanneal_Cost < -1 or > 1, that periodically calculates the initial and current cost temperatures or just the initial cost temperature, resp., as a deviation over a sample of cost functions.

These reannealing algorithms can be changed adaptively by the user as described below in the sections USER_REANNEAL_COST and USER_REANNEAL_PARAMETERS.

5.3. Efficiency Versus Necessity

ASA is not necessarily an "efficient" code. For example, if you know that your cost function to be optimized is something close to a parabola, then a simple gradient Newton search method most likely would be faster than ASA. ASA is believed to be faster and more robust than other simulated annealing techniques for *most* complex problems with multiple local optima; again, be careful to note that some problems are best treated by other algorithms. If you do not know much about the structure of your system, and especially if it has complex constraints, and you need to search for a global optimum, then this ASA code is heartily recommended to you.

In the context of efficiency and necessity, the user should be alert to recognize that any sampling or optimization program generally should be considered as complementary, not as a substitute, to gaining knowledge of a particular system. Unlike relatively "canned" codes that exist for (quasi–)linear systems, nonlinear systems typically are non-typical. Often some homework must be done to understand the system, and tuning often is required of numerical algorithms such as ASA. For example, while principal component analyses (PCA) often suffices to generate good (quasi–)orthogonal or (quasi–)independent sets of parameters, this is not true for general nonlinear systems. While such innovations as reannealing take good advantage of ASA which offers independent distributions for each parameter, this generally may not be a good substitute for a user–defined front–end, e.g., before the call to asa () or even embedded within the cost_function (), to interpret and define relevant parameters.

The ASA-NOTES file contains the sections @@Number of Generated States Required and @@Judging Importance-Sampling, recommending use of log-log plots to extrapolate the number of generated states required to attain a global minimum, possibly as a function of selected OPTIONS.

6. Outline of Use

Set up the ASA interface: Your program should be divided into two basic modules. (1) The user calling procedure, containing the cost function to be minimized (or its negative if you require a global maximum), is contained in asa_usr.c, asa_usr.h and asa_usr_cst.c. (2) The ASA optimization procedure, is contained in asa.c and asa.h. The file asa_usr_asa.h contains definitions and macros common to both asa.h and asa_usr.h. Furthermore, there are some options to explore/read below. It is assumed there will be no confusion over the standard uses of the term "parameter" in different contexts, e.g., as an element passed by a subroutine or as a physical coefficient in a cost function.

ASA has been run successfully on many machines under many compilers. To check out your own system, you can run 'make' (or the equivalent set of commands in the ASA–Makefile), and compare your asa_out and asa_usr_out files to the asa_test_asa and asa_test_usr files, respectively, provided with this code. No attempt was made to optimize any compiler, so that the test runs do not really signify any testing of compilers or architectures; rather they are meant to be used as a guide to determine what you might expect on your own machine.

The major sections below describe the compilation procedures, the Program Options available to you to control the code, the use of templates to set up your user module and interface to the asa module, and how to submit bug reports.

If you already have your own cost function defined, you can insert it into asa_usr_cst.c. If you wish to insert more OPTIONS, as a quick guide to get started, you can search through asa_usr.c and the ASA-Makefile for all occurrences of "MY_TEMPLATE_" to insert the necessary definitions required to run ASA. If you use both OPTIONS_FILE and OPTIONS_FILE_DATA set to TRUE, then usually most such information can be placed in the asa_opt file, and then only the cost_function () must be inserted. The place to insert the cost_function () is marked by "MY_TEMPLATE_cost."

7. ASA-Makefile/Compilation Procedures

The ASA–Makefile is intended to be a template for your own Makefile. For quick use, just copy this file to Makefile, which will be recognized by any standard make tool.

The PostScript® ASA-README.ps and ASCII ASA-README.txt and ASA-README+.txt files were generated using 'make doc'. The ASA-Makefile describes some options for formatting these files differently. Use 'make' or 'make all' to compile and run asa_run, the executable prepared for the test function. Examine the ASA-Makefile to determine the "clean" options available.

Since complex problems by their nature are often quite unique, it is unlikely that the default parameters are just right for your problem. However, experience has shown that if you *a priori* do not have any reason to determine your own parameters, then you might do just fine using these defaults, and these are recommended as a first–order guess. These defaults can be changed simply by adding to the DEFINE_OPTIONS line in the ASA–Makefile, by passing options on your command line, and by changing structure elements in the user or as a module as described below. Depending on how you integrate ASA into your own user modules, you may wish to modify this ASA–Makefile or at least use some of these options in your own compilation procedures.

Note that the ASA-Makefile is just a convenience, not a necessity, to use ASA. E.g., on systems which do not support this utility, you may simply compile the files following the guidelines in the ASA-Makefile, taking care to pass the correct DEFINE_OPTIONS to your compilation commands at your shell prompt. Still another way, albeit not as convenient, is to make the desired changes in the asa_usr_asa.h, and asa.h or asa_usr.h files as required.

Since the ASA-Makefile contains comments giving short descriptions of some options, it should be considered as an extension of this documentation file. For convenience, most of this information is repeated below. However, to see how they can be used in compilations, please read through the ASA-Makefile.

For example, to run the ASA test problem using the gcc compiler, you could just type at your "%" prompt:

```
% cp ASA-Makefile Makefile
```

% gcc -g -DASA_TEST=TRUE -o asa_run asa_usr.c asa_usr_cst.c asa.c -lm

% asa run

Note that if you use gcc across different levels of compiler optimization on different platforms, you may wish to invoke -ffloat-store to be sure to get the same precision in your results.

If you have defined your own cost function in asa_usr_cst.c or within the "MY_TEMPLATE_" guides in asa_usr.c, then ASA_TEST should be set to FALSE (the default if ASA_TEST is not defined in your compilation lines or in the ASA-Makefile). The code for ASA_TEST=TRUE is given just above these guides as a template to use for your own cost function.

The easiest way for many users to quickly use ASA likely is to invoke the COST_FILE, OPTIONS_FILE, and OPTIONS_FILE_DATA OPTIONS (the default), using the files asa_usr_cst.c and asa_opt as templates. This is further described below and illustrated in the https://www.ingber.com/asa_examples.txt file in the section Use of COST_FILE on Shubert Problem.

7.1. DLL ASA-Makefile

Under Cygwin (cygwin.com), set ASA_LIB to TRUE and INCL_STDOUT to FALSE (OPTIONS described below), with the command

% make asadll

to produce a DLL to call asa_main() as a DLL function under windows. (Ignore any undefined references to _WinMain.) Note that per instructions given in the ASA-Makefile, i686-pc-mingw32-gcc.exe should be used.

If paths are used to access files in code, under Windows use absolute paths with "\" (double back slash) to separate folders/directories, instead of relative paths with "/" (single forward slash) separators as on other Unix platforms.

8. Generic ASA Interfaces

The sections above describe how to quickly adapt ASA for use in many problems. However, complex projects often require sophisticated use of multiple languages to handle data and multiple algorithms. ASA has many OPTIONS that enable users to interface ASA with such complex projects.

ASA should compile under C++ as well as under C compilers. For example, I regularly test this by running projects under both gcc and g++. This can be very useful when ASA is called from other C++ programs, e.g., when using ASA_LIB set to TRUE. The ASA-NOTES file has additional information, e.g., when using SUNPRO CC, etc.

I have led many projects that required ASA to interface with Java, Maple, Matlab, MySQL, etc. The approach briefly described below can be applied to any language that permits a simple interface to C code. This definitely requires some expert experience in C, so you may have to find a local C guru, since I cannot help you with your specific project.

Some specific interfaces have been prepared by other people, and I have included some of them in the asa_contrib.txt file. The tradeoff for their simple use is that these approaches are limited to using just a few ASA OPTIONS as they typically have trimmed down the ASA code.

The generic approach is to utilize at least the OPTIONS ASA_LIB and OPTIONAL_DATA_PTR, setting them to TRUE. ASA_LIB permits the entire ASA code to be called as a simple function. Its sole parameter can be a struct defined by OPTIONAL_PTR_TYPE, e.g., OPTIONAL_PTR_TYPE PROJECT, defined in asa_usr_asa.h. A small include file common to asa_usr_asa.h and to the larger complex project, e.g., project.h, is used to define the constituents of the PROJECT struct. As described above in the DLL ASA—Makefile sub-Section of Section 7, ASA_LIB can be used to create a DLL to be called by Windows programs.

A small C function, e.g., project.c, is to be used for the interface between ASA and the other language. Similarly, another small function also may be used to interface the project to handle the interface, e.g., project.m, project.java, project.mpl, etc. Inversely, the interface may (also) be between the cost function, e.g., in asa_usr_cst.c or asa_usr.c, and the project. Then the application below is used to pass information between the cost function and the other language.

The other language passes information and data to project.c required by ASA, where it is packed into the struct defined by OPTIONAL_PTR_TYPE. Multiple or recursive calls to ASA can be handled by including a flag in this struct, e.g., to turn on different cost functions. Also added to this struct are placeholders for the output of ASA required by the project. This struct is passed to the ASA code by calling asa_main () defined in asa_usr.c with a parameter PROJECT *Project. In asa_main (), in the section defining properties of OPTIONAL_DATA_PTR, the pointer to Project struct is set to the pointer path to Asa_Data_Ptr. Asa_Data_Ptr is now passed throughout the entire ASA code via the OPTIONS pointer, project parameters can be adaptively changed, etc. See the discussion under Asa_Data_Ptr.

After the call to asa () in asa_usr.c, its output can be packed into the project struct, before memory is freed. The pointer Asa_Data_Ptr should be set to NULL instead of freed; see the comment in asa_usr.c at the place Asa_Data_Ptr is freed in the default code.

It is wise to create #define PROJECT and #endif pairs wherever changes to any ASA code are made, define PROJECT to TRUE in asa_usr_asa.h, so that it will be easy to modify updated ASA code, etc. Probably several such changes will have to be made in asa_usr.c. Control of OPTIONS likely will best be handled in asa usr asa.h than in the ASA–Makefile.

9. User Options

Program Options, i.e., the USER_DEFINES typedef on the OPTIONS, USER_OPTIONS, RECUR_USER_OPTIONS, etc., are turned on during the running of asa (). The DEFINE_OPTIONS are compiled in by the use of arguments to the compilation or by setting them in the asa_usr_asa.h file. An example of the former is Reanneal_Parameters, and an example of the latter is ASA_SAMPLE. The basic code is kept small for most users by using the Pre-Compile DEFINE_OPTIONS to pull in additional DEFINE_OPTIONS only if required. The Program Options are intended to be used adaptively and/or to pull in additional code for cases where repeated or recursive use, e.g., when using

SELF_OPTIMIZE, might be facilitated by having control of some Program Options at separate levels.

Note that even when the DEFINE_OPTIONS or Program Options are used to pull in new code, separate levels of control also can be achieved, albeit usually at the price of incurring some overhead in setting values at some levels of recursion or repeated calls. For example, in cases where new arrays or functions come into play, enough parameters are passed between the asa and user modules to calculate the defaults as well as different values adaptively. In some often used cases, separate DEFINE_OPTIONS are given, e.g., both OPTIONS_FILE and RECUR_OPTIONS_FILE exist. I have tried to strike some reasonable balance between these goals and constraints.

The DEFINE_OPTIONS are organized into two groups: Pre-Compile Options and (Pre-Compile) Printing Options. In addition, there are some alternatives to explore under Compiler Choices and Document Formatting. Below are the DEFINE_OPTIONS with their defaults. The Program Options are further discussed in other sections in this document.

Note that the Pre-Compile DEFINE_OPTIONS are all in capital letters, and the adaptive Program Options (under structure USER_OPTIONS in the user module and under structure OPTIONS in the asa module) are in capital and lower-case letters. In this file, often just the term OPTIONS may refer to the set of all options when the context is clear.

9.1. Pre-Compile DEFINE_OPTIONS

9.1.1. USER COST FUNCTION=cost function

The default name of the cost function is cost_function. This can be changed in asa_usr_asa.h (or the ASA-Makefile) by defining USER_COST_FUNCTION. This of course requires compiling in the new cost function and its prototype.

9.1.2. RECUR USER COST FUNCTION=recur cost function

When SELF_OPTIMIZE is TRUE, the default name of the recur cost function is recur_cost_function. This can be changed in asa_usr_asa.h (or the ASA-Makefile) by defining RECUR_USER_COST_FUNCTION. This of course requires compiling in the new cost function and its prototype.

9.1.3. OPTIONS FILE=TRUE

You can elect to read in many of the Program Options from asa_opt by setting OPTIONS_FILE=TRUE. OPTIONS_FILE=TRUE can be set in the ASA-Makefile in compilation commands or in asa usr asa.h.

9.1.4. OPTIONS_FILE_DATA=TRUE

If OPTIONS_FILE is set to TRUE, then setting OPTIONS_FILE_DATA to TRUE permits reading most initialization data from asa_opt, i.e., number of parameters, minimum and maximum ranges, initial values, and integer or real types. This should suffice for most applications, just requiring insertion of the user's cost_function into asa_usr_cst.c or asa_usr.c.

If OPTIONS_FILE, OPTIONS_FILE_DATA and QUENCH_COST are TRUE, then *User_Quench_Cost_Scale is read in from asa_opt. If OPTIONS_FILE, OPTIONS_FILE_DATA, QUENCH_COST, and QUENCH_PARAMETERS are TRUE, then *User_Quench_Cost_Scale and User_Quench_Param_Scale [] all are read in from asa_opt.

9.1.5. RECUR OPTIONS FILE=FALSE

When SELF_OPTIMIZE is TRUE, you can elect to read in many of the Program Options for the top-level program from asa_opt_recur (which you will have to create in the style of asa_opt), by setting RECUR_OPTIONS_FILE=TRUE.

9.1.6. RECUR OPTIONS FILE DATA=FALSE

When SELF_OPTIMIZE is TRUE, if RECUR_OPTIONS_FILE is set to TRUE, then setting RECUR_OPTIONS_FILE_DATA to TRUE permits reading most initialization data from asa_opt_recur (which you will have to create in the style of asa_opt), i.e., number of parameters, minimum and maximum ranges, initial values, and integer or real types.

If RECUR_OPTIONS_FILE, RECUR_OPTIONS_FILE_DATA and QUENCH_COST are TRUE, then *User_Quench_Cost_Scale is read in from asa_opt_recur. If RECUR_OPTIONS_FILE, RECUR_OPTIONS_FILE_DATA, QUENCH_COST, and QUENCH_PARAMETERS are TRUE, then *User_Quench_Cost_Scale and User_Quench_Param_Scale [] all are read in from asa_opt_recur.

9.1.7. COST_FILE=TRUE

If COST_FILE is set to TRUE, then you can use a separate file to define your cost function. When used together with OPTIONS_FILE and OPTIONS_FILE_DATA both set to TRUE, most users may be able to just use their own asa_usr_cst.c file for their cost_function () together with the asa_opt data file, and not have to work through some of the examples and templates contained in asa_usr.c.

When COST_FILE is set to TRUE, the file asa_usr_cst.c contains cost_function (). If you wish to change the name of cost_function () in asa_usr_cst.c, then you must also change this name in the call to asa () in asa_usr.c (search under "asa (") and in the prototype listing in asa_usr.h (in the HAVE_ANSI set to TRUE or FALSE section as appropriate). You may wish to copy the appropriate parameter list in asa_usr_cst.c just before the ASA_TEST problem to be sure of using the proper format expected by asa () in asa.c.

The https://www.ingber.com/asa_examples.txt file contains a section Use of COST_FILE on Shubert Problem which illustrates the simple modifications of ASA required to use COST_FILE.

9.1.8. ASA LIB=FALSE

Setting ASA_LIB=TRUE will facilitate your running asa () as a library call from another program, calling asa_main () in asa_usr.c. In the templates provided, all initializations and cost function definitions are set up in the user module. For example, you may wish to have some data read in to a module that calls asa_main (), then parses out this information to the arrays in asa_main () and initialize_parameters (and possibly recur_initialize_parameters). In conjunction with setting printout to stdout (see ASA_OUT and USER_ASA_OUT), this can be a convenient way of using the same asa_run executable for many runs

When ASA_LIB is TRUE, another function becomes available in asa_usr.c, asa_seed (), which can be used to change the initial seed used in runs made by asa_main (). If this routine is not called, then the default initial seed is used. An example of using this routine when calling asa_main () is given with ASA_TEMPLATE_LIB, using a main () at the end of the asa_usr.c file.

As described in the DLL ASA–Makefile sub-Section of Section 7, the ASA–Makefile and ASA_LIB can be used to create a DLL for Windows programs.

9.1.9. HAVE_ANSI=TRUE

Setting HAVE_ANSI=FALSE will permit you to use an older K&R C compiler. This option can be used if you do not have an ANSI compiler, overriding the default HAVE_ANSI=TRUE. If you use HAVE ANSI=FALSE, change CC and CDEBUGFLAGS as described in the ASA-Makefile.

9.1.10. IO PROTOTYPES=FALSE

Most newer operating systems do not like any other I/O prototyping other than those in their own include files. Other machines, like a Dec-3100 under Ultrix complain that the ANSI I/O prototypes were inconsistent. A Sun under 4.1.x gcc gave warnings if no I/O prototypes were present. The defaults in asa_usr_asa.h use newer system prototypes. IO_PROTOTYPES=TRUE will uncomment out declarations for such items as fprintf, fflush, fclose, exit, and fscanf.

9.1.11. TIME CALC=FALSE

Some systems do not have the time include files used here; others have different scales for time. Setting TIME_CALC=TRUE will permit use of the time routines.

9.1.12. TIME_STD=FALSE

Some systems, e.g., hpux and Cygwin (with i686-pc-mingw32-gcc.exe), use other Unix-standard macros to access time. Setting TIME_STD=TRUE when using TIME_CALC=TRUE will use these time routines instead.

9.1.13. TIME_GETRUSAGE=TRUE

An additional module for using TIME_CALC set to TRUE, setting TIME_GETRUSAGE to FALSE, is more portable to compile across some platforms, e.g., Cygwin (with i686-pc-mingw32-gcc.exe), but it can require different parameters for timing results. Comments have been placed in the code in asa.c.

9.1.14. INT LONG=TRUE

Some smaller systems choke on 'long int' and this option can be set to INT_LONG=FALSE to turn off warnings and possibly some errors. The cast LONG_INT is used to define 'int' or 'long int' appropriately.

9.1.15. INT ALLOC=FALSE

The cast on *number_parameters is set to ALLOC_INT which defaults to LONG_INT. On some machines, ALLOC_INT might have to be set to int if there is a strict requirement to use an (unsigned) int for calloc, while 'long int' still can be used for other aspects of ASA. If ALLOC_INT is to be set to int, set INT ALLOC to TRUE.

9.1.16. SMALL_FLOAT=1.0E-18

SMALL_FLOAT is a measure of accuracy permitted in log and divide operations in asa, i.e., which is not precisely equivalent to a given machine's precision. There also are Pre-Compile DEFINE_OPTIONS to separately set constants for minimum and maximum doubles and precision permitted by your machine. Experts who require the very best precision can fine—tune these parameters in the code.

Such issues arise because the fat tail of ASA, associated with high parameter temperatures, is very important for searching the breadth of the ranges especially in the initial stages of search. However, the parameter temperatures require small values at the final stages of the search to converge to the best solution, albeit this is reached very quickly given the exponential schedule proven in the referenced publications to be permissible with ASA. Note that the test problem in asa_usr_cst.c and asa_usr_c is a particularly nasty one, with 1E20 local minima and requiring ASA to search over 12 orders of magnitude of the cost function before correctly finding the global minimum. Thus, intermediate values disagree somewhat for SMALL_FLOAT=1.0E-12 from the settings using SMALL_FLOAT=1.0E-18 (the default); they agree if SMALL_FLOAT=1.0E-12 while also setting MIN_DOUBLE=1.0E-18. The results diverge when the parameter temperatures get down to the range of E-12, limiting the accuracy of the SMALL_FLOAT=1.0E-12 run.

On some machines that have register variables assigned inconsistently with other doubles, there can arise some numerical differences in some systems. There has been no such problem found on Sun/Solaris 2.x using gcc, but some problems have been noticed on some Intel chips using different gcc optimizations.

As mentioned above, note that the use of -ffloat-store with gcc/g++ is to maintain the same level of precision across levels of compiler optimization on different platforms.

9.1.17. MIN DOUBLE=SMALL FLOAT

You can define your own machine's minimum positive double here if you know it.

9.1.18. MAX_DOUBLE=1.0/SMALL_FLOAT

You can define your own machine's maximum double here if you know it.

9.1.19. EPS_DOUBLE=SMALL_FLOAT

You can define your own machine's maximum precision here if you know it.

9.1.20. CHECK_EXPONENT=FALSE

When CHECK_EXPONENT is set to TRUE, the macro EXPONENT_CHECK(x), defined in asa.h in terms of MIN_DOUBLE and MAX_DOUBLE, checks that an exponent x is within a valid range and, if not, adjusts its magnitude to fit in the range.

9.1.21. NO_PARAM_TEMP_TEST=FALSE

If NO_PARAM_TEMP_TEST is set to TRUE, then all parameter temperatures less than EPS_DOUBLE are set to EPS_DOUBLE, and no exit is called.

9.1.22. NO_COST_TEMP_TEST=FALSE

If NO_COST_TEMP_TEST is set to TRUE, then a cost temperature less than EPS_DOUBLE is set to EPS_DOUBLE, and no exit is called.

9.1.23. SELF OPTIMIZE=FALSE

The user module contains a template to illustrate how ASA may be used to self-optimize its Program Options. This can be very CPU-expensive and is of course dependent on how you define your recursive cost function (recur_cost_function in the user module). The example given returns from recur_cost_function the number of function evaluations taken to optimization the test cost_function, with the constraint to only accept optimizations of the cost_function that are lower than a specified value. A few lines of code can be uncommented in asa_usr.c to force a fast exit for this demo; search for FAST EXIT. (Note that this also could achieved by using OPTIONS->Immediate_Exit discussed below.)

The ASA_TEMPLATE_SELFOPT example uses OPTIONS_FILE=FALSE in the Pre-Compile Options. Note that DEFINE_OPTIONS OPTIONS_FILE=TRUE and OPTIONS_FILE_DATA=TRUE here would take data from asa_opt for the lower-level program using the cost_function (). Both DEFINE_OPTIONS RECUR_OPTIONS_FILE and RECUR_OPTIONS_FILE_DATA would have to be set to TRUE to use asa_opt_recur to read in both the OPTIONS and the recur_cost_parameters data (which you would have to write in the style of asa_opt) for the top-level recur_cost_function ().

This can be useful when approaching a new system, and it is suspected that the default ASA Program Options are not at all efficient for this system. It is suggested that a trimmed cost function or data set be used to get a reasonable guess for a good set of Program Options. ASA has demonstrated that it typically is quite robust under a given set of Program Options, so it might not make too much sense to spend lots of resources performing additional fine tuning of these options. Also, it is possible you might crash the code by permitting ranges of Program Options that cause your particular cost_function to return garbage to asa ().

9.1.24. ASA TEST=FALSE

Setting ASA_TEST to TRUE will permit running the ASA test problem. This has been added to the DEFINE_OPTIONS in the ASA-Makefile so that just running make will run the test problem for the new user. No attempt was made to optimize any OPTIONS for the ASA_TEST problem as it appears in the standard code.

9.1.25. ASA TEST POINT=FALSE

The code used for the ASA_TEST problem closely follows the reference given in asa_usr.c, and was rewritten from code given to the author in 1992. Other researchers have sent the author different code for this system, and all results agree within round-off errors.

However, note that the actual problem stated in the reference in asa_usr.c is harder, requiring the finding of an optimal point and not an optimal region. The code for that problem is given in asa_usr.c when ASA_TEST_POINT is set to TRUE (having the effect of setting COST_FILE to FALSE in asa_usr_asa.h). The https://www.ingber.com/asa_examples.txt file illustrates how that global minimum can be attained.

9.1.26. MY_TEMPLATE=TRUE

When MY_TEMPLATE is set to TRUE (the default), locations in asa_usr.c and asa_usr_asa.h become active sites for your own code. Searching asa_usr.c for "MY_TEMPLATE_" provides a guide for additional code to add for your own system. For example, just above the occurrence of the guides for MY_TEMPLATE_cost is the corresponding code for ASA_TEST=TRUE. Keeping the default of ASA_TEST set to FALSE permits such changes without overwriting the test example.

9.1.27. USER_INITIAL_COST_TEMP=FALSE

Setting USER_INITIAL_COST_TEMP to TRUE permits you to specify the initial cost temperature in the User_Cost_Temperature [] array. This can be useful in problems where you want to start the search at a specific scale.

9.1.28. RATIO TEMPERATURE SCALES=FALSE

Different rates of parameter annealing can be set with RATIO_TEMPERATURE_SCALES set to TRUE. This requires initializing the User_Temperature_Ratio [] array in the user module as discussed below.

9.1.29. USER INITIAL PARAMETERS TEMPS=FALSE

Setting USER_INITIAL_PARAMETERS_TEMPS to TRUE permits you to specify the initial parameter temperatures in the User_Parameter_Temperature [] array. This can be useful in constrained problems, where greater efficiency can be achieved in focussing the search than might be permitted just by setting upper and lower bounds.

9.1.30. DELTA PARAMETERS=FALSE

Different increments, used during reannealing to set each parameter's numerical derivatives, can be set with DELTA_PARAMETERS set to TRUE. This requires initializing the User_Delta_Parameter [] array in the user module as discussed below.

9.1.31. QUENCH_PARAMETERS=FALSE

This DEFINE_OPTIONS permits you to alter the basic algorithm to perform selective "quenching," i.e., faster temperature cooling than permitted by the ASA algorithm. This can be very useful, e.g., to quench the system down to some region of interest, and then to perform proper annealing for the rest of the run. However, note that once you decide to quench rather than to truly anneal, there no longer is any statistical guarantee of finding a global optimum.

Once you decide you can quench, there are many more alternative algorithms you might wish to choose for your system, e.g., creating a hybrid global-local adaptive quenching search algorithm, e.g., using USER_REANNEAL_PARAMETERS described below. Note that just using the quenching OPTIONS provided with ASA can be quite powerful, as demonstrated in the https://www.ingber.com/asa_examples.txt file.

Setting QUENCH_PARAMETERS to TRUE can be extremely useful in very large parameter dimensions; see the ASA-NOTES file under the section on Quenching. As discussed in the first 1989

VFSR paper, the heuristic statistical proof of finding the global optimum reduces to the following: The parameter temperature schedules must suffice to insure that the product of individual generating distributions,

$$g = \prod_{i} g^{i}$$
,

taken at all annealing times, indexed by k, of not generating a global optimum, given infinite time, is such that

$$\prod_{k} (1 - g_k) = 0 ,$$

which is equivalent to

$$\sum_{k} g_{k} = \infty .$$

For the ASA temperature schedule, this is satisfied as

$$\sum_{k} \prod^{D} 1/k^{-1/D} = \sum_{k} 1/k = \infty \ .$$

Now, if the temperature schedule above is redefined as

$$T_i(k_i) = T_{0i} \exp(-c_i k_i^{Q/D}) \; , \label{eq:Ti}$$

$$c_i = m_i \exp(-n_i Q/D)$$
,

in terms of the "quenching factor" Q, then the above proof fails if Q > 1 as

$$\sum_{k} \prod_{i=1}^{D} 1/k^{-Q/D} = \sum_{k} 1/k^{Q} \le \infty.$$

This simple calculation shows how the "curse of dimensionality" arises, and also gives a possible way of living with this disease which will be present in any algorithm that substantially samples the parameter space. In ASA, the influence of large dimensions becomes clearly focused on the exponential of the power of k being 1/D, as the annealing required to properly sample the space becomes prohibitively slow. So, if we cannot commit resources to properly sample the space ergodically, then for some systems perhaps the next best procedure would be to turn on quenching, whereby Q can become on the order of the size of number of dimensions. In some cases tried, a small system of only a few parameters can be used to determine some reasonable Program Options, and then these can be used for a much larger space scaled up to many parameters. This can work in some cases because of the independence of dimension of the generating functions.

If QUENCH_PARAMETERS is TRUE, then User_Quench_Param_Scale [] must be defined as described below.

If OPTIONS_FILE_DATA, QUENCH_COST, and QUENCH_PARAMETERS are TRUE, then *User_Quench_Cost_Scale and User_Quench_Param_Scale [] all are read in from asa_opt. If RECUR_OPTIONS_FILE_DATA, QUENCH_COST, and QUENCH_PARAMETERS are TRUE, then *User_Quench_Cost_Scale and User_Quench_Param_Scale [] all are read in from asa_opt_recur.

9.1.32. QUENCH_COST=FALSE

If QUENCH_COST is set to TRUE, the scale of the power of 1/D temperature schedule used for the acceptance function can be altered in a similar fashion to that described above when QUENCH_PARAMETERS is set to TRUE. However, note that this OPTIONS does not affect the annealing proof of ASA, and so this may be used without damaging the statistical ergodicity of the algorithm. Even greater functional changes can be made using the Pre-Compile DEFINE_OPTIONS USER_COST_SCHEDULE, USER_ACCEPT_ASYMP_EXP, USER_ACCEPT_THRESHOLD, or USER_ACCEPTANCE_TEST.

If QUENCH_COST is TRUE, then User_Quench_Cost_Scale [0] must be defined as described below.

If OPTIONS_FILE_DATA and QUENCH_COST are TRUE, then User_Quench_Cost_Scale [] is read in from asa_opt. If RECUR_OPTIONS_FILE_DATA and QUENCH_COST are TRUE, then *User_Quench_Cost_Scale is read in from asa_opt_recur.

Similarly as noted above for QUENCH_PARAMETERS, setting QUENCH_COST to TRUE can be extremely useful in very large parameter dimensions; see the ASA-NOTES file under the section on Quenching.

9.1.33. QUENCH_PARAMETERS_SCALE=TRUE

When QUENCH_PARAMETERS is TRUE, if QUENCH_PARAMETERS_SCALE is TRUE, then the temperature scales and the temperature indexes are affected by User_Quench_Param_Scale []. This can have the effects of User_Quench_Param_Scale [] appear contrary, as the effects on the temperatures from the temperature scales and the temperature indexes can have opposing effects. However, these defaults are perhaps most intuitive when the User_Quench_Param_Scale [] are on the order of the parameter dimension.

When QUENCH_PARAMETERS is TRUE, if QUENCH_PARAMETERS_SCALE is FALSE, only the temperature indexes are affected by User_Quench_Param_Scale []. The same effect could be managed by raising Temperature_Anneal_Scale to the appropriate power, but this may not be as convenient.

9.1.34. QUENCH_COST_SCALE=TRUE

When QUENCH_COST is TRUE, if QUENCH_COST_SCALE is TRUE, then the temperature scale and the temperature index are affected by User_Quench_Cost_Scale [0]. This can have the effects of User_Quench_Cost_Scale [0] appear contrary, as the effects on the temperature from the temperature scale and the temperature index can have opposing effects. However, these defaults are perhaps most intuitive when User_Quench_Cost_Scale [0] is on the order of the parameter dimension.

When QUENCH_COST is TRUE, if QUENCH_COST_SCALE is FALSE, only the temperature index is affected by User_Quench_Cost_Scale [0]. The same effect could be managed by raising Temperature_Anneal_Scale to the appropriate power, but this may not be as convenient.

9.1.35. ASA TEMPLATE=FALSE

There are several templates that come with the ASA code. To permit use of these OPTIONS without having to delete these extra tests, these templates are wrapped with ASA_TEMPLATE's. To use your own cost function, you likely will only have to write cost_function () in asa_usr_cst.c, and use the asa_opt file. If you wish to add more OPTIONS or code, you may need to write relevant portions of cost_function () and initialize_parameters () in asa_usr.c and asa_usr.h.

The ASA-Makefile has several examples of DEFINE_OPTIONS that will generate test examples using special ASA_TEMPLATE's set to TRUE. These are {ASA_TEMPLATE_LIB, ASA_TEMPLATE_ASA_OUT_PID, ASA_TEMPLATE_MULTIPLE, ASA_TEMPLATE_SELFOPT, ASA_TEMPLATE_SAMPLE, ASA_TEMPLATE_QUEUE, ASA_TEMPLATE_PARALLEL, ASA_TEMPLATE_SAVE}; the sets of Pre-Compile OPTIONS these use are defined in asa_usr_asa.h.

Lines marked off by ASA_TEMPLATE, with no additional suffix, are for specific examples only. ASA_TEMPLATE, with no suffix, should not be set to TRUE, else all groups of these examples will be brought into the code, likely not what is wanted.

9.1.36. OPTIONAL_DATA_DBL=FALSE

It can be useful to return/pass additional information to the user module from/through the asa module. When OPTIONAL_DATA_DBL is set to TRUE, an additional Program Option pointer, *Asa_Data_Dbl, and its dimension, Asa_Data_Dim_Dbl, are available in USER_DEFINES *USER OPTIONS to gather such data.

In the ASA_TEMPLATE_SELFOPT example provided (see the set of DEFINE_OPTIONS used in asa_usr_asa.h), OPTIONAL_DATA_DBL is used together with SELF_OPTIMIZE to find the set of ASA parameters giving the (statistically) smallest number of generated points to solve the ASA test problem, assuming this were run several times with different random seeds for randflt in asa_usr.c. Here, Asa_Data_Dbl [0] is used as a flag to print out Asa_Data_Dbl [1] in asa_usr.c, set to *best_number_generated_saved in asa.c.

If ASA_PARALLEL is TRUE, then N_Generated and *best_number_generated_saved will only be accurate within the scale of OPTIONS->Gener_Block, as these are updated in the acceptance function.

9.1.37. OPTIONAL DATA INT=FALSE

It can be useful to return/pass additional integer information to the user module from/through the asa module. When OPTIONAL_DATA_INT is set to TRUE, an additional Program Option pointer, *Asa_Data_Int, and its dimension, Asa_Data_Dim_Int, are available in USER_DEFINES *USER_OPTIONS to gather such data.

9.1.38. OPTIONAL_DATA_PTR=FALSE

It can be useful to return/pass additional array or structure information to the user module from/through the asa module (possibly containing other structures, e.g., useful when SELF_OPTIMIZE is TRUE). When OPTIONAL_DATA_PTR is set to TRUE, an additional Program Option pointer, *Asa_Data_Ptr, and its dimension, Asa_Data_Dim_Ptr, are available in USER_DEFINES *USER_OPTIONS to gather such data. The type of *Asa_Data_Dim_Ptr is a pre-compile OPTIONS set by OPTIONAL_PTR_TYPE. See examples under Asa_Data_Dim_Ptr and Asa_Data_Ptr.

If OPTIONAL_DATA_PTR is being used for RECUR_USER_OPTIONS as well as for USER_OPTIONS, you need not create (or free) additional memory in recur_cost_function() for Asa_Data_Dim_Ptr and Asa_Data_Ptr to be passed to the inner cost_function(), but rather link pointers to those in RECUR_USER_OPTIONS. In asa_usr.c, there are guidelines to set "#if TRUE" to "#if FALSE" at these points of the code. This is the proper technique to use if ASA_SAVE, ASA_SAVE_OPT, or ASA_SAVE_BACKUP is set to TRUE (since data is saved by asa() depending on the level of recursion)...

If ASA_SAVE, ASA_SAVE_OPT, and ASA_SAVE_BACKUP are not set to TRUE, then multiple levels of recursion can each have their own defined information indexed to different elements of the array of structures of size Asa_Data_Dim_Ptr.

The https://www.ingber.com/asa_examples.txt file contains some guidance of the use of OPTIONAL DATA PTR and Asa Data Ptr.

9.1.39. OPTIONAL PTR TYPE=USER TYPE

When OPTIONAL_DATA_PTR is set to TRUE, the type of *Asa_Data_Ptr is a pre-compile OPTIONS set by OPTIONAL_PTR_TYPE, e.g., changing the label USER_TYPE in asa_usr_asa.h. Be sure to place any non-standard types, like your own typedef struct, before the #define OPTIONAL_PTR_TYPE at the top of asa_usr_asa.h, e.g., under #if MY_TEMPLATE (since OPTIONAL_PTR_TYPE is tested below in asa_usr_asa.h). See the discussion under Asa_Data_Ptr.

9.1.40. USER_COST_SCHEDULE=FALSE

The function used to control the cost_function temperature schedule is of the form test_temperature in asa.c. If the user sets the Pre-Compile DEFINE_OPTIONS USER_COST_SCHEDULE to TRUE, then this function of test_temperature can be controlled, adaptively if desired, in asa_usr.c in Cost_Schedule () (and in recur_Cost_Schedule () if SELF_OPTIMIZE is TRUE) by setting USER_COST_SCHEDULE to TRUE. The names of these functions are set to the relevant pointer in asa_usr.c, and can be changed if desired, i.e.,

USER_OPTIONS->Cost_Schedule = user_cost_schedule;

RECUR_USER_OPTIONS->Cost_Schedule = recur_user_cost_schedule;

9.1.41. USER ACCEPT ASYMP EXP=FALSE

When USER_ACCEPT_ASYMP_EXP is TRUE, an asymptotic form of the exponential function as an alternative to the Boltzmann function becomes available for the acceptance test. A parameter OPTIONS->Asymp_Exp_Param becomes available, with a default of 1.0 in asa_usr.c giving the standard Boltzmann function. If you require a more moderate acceptance test, then negative Asymp_Exp_Param may be helpful.

9.1.42. USER ACCEPT THRESHOLD=FALSE

When USER_ACCEPT_THRESHOLD is TRUE, a simple alternative to the Boltzmann function becomes available for the acceptance test, simply defining the probability of acceptance to be 1 if

$$C(p_{k+1}) - C(p_k) \le T_{\text{cost}},$$

and 0 otherwise.

9.1.43. USER_ACCEPTANCE_TEST=FALSE

If the Pre-Compile DEFINE_OPTIONS USER_ACCEPTANCE_TEST is set to TRUE, the Boltzmann test probability function used in the acceptance criteria in asa.c can be changed, adaptively if desired, in asa_usr.c in user_acceptance_test () (and in recur_user_acceptance_test () if SELF_OPTIMIZE is TRUE). The names of these functions are set to the relevant pointer in asa_usr.c, and can be changed if desired, i.e.,

If both USER_ACCEPTANCE_TEST and USER_ACCEPT_ASYMP_EXP are set to TRUE, then the default OPTIONS->Asymp_Exp_Param = 1 can be used in asa_usr.c to duplicate the Boltzmann test in asa.c, e.g., as a template to further develop a new acceptance test.

USER_OPTIONS->Acceptance_Test = user_acceptance_test;

RECUR_USER_OPTIONS->Acceptance_Test = recur_user_acceptance_test;

When USER_ACCEPTANCE_TEST is TRUE, then any random numbers needed for the acceptance criteria are generated in the user module instead of in the asa module.

When USER_ACCEPTANCE_TEST is TRUE, additional OPTIONS are available to modify the acceptance criteria, either after the cost function is calculated or during its calculation:

USER OPTIONS->User Acceptance Flag

USER OPTIONS->Cost Acceptance Flag

USER OPTIONS->Last Cost

USER OPTIONS->Cost Temp Curr

USER_OPTIONS->Cost_Temp_Init

USER OPTIONS->Cost Temp Scale

USER OPTIONS->Prob Bias

USER OPTIONS->Random Seed

Failing the acceptance test is not equivalent to dropping generated states from consideration for testing with the acceptance criteria, e.g., if they fail some regional constraints. asa () is designed so that User_Acceptance_Flag is set to TRUE prior to calling the cost function whenever acceptance tests need not be performed, i.e., when using the cost function to generate initial conditions, when being used to calculate derivatives, or when samples are being generated to calculate the cost temperature; otherwise it is set to FALSE. The value of Cost_Acceptance_Flag always is set to FALSE before entering the cost function.

When entering the acceptance function, if Cost_Acceptance_Flag is TRUE, then the value of USER_OPTIONS->User_Acceptance_Flag (assuming *valid_state_generated_flag is TRUE) calculated user cost function () determines the value of the acceptance test. Otherwise, value USER_OPTIONS->Acceptance_Test () is called to calculate the USER_OPTIONS->User_Acceptance_Flag. Note that if the cost function is used to calculate the acceptance criteria, and it is acceptable (e.g., also *valid_state_generated_flag is TRUE), then both USER_OPTIONS->User_Acceptance_Flag and USER_OPTIONS->Cost_Acceptance_Flag must be set to TRUE.

For example, this can be useful if during the calculation of the cost function, without having to proceed to the final evaluation, it becomes clear that the acceptance criteria will not be passed. This might occur if the cost function is increasing during its calculation and an acceptance test is carried out using the uniform random number calculated at the top of the cost function. The partially evaluated cost function can be compared to the Last_Cost, using the Boltzmann criteria or whatever criteria is established in USER_OPTIONS->user_acceptance_test (). Then it is clear that the acceptance criteria will not be met (of course after checking that any constraints are met and setting *valid_state_generated_flag to TRUE if so), then USER_OPTIONS->User_Acceptance_Flag can be set to or left at FALSE, and then proceed to return to asa (). However, other information registered in the acceptance function still should be calculated, e.g., updating indices, information used for ASA_SAMPLE and ASA_PARALLEL, etc.

9.1.44. USER_GENERATING_FUNCTION=FALSE

The ASA generating probability function in asa.c can be changed if the user sets the Pre-Compile DEFINE_OPTIONS USER_GENERATING_FUNCTION to TRUE; then this function can be changed, adaptively if desired, in asa_usr.c in user_generating_distrib () (and in recur_user_generating_distrib () if SELF_OPTIMIZE is TRUE) by setting USER_GENERATING_FUNCTION to TRUE. The names of these functions are set to the relevant pointer in asa usr.c, and can be changed if desired, i.e.,

USER_OPTIONS->Generating_Distrib = user_generating_distrib;

RECUR_USER_OPTIONS->Generating_Distrib = recur_user_generating_distrib;

The parameters passed to these functions are further described below.

Several parameters additional to those required for the ASA distribution are passed to make it easier to install other common distributions. Note that range checks take place at multiple stages of search, so be sure your chosen ranges can take this into account.

9.1.45. USER REANNEAL COST=FALSE

In asa.c reannealing of the cost temperature is determined by the algorithm described above in the section Reannealing Temperature Schedule.

If the user sets the Pre-Compile DEFINE_OPTIONS USER_REANNEAL_COST to TRUE, while Reanneal_Cost is not 0 or -1, then the function controlling the new reannealed cost temperature can be controlled, adaptively if desired using USER_OPTIONS, in asa_usr.c in user_reanneal_cost (), and in recur_user_reanneal_cost () if SELF_OPTIMIZE is TRUE. The names of these functions are set to the relevant pointer in asa_usr.c, and can be changed if desired, i.e.,

USER_OPTIONS->Reanneal_Cost_Function = user_reanneal_cost;

RECUR_USER_OPTIONS->Reanneal_Cost_Function = recur_user_reanneal_cost;

In these functions, the variables *current_cost_temperature, *initial_cost_temperature, and the best and last saved cost function can be altered, and the returned integer value of TRUE or FALSE determines whether to use the best saved cost function as the current cost temperature.

Since these functions can be called every value of Acceptance_Frequency_Modulus, Generated_Frequency_Modulus, or when the ratio of accepted to generated points is less than Accepted_To_Generated_Ratio, this opportunity also can be used to adaptively change other OPTIONS. This can be very useful for systems where the scales of the acceptance criteria do not simply correlate the cost temperature with the current best value of the cost function.

For example, this function could be used when the last saved cost function is so close to zero that the effect would be to set the *initial_cost_temperature to that value, but the best value for the cost function is known to be less than zero. (An alternative moving average example is given in asa_usr.c.) Other alternatives are to use USER_REANNEAL_COST with default FALSE and Reanneal_Cost > 1 or < -1, as described below.

9.1.46. USER_REANNEAL_PARAMETERS=FALSE

In asa.h, the macro #define \

FUNCTION REANNEAL PARAMS(temperature, tangent, max_tangent) \

(temperature * (max_tangent / tangent))

is used to determine the new temperature, subject to further tests in reanneal (). This is the default if USER REANNEAL PARAMETERS is FALSE.

If the user sets the Pre-Compile DEFINE_OPTIONS USER_REANNEAL_PARAMETERS to TRUE, then the function controlling the new reannealed temperature can be controlled, adaptively if desired using USER_OPTIONS, in asa_usr.c in user_reanneal_params (), and in recur_user_reanneal_params () if SELF_OPTIMIZE is TRUE. The names of these functions are set to the relevant pointer in asa_usr.c, and can be changed if desired, i.e.,

USER_OPTIONS->Reanneal_Params_Function = user_reanneal_params;

RECUR_USER_OPTIONS->Reanneal_Params_Function = recur_user_reanneal_params;

Since FUNCTION_REANNEAL_PARAMS () can be called every value of Acceptance_Frequency_Modulus, Generated_Frequency_Modulus, or when the ratio of accepted to generated points is less than Accepted_To_Generated_Ratio, this opportunity also can be used to adaptively change other OPTIONS. For example, if the QUENCH_PARAMETERS OPTIONS is set to TRUE, as discussed above, it may useful to create a hybrid global—local adaptive quenching search algorithm.

9.1.47. MAXIMUM REANNEAL INDEX=50000

The maximum index (number of steps) at which the initial temperature and the index of the temperature are rescaled to avoid losing machine precision. ASA typically is quite insensitive to the value used due to the dual rescaling.

9.1.48. REANNEAL_SCALE=10.0

The reannealing scale used when MAXIMUM REANNEAL INDEX is exceeded.

9.1.49. ASA SAMPLE=FALSE

When ASA_SAMPLE is set to TRUE, data is collected by ASA during its global optimization process to importance–sample the user's variables. Four OPTIONS become available to monitor the sampling: Bias Acceptance, *Bias Generated, Average Weights, and Limit Weights.

If Average_Weights exceeds the user's choice of Limit_Weights, then the ASA_OUT file will contain additional detailed information, including temperatures and biases for each current parameter. To facilitate extracting importance–sampled information from the file printed out by the asa module, all relevant lines start with :SAMPLE[|:|#|+]. A sample () function in asa_usr.c illustrates the use of these tags.

Many Monte Carlo sampling techniques require the user to guess an appropriately decreasing "window" to sample the variable space. The fat tail of the ASA generating function, and the decreasing effective range of newly accepted points driven by exponentially decreasing temperature schedules, removes this arbitrary aspect of such sampling.

However, note that, albeit local optima are sampled, the efficiency of ASA optimization most often leads to poor sampling in regions whose cost function is far from the optimal point; many such points may be important contributions to algorithms like integrals. Accordingly, ASA_SAMPLE likely is best used to explore new regions and new systems.

To increase the sampling rate and thereby to possibly increase the accuracy of this algorithm, use one or a combination of the various OPTIONS available for slowing down the annealing performed by ASA. However, the selected OPTIONS still must yield good convergence if the optimal region is to be properly sampled.

9.1.50. ADAPTIVE_OPTIONS=FALSE

When ADAPTIVE_OPTIONS is set to TRUE, it is possible to change relevant OPTIONS in the USER_DEFINES struct in asa_usr_asa.h during run time, by including an asa_adaptive_options file.

This can be very useful to efficiently determine how to tune long runs.

The function call to adaptive_options(USER_OPTIONS) should be placed at the top of your cost function. Some examples of the format of lines in the comma-delimited asa_adaptive_options file are given just above the adaptive_options function in asa_usr.c.

9.1.51. ASA_BYPASS=FALSE

In asa_usr.c ASA_BYPASS has been added to simply bypass asa(). At the top of asa_usr.c the trigger is

#ifndef ASA_BYPASS

#define ASA_BYPASS FALSE

#endif

which can be simply changed. This can be useful to debug codes, to run the FITLOC simplex code after a good region is developed using asa(), etc. A nominal value of cost_value of 10000 has been set, which can easily be changed.

9.1.52. ASA_QUEUE=FALSE

When ASA_QUEUE is set to TRUE, a first-in first-out (FIFO) queue, of size USER_OPTIONS->Queue_Size, is used to collect generated states. When a new state is generated, its parameters are tested, within specified resolutions of USER_OPTIONS->Queue_Resolution [] (the absolute values of each of the differences between the parameters of the current generated state and those in the queue). If a previous state is already represented, then the stored values of the cost function and the cost flag are returned, instead of calling the cost function again. Note that the size of the array required to store the queued parameters is Queue_Size times the number of parameters, and this can consume a lot of CPU time as well as storage, so this OPTIONS is only useful for cost functions that are themselves very costly to evaluate. Setting ASA_TEMPLATE_QUEUE to TRUE will run an example using the ASA_TEST problem.

The ASA_QUEUE DEFINE_OPTIONS also can be used to coarse-grain a fit, by setting high values of Queue_Resolution []. Note the difference between the operations of this DEFINE_OPTIONS and ASA_RESOLUTION.

If ASA_QUEUE is TRUE and ASA_RESOLUTION is FALSE, machine precision is used for type double variables, the queue is created and subsequent variables are tested against this queue. If ASA_RESOLUTION and ASA_QUEUE are both TRUE, then the Coarse_Resolution [] array is used for Queue_Resolution [], ASA_RESOLUTION is enforced from the very first call to the cost function, and the queue is created using these coarse variables.

The default in asa.c for the FIFO queue uses a simple search among stored parameter values, under the assumption that for most complex systems for which ASA_QUEUE=TRUE is useful, the bottleneck is in the evaluation of the cost functions. If you think this is not true for you, and you need to conserve CPU time in using lists, the https://www.ingber.com/asa_contrib.txt file gives code that uses doubly—linked and hashed lists.

If ASA_QUEUE and ASA_PRINT_MORE are TRUE then, whenever a queued cost function is used, this is recorded in asa_out.

When ASA_QUEUE is TRUE, then the value of *cost_flag is tested for FALSE in previous states in queue and an invalid statement is available in the printout.

9.1.53. ASA_RESOLUTION=FALSE

When ASA_RESOLUTION is set to TRUE, parameters are resolved to a user-defined resolution set in USER_OPTIONS->Coarse_Resolution [], i.e., within plus or minus the values of Coarse_Resolution []. This is performed as soon as candidate values are generated, for each parameter for which Coarse_Resolution [] is greater than zero. Note the difference between the operations of this OPTIONS and ASA_QUEUE.

If ASA_QUEUE is TRUE and ASA_RESOLUTION is FALSE, machine precision is used for type double variables, the queue is created and subsequent variables are tested against this queue. If ASA_RESOLUTION and ASA_QUEUE are both TRUE, then the Coarse_Resolution [] array is used for Queue_Resolution [], ASA_RESOLUTION is enforced from the very first call to the cost function, and the queue is created using these coarse variables.

When USER_OPTIONS->Coarse_Resolution [] is > 0 and parameter_type [] is > 0 (specifying an integer parameter), ASA_RESOLUTION takes precedence over parameter_type [] when calculating new generated parameters.

9.1.54. ASA FUZZY=FALSE

When ASA_FUZZY is set to TRUE, a modified fuzzy-logic algorithm originally developed by Hime Junior performs an adaptive tuning of selected ASA OPTIONS. ASA_FUZZY has several adaptive parameters defined in asa_usr_asa.h, and some fixed #define parameters in the several subroutines at the bottom of asa_usr.c. The default functions are set to tune QUENCH_COST and QUENCH_PARAMETERS parameters, so these are tested to be TRUE in the default asa_usr_asa.h. If you change any defaults, pay attention to sections labeled ASA_FUZZY at top and bottom sections of asa_usr.c and asa_usr.h.

Initial QUENCH settings are saved in *ASA_FUZZY_Init_User_Quench_Param_Scale and *ASA_FUZZY_Init_User_Quench_Cost_Scale which may be used to restore regular annealing schedules under some conditions. In asa_usr.c they can be reset by the user, e.g., to 1.0, etc.

Often ASA_FUZZY will turn on QUENCHing > 1, violating the proof of ASA, as discussed above. For many systems, this speeding up of the sampling process can be a welcome efficiency, but in some systems global minima may be missed, e.g., in systems with multiple scales.

9.1.55. ASA FUZZY PRINT=FALSE

When ASA_FUZZY is set to TRUE, if ASA_FUZZY_PRINT is TRUE, then notifications will be printed out from FuzzyControl () and/or AlterQuench () in the user module.

9.1.56. FITLOC=FALSE

When FITLOC is set to TRUE, three subroutines become active to perform a local fit after leaving asa (). This can be useful to shunt asa () to a local code after the region of the global fit is known with some confidence, which many times is an efficient procedure.

Any robust quasi-linear optimization code may work well for this purpose. To illustrate this procedure, the user module contains fitloc () which sets up the calls to simplex (). simplex () calls calcf () which calls cost_function (), and adds USER_OPTIONS->Penalty whenever simplex () asks for parameters out of ranges of the parameters or whenever a constraint in cost_function () is violated.

ASA parameters, the OPTIONS, are raised to a high level of view for direct control by the user. However, most optimization codes have their own parameters that may not be apparent to the user. For example, fitloc () calls simplex () which contains parameters such as {tol1, tol2, no_progress, alpha, beta1, beta2, gamma, delta, iters}. Many problems will require tuning of these parameters to achieve good results just from this simplex () algorithm.

USER_OPTIONS->Fit_Local is passed to cost_function (). This provides additional flexibility in deciding when to shunt asa () over to fitloc (), e.g., during multiple or recursive optimizations. USER_OPTIONS->Iter_Max determines the maximum iterations of the cost_function () by simplex (). USER_OPTIONS->Penalty determines how to weight violation of constraints, exceeding boundaries, etc.

9.1.57. FITLOC ROUND=TRUE

If FITLOC is set to TRUE and FITLOC_ROUND is TRUE, then each time parameters are passed to or between the local routines, simplex (), calcf (), and fitloc (), they are first processed by rounding integers or respecting rounding according to ASA_RESOLUTION constraints prior to any further

calculations. I.e., all values of a parameter within a given resolution are considered to be equivalent for calculating the cost function.

9.1.58. FITLOC_PRINT=TRUE

When FITLOC is set to TRUE, if FITLOC_PRINT is TRUE, then intermediate calculations will be printed out from fitloc () and simplex () in the user module.

9.1.59. MULTI MIN=FALSE

When MULTI MIN is set to TRUE, the lowest USER OPTIONS->Multi Number values of the cost function, determined to be the best-generated during the sampling process, of the cost function and their parameters are saved. These can be read out just after asa () returns after its fit. The pre-compile number USER_OPTIONS->Multi_Number and OPTIONS *Multi_Cost, **Multi_Params, *Multi_Grid, Multi_Specify become available. asa_usr.c, In memory for USER_OPTIONS->Multi_Cost [USER_OPTIONS->Multi_Number][*parameter_dimension], USER OPTIONS->Multi Params [USER OPTIONS->Multi Number][*parameter dimension], and USER_OPTIONS->Multi_Grid [*parameter_dimension] are set. Multi_Grid values must be set by the user, but may be overridden as explained below under USER_OPTIONS->Multi Grid.

If OPTIONS->Curvature_0 is FALSE, all USER_OPTIONS->Multi_Number tangents and curvatures are calculated. This can be useful for some calculations requiring the shapes of the local minima.

This procedure selects local minima that statistically have maintained some quasi-stability during sampling. Note that this procedure does not guarantee that the USER_OPTIONS->Multi_Number lowest sampled values of the cost function will be saved, only those that were selected to be the best-generated during the sampling process.

If OPTIONS->Multi_Specify is set to 0, the selection of best-generated states includes all sampled instances of the cost functions. If OPTIONS->Multi_Specify is set to 1, the selection of best-generated states is constrained to include only those with different values of the cost function.

The https://www.ingber.com/asa_examples.txt file contains some guidance of the use of MULTI_MIN (and Multi_[] OPTIONS).

9.1.60. ASA PARALLEL=FALSE

The parallelization procedure employed here does *not* destroy the sampling properties of ASA. When ASA_PARALLEL is set to TRUE, parallel blocks of generated states are calculated of number equal to the minimum of USER_OPTIONS->Gener_Block and USER_OPTIONS->Gener_Block_Max. For most systems with complex nonlinear cost functions that require the fat tail of the ASA distribution, leading to high generated to acceptance ratios, this is the most CPU intensive part of ASA that can benefit from parallelization.

USER_OPTIONS->Gener_Block is calculated as a moving average over a sample of USER_OPTIONS->Gener_Mov_Avr, of the previous numbers of generated states required to find a new best accepted state. If and when USER_OPTIONS->Gener_Mov_Avr is set to 0, then USER_OPTIONS->Gener_Block is not changed thereafter.

Each block of generated states is sorted to permit the lowest cost functions to pass first through the acceptance test.

There are hooks in asa.c to spawn off multiple processors. Parallel code should be inserted in asa.c between the lines:

```
/* *** ENTER CODE TO SPAWN OFF PARALLEL GENERATED STATES *** */
...
/* *** EXIT CODE SPAWNING OFF PARALLEL GENERATED STATES *** */
```

The ASA_TEMPLATE_PARALLEL example given in asa_usr.c illustrates how the the USER_OPTIONS are set. Note that since the random number generator is called differently, generating some extra states as described above, the results are not identical to the serial ASA_TEST calculation.

When ASA_PARALLEL is set to TRUE, the do loop around generated states that tests for invalid states is taken over, not within, blocks of parallel calculated cost functions for these generated states.

When ASA_PARALLEL is set to TRUE, code in ASA-MAKEFILE, asa.c, asa_usr.c, and asa_usr_asa.h is set to run under OpenMP using the gcc compiler; search for instances of _OPENMP. Change/add lines of code to correspond to your choice of parallel algorithm and compiler. The entire ASA_PARALLEL section makes assignments to indexed variables to afford flexibility for other such algorithms.

When ASA_PARALLEL is TRUE, OPTIONS—>parallel_id is available just before the call to each user_cost_function(), e.g., in case it is needed by the cost function or its own calls to other user functions. The default before any calls within the SPAWN block in asa.c is parallel_id = -1.

9.1.61. FDLIBM_POW=FALSE

When FDLIBM_POW is set to TRUE, a user-defined function s_pow () is used instead of pow (). This may be desirable on some machines when a speed-up can be realized. Some code in https://www.ingber.com/asa_contrib.txt should first be tested with the standard ASA_TEST OPTIONS to see if the resulting asa_out file agrees with the asa_test_asa file.

9.1.62. FDLIBM_LOG=FALSE

When FDLIBM_LOG is set to TRUE, a user-defined function s_log () is used instead of log (). This may be desirable on some machines when a speed-up can be realized. Some code in https://www.ingber.com/asa_contrib.txt should first be tested with the standard ASA_TEST OPTIONS to see if the resulting asa_out file agrees with the asa_test_asa file.

9.1.63. FDLIBM EXP=FALSE

When FDLIBM_EXP is set to TRUE, a user-defined function s_exp () is used instead of exp (). This may be desirable on some machines when a speed-up can be realized. Some code in https://www.ingber.com/asa_contrib.txt should first be tested with the standard ASA_TEST OPTIONS to see if the resulting asa_out file agrees with the asa_test_asa file.

9.2. Printing DEFINE_OPTIONS

9.2.1. USER OUT=\"asa usr out\"

The name of the output file containing all printing from asa_usr.c. If you wish to attach a process number use USER_OUT=\"asa_usr_out_\". (Use USER_OUT=\"asa_usr_out_\" if this is set in the ASA-Makefile.) If USER_OUT=\"STDOUT\" then asa_usr.c will print to stdout.

9.2.2. USER_ASA_USR_OUT=FALSE

When USER_ASA_USR_OUT is set to TRUE, an additional Program Option pointer, *Asa_Usr_Out_File, must be used to dynamically set the name(s) of the file(s) printed out by the user module. (This overrides any USER_OUT settings.)

9.2.3. INCL STDOUT=TRUE

Some compilers on some systems under some levels of optimization will not compile if "stdout" is present in the code. All instances of "stdout" and "printf" in the user and the asa modules can be commented out by setting INCL_STDOUT to FALSE. Note that this also will suppress some output from such OPTIONS as ASA PIPE, TIME CALC, etc.

The use of INCL_STDOUT set to FALSE is recommended for creating a DLL as described in the DLL ASA–Makefile sub-Section of Section 7.

9.2.4. ASA PRINT=TRUE

Setting this to FALSE will suppress all printing within asa.

9.2.5. ASA_OUT=\"asa_out\"

The name of the output file containing all printing from asa. If you wish to attach a process number use ASA_OUT=\"asa_out_\$\$\". (Use ASA_OUT=\"asa_out_\$\$\$\" if this is set in the ASA-Makefile.) If ASA_OUT=\"STDOUT\" then ASA will print to stdout. See the discussion of the use of ASA_TEMPLATE_ASA_OUT_PID in the section USER_ASA_OUT below to obtain multiple output files numbered according to the system pid.

9.2.6. USER_ASA_OUT=FALSE

When USER_ASA_OUT is set to TRUE, an additional Program Option pointer, *Asa_Out_File, is used to dynamically set the name(s) of the file(s) printed out by the asa module. (This overrides any ASA_OUT settings.) In asa_usr.c, if USER_OPTIONS->Asa_Out_File = "STDOUT";, then ASA will print to stdout.

In the ASA_TEMPLATE_MULTIPLE example provided (see the set of DEFINE_OPTIONS used in asa_usr_asa.h), USER_ASA_OUT is used to generate multiple files of separate ASA runs. (If QUENCH_PARAMETERS and/or QUENCH_COST is set to TRUE, then this example will separate runs with different quenching values.)

In the ASA_TEMPLATE_ASA_OUT_PID example provided (see the set of DEFINE_OPTIONS used in asa_usr_asa.h), USER_ASA_OUT is used to generate ASA_OUT files of the form asa_out_x and asa_usr_out_x, where x is the system pid. This can be useful for a series of runs just changing parameters in asa_opt, getting different output files without recompiling. Depending on your system, you may have to change the include file and the prototype of getpid () in asa_usr.h under ASA_TEMPLATE_ASA_OUT_PID, and possibly the int declaration of pid_int in asa_usr.c.

9.2.7. ASA_PRINT_INTERMED=TRUE

This option is only effective if ASA_PRINT is TRUE. Setting ASA_PRINT_INTERMED to FALSE will suppress much intermediate printing within asa, especially arrays which can be large when the number of parameters is large. Printing at intermediate stages of testing/reannealing has been turned off when SELF_OPTIMIZE is set to TRUE, since there likely can be quite a bit of data generated; this can be changed by explicitly setting ASA_PRINT_INTERMED to TRUE in the ASA-Makefile or on your compilation command lines.

9.2.8. ASA_PRINT_MORE=FALSE

Setting ASA_PRINT_MORE to TRUE will print out more intermediate information, e.g., new parameters whenever a new minimum is reported. As is the case whenever tangents are not calculated by choosing some ASA options, normally the intermediate values of tangents will not be up to date.

The section above, Use of Documentation for Tuning, emphasizes the importance of using ASA_PRINT_MORE set to TRUE to help determine optimal tuning of ASA on specific problems.

9.2.9. **G_FIELD=12 & G_PRECISION=7**

The field width and precision of doubles is specified in asa.c as G_FIELD.G_PRECISION, e.g., as %gG_FIELD.G_PRECISION or %g-G_FIELD.G_PRECISION. These two Printing DEFINE OPTIONS are available to change the default of 12.7.

9.2.10. ASA_SAVE=FALSE

When ASA_SAVE is set to TRUE, as a saves enough information in file asa_save after each newly best accepted state, to restart from the point entering the main annealing loop, continue thereafter from the best accepted state in asa_save. Of course, this use of I/O takes CPU resources, and can appreciably slow down your runs. When SYSTEM_CALL is set to TRUE, for extra protection, e.g., in case the run

aborts during a write of asa_save, each time a file asa_save is written, it also is copied to a new file asa save.old.

In order to store the whole block of random numbers used at any time, the number USER USER_OPTIONS->Random_Array_Dim and array USER_OPTIONS->Random_Array are required. These may be changed by the user in asa_usr.c for different random number generators and shuffling algorithms. The default is to use SHUFFLE defined in asa_usr.h for Random_Array_Dim in the default random number generator in asa_usr.c, and the pointer Random_Array is set to the pointer of the static array random_array at the top of asa_usr.c.

Just restart the run by executing asa_run. When ASA_SAVE is set to TRUE, the existence of file asa_save is used to determine whether a new run or a rerun is to proceed. Therefore, be sure your ASA directory does not have any old asa_save file present if a new run is to start.

The asa_opt file is included just after asa_save files are read into the code. Therefore, any new C code you wish to have override information read in from asa_save can be simply added to the bottom of asa_opt. Be sure you write the names of these variables as they are used in the asa.c file, which can differ from their counterparts in asa_usr.c file. Some example are given at the end of asa_opt before the #endif statement. Each time you add new information to be compiled, be sure to enforce a new recompile of asa.c and asa_run. In most cases this can be done simply by removing asa.o before using a make or recompiling the executable. However, see ASA_SAVE_OPT for changes that may be made without any recompilation.

When ASA is run at several levels of recursion, if USER_OPTIONS->Asa_Recursive_Level is properly incremented from 0 at the innermost shell, the outermost shell at level n will create files asa_save_{n}.

9.2.11. ASA SAVE OPT=FALSE

When ASA_SAVE_OPT is set to TRUE, when asa is restarted, if the file asa_opt_save is present in the same directory as asa_opt, then new values of ASA parameters and OPTIONS are read in after initializing to the point of the last writing of asa_save.

No recompilation of the code is necessary, and only warnings are issued if asa_save_opt is not present. The file asa_save_opt should be created as an exact copy of asa_opt before changes in values of parameters and OPTIONS are made. When ASA_SAVE_OPT is TRUE, ASA_SAVE is automatically set to TRUE in asa_usr_asa.h.

9.2.12. ASA_SAVE_BACKUP=FALSE

When ASA_SAVE_BACKUP is set to TRUE, as a saves enough information after each newly best accepted state, creating a file asa_save.{N_Accepted}, to enable the user to restart from any previous best accepted state when that asa_save.{best_state} is copied to asa_save.

When used with ASA_PIPE and/or ASA_PIPE_FILE, ASA_SAVE_BACKUP permits the user to interactively tune the optimization process without having to start new runs. Read the above ASA_SAVE section on the use of the asa_opt file to modify code before reading in the asa_save file.

When ASA_SAVE_BACKUP is TRUE, ASA_SAVE is automatically set to TRUE in asa_usr_asa.h.

When ASA is run at several levels of recursion, if USER_OPTIONS->Asa_Recursive_Level is properly incremented from 0 at the innermost shell, the outermost shell at level n will create files asa_save_{n}.{N_Accepted}.

9.2.13. ASA_PIPE=FALSE

When ASA_PIPE is set to TRUE, as prints to STDOUT lines of data after calls to the cost function, which can be used to update databases or graphs in real time. This information is {number of valid generated states, number of accepted states, best cost function, best parameter values, current cost temperature, current parameter temperatures, last cost function}.

9.2.14. ASA PIPE FILE=FALSE

When ASA_PIPE_FILE is set to TRUE, as a prints to asa_pipe lines of data that can be used to examine run data. This can be used complementary to ASA_PIPE.

9.2.15. ASA_EXIT_ANYTIME=FALSE

When ASA_EXIT_ANYTIME is set to TRUE, the first call to asa() creates the file asa_exit_anytime. To permit IMMEDIATE_EXIT before calculating the next generated state, simply delete the file asa_exit_anytime, and asa() will exit with code IMMEDIATE_EXIT. This can be useful on long runs.

9.2.16. SYSTEM_CALL=TRUE

When SYSTEM_CALL is set to FALSE, as a avoids popen () commands. This is useful on machines that do not permit these commands. For example, when ASA_SAVE is set to TRUE, as a uses a popen call in asa.c, to copy asa_save to asa_save.old. This also is required to use ASA_SAVE_BACKUP set to TRUE.

9.2.17. noEXIT_INVALID_COST_FUNCTION=TRUE

The default to exit ASA when an invalid cost function was calculated, giving an exit code of 8 to report an INVALID_COST_FUNCTION, has been replaced by an OPTIONS to reject the cost function as invalid, adding to the count of *number_invalid in the asa_out file, which can exit subject to the OPTIONS Limit_Invalid_Generated_States. Sometimes only a small percentage of generated states has parameters that cause this condition, and this is a reasonable way of continuing the sampling. Note that when Limit Invalid Generated States is set to -1, this test is bypassed.

When noEXIT_INVALID_COST_FUNCTION is set to FALSE, then the previous conditions are enforced, and ASA exits whenever an improper cost function is calculated.

```
9.3. Program OPTIONS
typedef struct
          LONG INT Limit_Acceptances;
          LONG INT Limit Generated;
          int Limit Invalid Generated States;
          double Accepted To Generated Ratio;
          double Cost Precision;
          int Maximum Cost Repeat;
          int Number Cost Samples:
          double Temperature Ratio Scale;
          double Cost Parameter Scale Ratio;
          double Temperature_Anneal_Scale;
#if USER INITIAL COST TEMP
          double *User Cost Temperature;
#endif
          int Include_Integer_Parameters;
          int User Initial Parameters;
          ALLOC INT Sequential Parameters;
          double Initial Parameter Temperature:
#if RATIO TEMPERATURE SCALES
          double *User_Temperature_Ratio;
#endif
#if USER INITIAL PARAMETERS TEMPS
          double *User Parameter Temperature;
```

#endif

int Acceptance_Frequency_Modulus; int Generated_Frequency_Modulus; int Reanneal_Cost; int Reanneal_Parameters;

double Delta_X;

#if DELTA PARAMETERS

double *User_Delta_Parameter;

#endif

int User_Tangents;
int Curvature_0;

#if QUENCH PARAMETERS

double *User Quench Param Scale;

#endif

#if QUENCH_COST

double *User_Quench_Cost_Scale;

#endif

LONG_INT N_Accepted; LONG_INT N_Generated; int Locate_Cost; int Immediate_Exit;

double *Best_Cost;
double *Best_Parameters;
double *Last Cost;

double Last_Cost;

double *Last_Parameters;

#if OPTIONAL DATA DBL

ALLOC_INT Asa_Data_Dim_Dbl; double *Asa_Data_Dbl;

#endif

#if OPTIONAL_DATA_INT

ALLOC_INT Asa_Data_Dim_Int; double *Asa_Data_Int;

#endif

#if OPTIONAL DATA PTR

ALLOC_INT Asa_Data_Dim_Ptr; OPTIONAL_PTR_TYPE *Asa_Data_Ptr;

#endif

#if USER_ASA_OUT

char *Asa_Out_File;

#endif

#if USER_ASA_USR_OUT

char *Asa_Out_Usr_File;

#endif

#if USER COST SCHEDULE

double (*Cost_Schedule) ();

#endif

#if USER_ACCEPT_ASYMP_EXP

double Asymp_Exp_Param;

```
#endif
#if USER_ACCEPTANCE_TEST
          void ( *Acceptance_Test ) ();
          int User Acceptance Flag;
          int Cost Acceptance Flag;
          double Last Cost;
          double Cost_Temp_Curr;
          double Cost_Temp_Init;
          double Cost_Temp_Scale;
          double Prob Bias;
          LONG_INT *Random_Seed;
#endif
#if USER GENERATING FUNCTION
          double ( *Generating_Distrib ) ();
#endif
#if USER REANNEAL COST
          int (*Reanneal_Cost_Function)();
#endif
#if USER_REANNEAL_PARAMETERS
          double ( *Reanneal_Params_Function ) ();
#endif
#if ASA SAMPLE
          double Bias_Acceptance;
          double *Bias_Generated;
          double Average_Weights;
          double Limit Weights;
#endif
#if ASA_QUEUE
          ALLOC INT Queue Size;
          double *Queue_Resolution;
#endif
#if ASA RESOLUTION
          double *Coarse_Resolution;
#endif
#if FITLOC
          int Fit_Local;
          int Iter Max;
          double Penalty;
#endif
#if MULTI MIN
          double *Multi_Cost;
          double **Multi_Params;
          double *Multi Grid;
          int Multi_Specify;
#endif
#if ASA_PARALLEL
          int parallel_id;
          int Gener_Mov_Avr;
          LONG INT Gener Block;
          LONG_INT Gener_Block_Max;
#endif
#if ASA_SAVE
          ALLOC_INT Random_Array_Dim;
          LONG_INT *Random_Array;
```

#endif

int Asa_Recursive_Level;

USER DEFINES;

Note that two ways are maintained for passing the Program Options. Check the comments in the ASA-NOTES file. It may be necessary to change some of the options for some systems. Read the https://www.ingber.com/asa_examples.txt file for some ongoing discussions and suggestions on how to try to optimally set these options. Note the distinction between trying to speed up annealing/quenching versus trying to slow down annealing (which sometimes can speed up the search by avoiding spending too much time in some local optimal regions). Templates are set up in ASA to accommodate all alternatives. Below, the defaults are given in square brackets [].

(A) asa_usr.c file

When using ASA as part of a large library, it likely is easiest to make these changes within the user module, e.g., using the template placed in asa_usr.c. In the user module, the Program Options are stored in the structure USER_DEFINES *USER_OPTIONS (and in USER_DEFINES *RECUR_USER_OPTIONS if SELF_OPTIMIZE is TRUE).

(B) asa_opt file

It likely is most efficient to use a separate data file avoiding repeated compilations of the code, to test various combinations of Program Options, e.g., using the file asa_opt when OPTIONS_FILE and OPTIONS_FILE_DATA are set to TRUE in the ASA-Makefile or on your compilation command lines.

In the asa module (which can be called recursively) the structure is called USER_DEFINES *OPTIONS. For the rest of this file, where no confusion can reasonably arise, the Program Options will be referred to as USER_DEFINES *OPTIONS.

9.3.1. OPTIONS->Limit_Acceptances[10000]

The maximum number of states accepted before quitting. All the templates in ASA have been set to use Limit_Acceptances=1000 to illustrate the way these options can be changed. If Limit_Acceptances is set to 0, then no limit is observed. This can be useful for some systems that cannot handle large integers.

9.3.2. OPTIONS->Limit Generated[99999]

The maximum number of states generated before quitting. If Limit_Generated is set to 0, then no limit is observed. This can be useful for some systems that cannot handle large integers.

9.3.3. OPTIONS->Limit_Invalid_Generated_States[1000]

This sets limits of repetitive invalid generated states, e.g., when using this method to include constraints. This also can be useful to quickly exit asa () if this is requested by your cost function: Setting the value of Limit_Invalid_Generated_States to 0 will exit at the next calculation of the cost function (possibly after a few more exiting calls to calculate tangents and curvatures). For example, to exit asa () at a specific number of generated points, set up a counter in your cost function, e.g., similar to the one in the test function in asa_usr.c. For all calls >= the limit of the number of calls to the cost function, terminate by setting OPTIONS->Limit_Invalid_Generated_States = 0 and setting *cost_flag = FALSE. Note that a quick exit also can be achieved using OPTIONS->Immediate_Exit. Also note that when Limit_Invalid_Generated_States is set to -1, this test is bypassed.

9.3.4. OPTIONS->Accepted_To_Generated_Ratio[1.0E-6]

The least ratio of accepted to generated states. If this value is encountered, then the usual tests, including possible reannealing, are initiated even if the timing does not coincide with Acceptance_Frequency_Modulus or Generated_Frequency_Modulus (defined below). All the templates in ASA have been set to use Accepted_To_Generated_Ratio=1.0E-4 to illustrate the way these options can be changed.

9.3.5. OPTIONS->Cost_Precision[1.0E-18]

This sets the precision required of the cost function if exiting because of reaching Maximum_Cost_Repeat, which is effective as long as Maximum_Cost_Repeat > 0.

9.3.6. OPTIONS->Maximum_Cost_Repeat[5]

The maximum number of times that the cost function repeats itself, within limits set by Cost_Precision, before quitting. This test is performed only when Acceptance_Frequency_Modulus or Generated_Frequency_Modulus is invoked, or when the ratio of accepted to generated points is less than Accepted_To_Generated_Ratio, in order to help prevent exiting prematurely in a local minimum. If Maximum_Cost_Repeat is 0, this test is bypassed.

9.3.7. OPTIONS->Number_Cost_Samples[5]

When Number_Cost_Samples > 0, the initial cost temperature is calculated as the average of the absolute values of Number_Cost_Samples sampled cost functions.

When Number_Cost_Samples < -1, the initial cost temperature is calculated as the deviation over a sample of -Number_Cost_Samples number of cost functions, i.e., the square-root of the difference of the second moment and the square of the first moment, normalized by the ratio of -Number_Cost_Samples to -Number_Cost_Samples - 1.

When ASA_SAVE is set to TRUE, Number_Cost_Samples is set to 1 after the initial run since all the required information for subsequent runs already has been collected.

If the calculated initial cost temperature is not set using USER_INITIAL_COST_TEMP, and if it is essentially zero (using a test with SMALL_FLOAT), then it is reset to e = 2.718. If ASA_PRINT is TRUE a warning is printed.

See Reanneal_Cost for similar treatment of the reannealed cost temperature.

9.3.8. OPTIONS->Temperature_Ratio_Scale[1.0E-5]

This scale is a guide to the expected cost temperature of convergence within a small range of the global minimum. As explained in the ASA papers, and as outlined in the ASA–NOTES, this is used to set the rates of annealing. Here is a brief description in terms of the temperature schedule outlined above.

As a useful physical guide, the temperature is further parameterized in terms of quantities m_i and n_i , derived from an "expected" final temperature (which is not enforced in ASA), T_{fi} ,

$$T_{fi} = T_{0i} \exp(-m_i)$$
 when $k_{fi} = \exp n_i$,

$$c_i = m_i \exp(-n_i/D) .$$

However, note that since the initial temperatures and generating indices, T_{0i} and k_i , are independently scaled for each parameter, it usually is reasonable to simply take $\{c_i, m_i, n_i\}$ to be independent of the index i, i.e., to be $\{c, m, n\}$ for all i.

In asa.c,

```
m = -\log(\text{Temperature\_Ratio\_Scale}).
```

This can be overridden if RATIO_TEMPERATURE_SCALES (further discussed below) is set to TRUE, and then values of multipliers of <code>-log(Temperature_Ratio_Scale)</code> are used in asa.c. These multipliers are calculated in the user module as <code>OPTIONS->User_Temperature_Ratio[]</code>. Then,

```
m_i = m \text{ OPTIONS} - \text{> User Temperature Ratio[i]}.
```

For large numbers of parameters, Temperature_Ratio_Scale is a very influential Program Option in determining the scale of parameter annealing. It likely would be best to start with a larger value than the default, to slow down the annealing.

The ASA-NOTES contain a section giving a little more explanation on the use of Temperature_Ratio_Scale.

9.3.9. OPTIONS->Cost Parameter Scale Ratio[1.0]

This is the ratio of cost:parameter temperature annealing scales. As explained in the ASA papers, and as outlined in the ASA–NOTES, this is used to set the rates of annealing.

In terms of the algebraic development given above for the Temperature_Ratio_Scale, in asa.c,

 $c_{\text{cost}} = c \text{ Cost_Parameter_Scale_Ratio}$.

Cost_Parameter_Scale_Ratio is a very influential Program Option in determining the scale of annealing of the cost function.

9.3.10. OPTIONS->Temperature Anneal Scale[100.0]

This scale is a guide to achieve the expected cost temperature sought by Temperature_Ratio_Scale within the limits expected by Limit_Acceptances. As explained in the ASA papers, and as outlined in the ASA-NOTES, this is used to set the rates of annealing.

In terms of the algebraic development given above for the Temperature Ratio Scale, in asa.c,

 $n = \log(\text{Temperature_Anneal_Scale})$.

For large numbers of parameters, Temperature_Anneal_Scale probably should at least initially be set to values greater than *number_parameters, although it will not be as influential as Temperature_Ratio_Scale.

9.3.11. OPTIONS->User_Cost_Temperature

If USER_INITIAL_COST_TEMP is TRUE, a pointer, OPTIONS->User_Cost_Temperature, is used to adaptively initialize the cost temperature. If this choice is elected, then User_Cost_Temperature [] must be initialized.

9.3.12. OPTIONS->Include_Integer_Parameters[FALSE]

If Include_Integer_Parameters is TRUE, include integer parameters in derivative and reannealing calculations, except those with INTEGER_TYPE (2). This is useful when the parameters can be analytically continued between their integer values, or if you set the parameter increments to integral values by setting ASA_RESOLUTION to TRUE, as discussed further below.

9.3.13. OPTIONS->User_Initial_Parameters[FALSE]

ASA always requests that the user guess initial values of starting parameters, since that guess is as good as any random guess the code might make. The default is to use the ASA distribution about this point to generate an initial state of parameters and value of the cost function that satisfy the user's constraints. If User_Initial_Parameters is set to TRUE, then the first user's guess is used to calculate this first state.

9.3.14. OPTIONS->Sequential_Parameters[-1]

The ASA default for generating new points in parameter space is to find a new point in the full space, rather than to sample the space one parameter at a time as do most other algorithms. This is in accord with the general philosophy of sampling the space without any prior knowledge of ordering of the parameters. However, if you have reason to believe that at some stage(s) of search there might be some benefit to sampling the parameters sequentially, then set Sequential_Parameters to the parameter number you wish to start your annealing cycle, i.e., ranging from 0 to (*parameter dimension - 1). Then, ASA will cycle through your parameters in the order you have placed them in all arrays defining their track of which parameter is actively being OPTIONS->Sequential_Parameters, thereby permitting adaptive changes. Any negative value for Sequential_Parameters will use the default ASA algorithm. Upon exiting asa (), Sequential_Parameters is reset back to its initial value.

9.3.15. OPTIONS->Initial_Parameter_Temperature[1.0]

The initial temperature for all parameters. This is overridden by use of the USER_INITIAL_PARAMETERS_TEMPS option.

9.3.16. OPTIONS->User_Temperature_Ratio

If RATIO_TEMPERATURE_SCALES is TRUE, a pointer, OPTIONS—>User_Temperature_Ratio, is used to adaptively set ratios of scales used to anneal the parameters in the cost function. This can be useful when some parameters are not being reannealed, or when setting the initial temperatures (using USER_INITIAL_PARAMETERS_TEMPS set to TRUE) is not sufficient to handle all your parameters properly. This typically is not encountered, so it is advised to look elsewhere at first to improve your search. If this choice is elected, then User_Temperature_Ratio[] must be initialized.

9.3.17. OPTIONS->User_Parameter_Temperature

If USER_INITIAL_PARAMETERS_TEMPS is TRUE, a pointer, OPTIONS->User_Parameter_Temperature, is used to adaptively initialize parameters temperatures. If this choice is elected, then User_Parameter_Temperature [] must be initialized.

9.3.18. OPTIONS->Acceptance_Frequency_Modulus[100]

The frequency of testing for periodic testing and reannealing, dependent on the number of accepted states. If Acceptance_Frequency_Modulus is set to 0, then this test is not performed.

9.3.19. OPTIONS->Generated_Frequency_Modulus[10000]

The frequency of testing for periodic testing and reannealing, dependent on the number of generated states. If Generated_Frequency_Modulus is set to 0, then this test is not performed.

9.3.20. OPTIONS->Reanneal_Cost[1]

A value of Reanneal_Cost set to FALSE=0 bypasses reannealing of the cost temperature. This might be done for systems where such reannealing is not useful. Note that the use of USER_REANNEAL_COST permits users to define their own cost temperature reannealing algorithm when Reanneal_Cost is not 0 or -1.

A value of Reanneal_Cost = 1 permits the default reannealing of the cost temperature to be part of the fitting process, correlating the cost temperature with the current last and best values of the cost function as described above.

If Reanneal_Cost > 1, then the reannealed initial cost temperature is calculated as the deviation over a sample of -Reanneal_Cost number of cost functions, i.e., the square—root of the difference of the second moment and the square of the first moment, normalized by the ratio of Reanneal_Cost to Reanneal_Cost - 1. For example, if the initial cost temperature is reannealed to a larger value, this increases the effective index of the current cost temperature, effectively slowing down the rate of decrease of future current cost temperatures as this index is increased for each acceptance test.

If Reanneal_Cost < -1, then the cost index is reset to 1, and the initial and current cost temperatures are calculated as the deviation over a sample of -Reanneal_Cost number of cost functions, i.e., the square—root of the difference of the second moment and the square of the first moment, normalized by the ratio of -Reanneal_Cost to -Reanneal_Cost - 1. This often gives rise to fluctuating current cost temperatures, sometimes diminishing the value of the acceptance test. However, for some systems that have different behavior at different scales, this can be a very useful OPTIONS.

The algorithms with Reanneal_Cost > 1 or < -1 typically require more calls to the cost function than the default of Reanneal_Cost = 1. This typically is even more so when Reanneal_Cost < -1 than when Reanneal_Cost > 1 due to the resetting of the current cost temperature as well as the initial cost temperature. Because of the fat tail of the parameter distributions, quite often relatively large values of the cost function will be included in the periodic sampling. However, of course the parameter temperatures continue to diminish, focusing the fit towards the global optimal value.

Note that Number_Cost_Samples can be used similarly for calculating the initial cost temperature.

9.3.21. OPTIONS->Reanneal_Parameters[TRUE]

This permits reannealing of the parameter temperatures to be part of the fitting process. This might have to be set to FALSE for systems with very large numbers of parameters just to decrease the number of function calls.

9.3.22. OPTIONS->Delta X[0.001]

The fractional increment of parameters used to take numerical derivatives when calculating tangents for reannealing, for each parameter chosen to be reannealed. This is overridden when DELTA_PARAMETERS is set to TRUE. If Delta_X is set to 0, then no tangents are calculated.

Note, that for second-derivative off-diagonal curvature calculations, the algorithm used may cause evaluations of your cost function outside a range when a parameter being sampled is at the boundary. However, only values of parameters within the ranges set by the user are actually used for acceptance tests. Note that the user may set User_Tangents to TRUE, as discussed below, to choose any other algorithm to calculate derivatives or other indicators to be used for reannealing.

When calculating derivatives, an invalid cost function, i.e., returning *cost_flag = FALSE from the cost function, will exit asa() with *exit_code INVALID_COST_FUNCTION_DERIV. Also, when calculating derivatives, no extra test is performed to check that parameters are within their lower and upper bounds (since meshes for derivatives may exceed these bounds). If this is not desired, then within the cost function, within a test for USER_OPTIONS->Locate_Cost == 3, a decision may be made whether to return this *exit_code.

9.3.23. OPTIONS->User Delta Parameter

If DELTA_PARAMETERS is TRUE, a pointer, OPTIONS->User_Delta_Parameter, is used to adaptively set increments of parameters used to take pseudo-derivatives (numerical derivatives), for each parameter chosen to be reannealed. For example, this can be useful to reanneal integer parameters when a choice is made to permit their derivatives to be taken. If this choice is elected, then OPTIONS->User_Delta_Parameter[] must be initialized. No tangents are calculated for any parameter if User_Delta_Parameter[] is 0.

9.3.24. OPTIONS->User Tangents[FALSE]

By default, asa () calculates numerical tangents (first derivatives) of the cost function for use in reannealing and to provide this information to the user. However, if User_Tangents is set to TRUE, then when asa () requires tangents to be calculated, a value of *valid_state_generated_flag (called *cost_flag in ASA_TEST in asa_usr.c and asa_usr_cst.c) of -1 is set and the cost function is called. The user is expected to set up a test in the beginning of the cost function to sense this value, and then calculate the tangents [] array (containing the derivatives of the cost function, or whatever sensitivity measure is desired to be used for reannealing) to be returned to asa (). An example is provided with a ASA_TEMPLATE_SAMPLE.

9.3.25. OPTIONS->Curvature 0[FALSE]

If the curvature array is quite large for your system, and you really do not use this information, you can set Curvature_0 to TRUE which just requires a one-dimensional curvature [0] to be defined to pass to the asa module (to avoid problems with some systems). This is most useful, and typically is necessary, when minimizing systems with large numbers of parameters since the curvature array is of size number of parameters squared.

If you wish to calculate the curvature array periodically, every reannealing cycle determined by Acceptance_Frequency_Modulus, Generated_Frequency_Modulus, or Accepted_To_Generated_Ratio, then set OPTIONS->Curvature_0 to -1.

9.3.26. OPTIONS->User Quench Param Scale

If QUENCH_PARAMETERS is TRUE, a pointer, OPTIONS->User_Quench_Param_Scale, is used to adaptively set the scale of the temperature schedule. If this choice is elected, then OPTIONS->User_Quench_Param_Scale [] must be initialized, and values defined for each dimension. The default in the asa module is to assign the annealing value of 1 to all elements that might be defined otherwise. If values are selected greater than 1 using this Program Option, then quenching is enforced.

Note that you can use this control quite differently, to slow down the annealing process by setting OPTIONS->User_Quench_Param_Scale [] to values less than 1. This can be useful in problems where the global optimal point is at a quite different scale from other local optima, masking its presence.

If OPTIONS_FILE_DATA, QUENCH_COST, and QUENCH_PARAMETERS are TRUE, then *User_Quench_Cost_Scale and User_Quench_Param_Scale [] all are read in from asa_opt. If RECUR_OPTIONS_FILE_DATA, QUENCH_COST, and QUENCH_PARAMETERS are TRUE, then *User_Quench_Cost_Scale and User_Quench_Param_Scale [] all are read in from asa_opt_recur.

9.3.27. OPTIONS->User_Quench_Cost_Scale

If QUENCH_COST is TRUE, a pointer, OPTIONS->User_Quench_Cost_Scale, is used to adaptively set the scale of the temperature schedule. If this choice is elected, then OPTIONS->User_Quench_Cost_Scale [0] must be initialized. The default in the asa module is to assign the annealing value of 1 to this element that might be defined otherwise.

OPTIONS->User_Quench_Cost_Scale may be changed adaptively without affecting the ergodicity of the algorithm, within reason of course. This might be useful for some systems that require different approaches to the cost function in different ranges of its parameters. Note that increasing this parameter beyond its default of 1.0 can result in rapidly locking in the search to a small region of the cost function, severely handicapping the algorithm. On the contrary, you may find that slowing the cost temperature schedule, by setting this parameter to a value less than 1.0, may work better for your system.

If OPTIONS_FILE_DATA and QUENCH_COST are TRUE, then *User_Quench_Cost_Scale is read in from asa_opt. If RECUR_OPTIONS_FILE_DATA and QUENCH_COST are TRUE, then *User_Quench_Cost_Scale is read in from asa_opt_recur.

9.3.28. OPTIONS->N_Accepted

 N_A ccepted contains the current number of points saved by the acceptance criteria. This can be used to monitor the fit. On exiting from asa () N_A ccepted contains the value of *best_number_accepted_saved in asa.c. Note that the value of N_A ccepted typically will be less in the cost function than in ASA_OUT, as the value of the returned cost must be tested back in asa () to see if N_A ccepted should be incremented.

9.3.29. OPTIONS->N_Generated

N_Generated contains the current number of generated states. This can be used to monitor the fit. On exiting from asa () N_Generated contains the value of *best_number_generated_saved in asa.c. Note that the value of N_Generated typically will be less in the cost function than in ASA_OUT, as this value is only incremented upon returning to asa() if some tests are passed.

If ASA_PARALLEL is TRUE, then N_Generated and *best_number_generated_saved will only be accurate within the scale of OPTIONS->Gener_Block, as these are updated in the acceptance function.

9.3.30. OPTIONS->Locate Cost

Locate_Cost is a flag set in asa (), telling at what point the cost function is being called. This can be useful for determining when to perform tests while in your cost function. When ASA_PRINT is TRUE, the value is printed out upon exiting asa (). Note that there are several possible values that be reasonable, depending on from where the final exit was called.

Locate_Cost = 0. The cost function is being used for the initial cost temperature.

Locate_Cost = 1. The cost function is being used for the initial cost value.

Locate_Cost = 2. The cost function is being used for a new generated state.

Locate_Cost = 12. The cost function is being used for a new generated state just after a new best state was achieved.

Locate_Cost = 3. The cost function is being used for the cost derivatives to reanneal the parameters.

Locate Cost = 4. The cost function is being used for reannealing the cost temperature.

Locate_Cost = 5. The cost function is being used for the exiting of asa () to calculate final curvatures.

Locate_Cost = -1. Exited main loop of asa () because of user-defined constraints in OPTIONS, e.g., Acceptance_Frequency_Modulus, Generated_Frequency_Modulus, or Accepted_To_Generated_Ratio.

When calculating derivatives, an invalid cost function, i.e., returning *cost_flag = FALSE from the cost function, will exit asa() with *exit_code INVALID_COST_FUNCTION_DERIV. Also, when calculating derivatives, no extra test is performed to check that parameters are within their lower and upper bounds (since meshes for derivatives may exceed these bounds). If this is not desired, then within the cost function, within a test for USER_OPTIONS->Locate_Cost == 3, a decision may be made whether to return this *exit_code.

9.3.31. OPTIONS->Immediate_Exit[FALSE]

OPTIONS->Immediate_Exit is initialized to FALSE when entering asa (). At any time during the fit, except while the call to cost_function () from asa () is being used simply to calculate derivatives or the cost temperature, if the user sets Immediate_Exit to TRUE, then just after bookkeeping is performed by the acceptance test, asa () will exit with code IMMEDIATE_EXIT. All closing calculations of current cost_tangents [] and cost_curvature [] are bypassed.

9.3.32. OPTIONS->Best Cost

In asa_usr_asa.h, the OPTIONS *Best_Cost is a pointer to the value of the cost function of the saved best state calculated in asa (). E.g., together with *Best_Parameters and Locate_Cost, these OPTIONS can aid several adaptive features of ASA, e.g., automating the diminishing of ranges each time a new best state is achieved, as illustrated in the ASA_TEMPLATE just after the comment MY_TEMPLATE_diminishing_ranges in asa_usr.c. This OPTIONS is to be used read—only by the cost function, as calculated in asa (); do not change it in the user module unless you so wish to modify the sampling.

9.3.33. OPTIONS->Best Parameters

In asa_usr_asa.h, the OPTIONS *Best_Parameters is a pointer to the values of the parameters of the saved best state calculated in asa (). E.g., together with *Best_Cost and Locate_Cost, these OPTIONS can aid several adaptive features of ASA, e.g., automating the diminishing of ranges each time a new best state is achieved, as illustrated in the ASA_TEMPLATE just after the comment MY_TEMPLATE_diminishing_ranges in asa_usr.c. This OPTIONS is to be used read—only by the cost function, as calculated in asa (); do not change it in the user module unless you so wish to modify the sampling.

9.3.34. OPTIONS->Last_Cost

In asa_usr_asa.h, the OPTIONS *Last_Cost is a pointer to the value of the cost function of the last saved state calculated in asa (). This can be compared to *Best_Cost as a measure of fluctuations among local minima. An example of use in a user-defined acceptance test is in asa_usr.c when USER_ACCEPTANCE_TEST is TRUE. This OPTIONS is to be used read-only by the cost function, as calculated in asa (); do not change it in the user module unless you so wish to modify the sampling.

9.3.35. OPTIONS->Last Parameters

In asa_usr_asa.h, the OPTIONS *Last_Parameters is a pointer to the values of the parameters of the last saved state calculated in asa (). This can be compared to *Best_Parameters as a measure of fluctuations among local minima. This OPTIONS is to be used read—only by the cost function, as calculated in asa (); do not change it in the user module unless you so wish to modify the sampling.

9.3.36. OPTIONS->Asa_Data_Dim_Dbl

If the Pre-Compile Option OPTIONAL_DATA_DBL [FALSE] is set to TRUE, an additional Program Option, OPTIONS->Asa_Data_Dim_Dbl, becomes available to define the dimension of OPTIONS->Asa_Data_Dbl [].

9.3.37. OPTIONS->Asa_Data_Dbl

If the Pre-Compile Option OPTIONAL_DATA_DBL [FALSE] is set to TRUE, an additional Program Option pointer, OPTIONS->Asa_Data_Dbl, becomes available to return additional information to the user module from the asa module. This information communicates with the asa module, and memory must be allocated for it in the user module. An example is given in asa_usr.c when SELF_OPTIMIZE is TRUE.

9.3.38. OPTIONS->Asa_Data_Dim_Int

If the Pre-Compile Option OPTIONAL_DATA_INT [FALSE] is set to TRUE, an additional Program Option, OPTIONS->Asa_Data_Dim_Int, becomes available to define the dimension of OPTIONS->Asa_Data_Int [].

9.3.39. OPTIONS->Asa Data Int

If the Pre-Compile Option OPTIONAL_DATA_INT [FALSE] is set to TRUE, an additional Program Option pointer, OPTIONS->Asa_Data_Int, becomes available to return additional integer information to the user module from the asa module. This information communicates with the asa module, and memory must be allocated for it in the user module.

9.3.40. OPTIONS->Asa Data Dim Ptr

If the Pre-Compile Option OPTIONAL_DATA_PTR [FALSE] is set to TRUE, an additional Program Option, OPTIONS->Asa_Data_Dim_Ptr, becomes available to define the dimension of OPTIONS->Asa_Data_Ptr.

For example, a value of Asa_Data_Dim_Ptr = 2 might be used to set different entries in data arrays at two levels of recursion. See the discussion under OPTIONAL_DATA_PTR and Asa_Data_Ptr for use in multiple recursion.

9.3.41. OPTIONS->Asa_Data_Ptr

If the Pre-Compile Option OPTIONAL_DATA_PTR [FALSE] is set to TRUE, an additional Program Option pointer, OPTIONS->Asa_Data_Ptr, becomes available to define an array, of type OPTIONAL_PTR_TYPE defined by the user, which can be used to pass arbitrary arrays or structures to the user module from the asa module. This information communicates with the asa module, and memory must be allocated for it in the user module.

For example, struct DATA might contain an array data[10] to be used in the cost_function. Asa_Data_Dim_Ptr might have a value 2. Set #define OPTIONAL_PTR_TYPE DATA. Then, data[3] in struct Asa_Data_Ptr[1] could be set and accessed as USER_OPTIONS->Asa_Data_Ptr[1].data[3] in the cost function.

For example, your main program that calls asa_main() would have developed a struct SelectedType *SelectedPointer, and you can call asa_main(SelectedPointer, ...). In asa_usr_asa.h, you would have

#define OPTIONAL_PTR_TYPE SelectedType

In asa_usr.c (and asa_usr.h) you would develop int asa_main(OPTIONAL_PTR_TYPE *OptionalPointer,

...) and, close to the appropriate ASA_TEMPLATE, you would set

USER OPTIONS->Asa Data Ptr = OptionalPointer;

if Asa_Data_Dim_Ptr = 1 were being passed to ASA; otherwise, if Asa_Data_Dim_Ptr > 1, as illustrated in the ASA_TEMPLATE in asa_usr.c, memory must be allocated (and then freed) for Asa_Data_Ptr, and each dimension must be properly aligned with the structs being passed from the main program.

Note that the typedef of the SelectedType struct (whether or not you explicitly use the term typedef), developed in your main program, cannot be passed by its name to asa_main(), and so it must be declared as well in the asa module. In asa_usr_asa.h, the SelectedType struct can be included with the use of an #include SelectedStruct.h common to your main program and asa usr asa.h.

As noted in asa_usr.c, before exiting asa_main(), instead of freeing Asa_Data_Ptr, if memory has been allocated outside ASA, e.g., by the use of ASA_LIB, use

USER_OPTIONS->Asa_Data_Ptr = NULL;

The https://www.ingber.com/asa_examples.txt file contains some guidance of the use of OPTIONAL_DATA_PTR and Asa_Data_Ptr.

9.3.42. OPTIONS->Asa Out File

If you wish to have the printing from the asa module be sent to a file determined dynamically from the user module, set the Pre-Compile Printing Option USER_ASA_OUT [FALSE] to TRUE, and define the Program Option *Asa_Out_File in the user module. (This overrides any ASA_OUT settings.) An example of this use for multiple asa () runs is given in the user module.

9.3.43. OPTIONS->Asa_Usr_Out_File

If you wish to have the printing from the user module be sent to a file determined dynamically from the user module, set the Pre-Compile Printing Option USER_ASA_USR_OUT [FALSE] to TRUE, and define the Program Option *Asa_Usr_Out_File in the user module. (This overrides any USER_OUT settings.)

9.3.44. OPTIONS->Cost Schedule

If USER_COST_SCHEDULE [FALSE] is set to TRUE, then (*Cost_Schedule) () is created as a pointer to the function user_cost_schedule () in asa_usr.c, and to recur_user_cost_schedule () if SELF_OPTIMIZE is set to TRUE.

9.3.45. OPTIONS->Asymp Exp Param

When USER_ACCEPT_ASYMP_EXP [FALSE] is TRUE, an asymptotic form of the exponential function as an alternative to the Boltzmann function becomes available for the acceptance test. A parameter OPTIONS->Asymp_Exp_Param becomes available, with a default of 1.0 in asa_usr.c giving the standard Boltzmann function. The asymptotic approximation to the exp function used for the acceptance distribution is

$$\exp(-x) \to [1 - (1 - q)x]^{1/(1-q)}$$
.

If you require a more moderate acceptance test, then negative Asymp Exp Param may be helpful.

9.3.46. OPTIONS->Acceptance_Test

If USER_ACCEPTANCE_TEST [FALSE] is set to TRUE, then (*Acceptance_Test) () is created as a pointer to the function user_acceptance_test () in asa_usr.c, and to recur_user_acceptance_test () if SELF OPTIMIZE is set to TRUE.

9.3.47. OPTIONS->User_Acceptance_Flag

If USER_ACCEPTANCE_TEST [FALSE] is set to TRUE, then User_Acceptance_Flag is created. In asa (), User_Acceptance_Flag is set to TRUE prior to calling the cost function whenever acceptance tests need not be performed, i.e., when using the cost function to generate initial conditions, when being used to calculate derivatives, or when samples are being generated to calculate the cost temperature;

otherwise it is set to FALSE. If User_Acceptance_Flag is returned from the cost function as FALSE, then it is assumed that the cost function will fail the acceptance criteria, but other data is still collected in the acceptance function. When entering the acceptance test in asa, a test is done to see if the acceptance test has already been determined by the cost function; if not, then OPTIONS—>Acceptance_Test() is called to calculate the acceptance test to determine the resulting value of User_Acceptance_Flag.

9.3.48. OPTIONS->Cost_Acceptance_Flag

If USER_ACCEPTANCE_TEST [FALSE] is set to TRUE, then Cost_Acceptance_Flag is created. In asa (), Cost_Acceptance_Flag is set to a default of FALSE before entering the cost function. If both Cost_Acceptance_Flag and User_Acceptance_Flag are returned from the cost function as TRUE, then it is assumed that the cost function has decided that the acceptance criteria is passed, and other data is collected in the acceptance function.

9.3.49. OPTIONS->Cost Temp Curr

If USER_ACCEPTANCE_TEST [FALSE] is set to TRUE, then Cost_Temp_Curr is available to user_cost_function and/or to OPTIONS->Acceptance_Test to calculate the acceptance criteria.

9.3.50. OPTIONS->Cost_Temp_Init

If USER_ACCEPTANCE_TEST [FALSE] is set to TRUE, then Cost_Temp_Init is available to user_cost_function and/or to OPTIONS->Acceptance_Test to calculate the acceptance criteria.

9.3.51. OPTIONS->Cost_Temp_Scale

If USER_ACCEPTANCE_TEST [FALSE] is set to TRUE, then Cost_Temp_Scale is available to user_cost_function and/or to OPTIONS->Acceptance_Test to calculate the acceptance criteria.

9.3.52. OPTIONS->Prob Bias

If USER_ACCEPTANCE_TEST [FALSE] is set to TRUE, then Prob_Bias is returned by the user module to the asa module. This usually is the Boltzmann test term which is compared with a uniform random number to determine acceptance, and its value can be required for other OPTIONS such as ASA SAMPLE.

9.3.53. OPTIONS->Random Seed

If USER_ACCEPTANCE_TEST [FALSE] is set to TRUE, then in asa () OPTIONS—>Random_Seed is set to the address of the random seed throughout asa, to synchronize the random number generator with the rest of the run, e.g., permitting *Random_Seed to be used in user_cost_function ().

9.3.54. OPTIONS->Generating Distrib

If USER_GENERATING_FUNCTION [FALSE] is set to TRUE, then (*Generating_Distrib) () is created as a pointer to the function user_generating_distrib () in asa_usr.c, and to recur_user_generating_distrib () if SELF_OPTIMIZE is set to TRUE. The parameters passed to these functions are further described below.

9.3.55. OPTIONS->Reanneal_Cost_Function

If USER_REANNEAL_COST [FALSE] is set to TRUE, then (*Reanneal_Cost_Function) () is created as a pointer to the function user_reanneal_cost () in asa_usr.c, and to recur_user_reanneal_cost () if SELF_OPTIMIZE is set to TRUE.

9.3.56. OPTIONS->Reanneal_Params_Function

If USER_REANNEAL_PARAMETERS [FALSE] is set to TRUE, then (*Reanneal_Params_Function) () is created as a pointer to the function user_reanneal_params () in asa_usr.c, and to recur_user_reanneal_params () if SELF_OPTIMIZE is set to TRUE.

9.3.57. OPTIONS->Bias_Acceptance

If ASA_SAMPLE is TRUE, this is the bias of the current state from the Boltzmann acceptance test described above, taken to be the minimum of one and the Boltzmann factor if the new point is accepted, and one minus this number if it is rejected.

9.3.58. OPTIONS->Bias_Generated

If ASA_SAMPLE is TRUE, a pointer, OPTIONS->Bias_Generated, contains the the biases of the current state from the generating distributions of all active parameters, described above. Memory for OPTIONS->Bias_Generated[] must be created in the user module.

9.3.59. OPTIONS->Average_Weights

IF ASA_SAMPLE is TRUE, this is the average of the weight array holding the products of the inverse asa generating distributions of all active parameters.

For example, OPTIONS->N_Accepted can be used to monitor changes in a new saved point in the cost function, and when OPTIONS->Average_Weights reaches a specified number (perhaps repeated several times), the cost function could return an invalid flag from the cost function to terminate the run. When the Average_Weights is very small, then additional sampled points likely will not substantially contribute more information.

9.3.60. OPTIONS->Limit_Weights

If ASA_SAMPLE is set to TRUE, Limit_Weights is a limit on the value of the average of the weight array holding the inverse as a generating distribution. When this lower limit is crossed, as a will no longer send sampling output to be printed out, although it still will be calculated. As the run progresses, this average will decrease until contributions from further sampling become relatively unimportant.

9.3.61. OPTIONS->Queue_Size

If ASA_QUEUE is set to TRUE, Queue_Size is a limit on the size of the FIFO queue used to store generated states. This array must be defined in the user module.

After Queue_Size has been set, and memory created in asa (), it may be changed adaptively to any number less than this. If Queue_Size is 0, then no queue is used.

9.3.62. OPTIONS->Queue_Resolution

If ASA_QUEUE is set to TRUE, Queue_Resolution is a pointer to an array of resolutions used to compare the currently generated parameters to those in the queue. This array must have space allocated and be defined in the user module. See the discussions on ASA_QUEUE and ASA_RESOLUTION on the differences in operations of these two OPTIONS.

The ASA_QUEUE OPTIONS also can be used to coarse-grain a fit, by setting high values of Queue_Resolution [].

9.3.63. OPTIONS->Coarse_Resolution

If ASA_RESOLUTION is set to TRUE, Coarse_Resolution is a pointer to an array of resolutions used to resolve the values of generated parameters. This array must have space allocated and be defined in the user module. See the discussions on ASA_QUEUE and ASA_RESOLUTION on the differences in operations of these two OPTIONS.

9.3.64. OPTIONS->Fit Local

If FITLOC is TRUE, OPTIONS->Fit_Local is passed to cost_function (). This provides additional flexibility in deciding when to shunt asa () over to fitloc (), e.g., during multiple or recursive optimizations. As used in asa_usr.c, a value of Fit_Local set >= 1 is required to enter the local code.

If Fit_Local is set to 2, any better fit found by the local code better than asa () is ignored if that local fit is achieved by violating the ranges of the parameters. This additional test is much stricter than that

imposed by OPTIONS->Penalty,

9.3.65. OPTIONS->Iter_Max

When FITLOC is TRUE, OPTIONS->Iter_Max determines the maximum iterations of the cost_function () by simplex ().

9.3.66. OPTIONS->Penalty

If FITLOC is TRUE, OPTIONS->Penalty determines how to weight violation of constraints, exceeding boundaries, etc.

9.3.67. Multi_Number

If MULTI_MIN is set to TRUE, then OPTIONS->Multi_Number, the number of best-generated states of the lowest sampled values of the cost function, must be set by the user. Of course this can be changed adaptively.

Note that OPTIONS->Multi_Cost and OPTIONS->Multi_Params are seeded with the highest cost function state just before entering the main annealing loop. Therefore, if OPTIONS->Multi_Number is larger than the actual number of accepted best states (new lowest values), or if a set of states are identical within the precision reported, then there will be a set of duplicate states returned which should be filtered via a unique macro.

9.3.68. OPTIONS->Multi Cost

If MULTI_MIN is TRUE, OPTIONS->Multi_Cost save the lowest OPTIONS->Multi_Number values of the cost function.

9.3.69. OPTIONS->Multi Params

If MULTI_MIN is TRUE, OPTIONS->Multi_Params save the parameters of the lowest OPTIONS->Multi_Number values of the cost function.

9.3.70. OPTIONS->Multi Grid

If MULTI_MIN is TRUE, OPTIONS->Multi_Grid must be set by the user to define the resolution permitted to distinguish among parameter values of the best-generated states. However, this is overridden in asa.c, to ensure that Multi_Grid is greater or equal to EPS_DOUBLE and to OPTIONS->Coarse_Resolution if ASA_RESOLUTION is TRUE.

9.3.71. OPTIONS->Multi_Specify

If MULTI_MIN is TRUE, and if OPTIONS->Multi_Specify is set to 0, the selection of best-generated states includes all sampled instances of the cost functions. If Multi_Specify is set to 1, the selection of best-generated states is constrained to include only those with different values of the cost function.

If Multi_Specify is set to 0, and multiple best cost functions are found, the parameters of the final cost function reported will be the parameters of the last best cost function. All new best cost functions will be reported, but if ASA_PRINT_MORE is TRUE only new lower best cost functions will have additional print out.

The final values in asa.c of *best_number_generated_saved and *best_number_accepted_saved at the best cost function will be those of the first instance of the final best value. If Multi_Specify is set to 0, the parameters corresponding to this final value may not be among the set of final Multi_Params if there are more best cost functions than Multi_Number.

If ASA_PARALLEL is TRUE, then N_Generated and *best_number_generated_saved will only be accurate within the scale of OPTIONS->Gener_Block, as these are updated in the acceptance function.

9.3.72. OPTIONS->Gener Mov Avr

If ASA_PARALLEL is set to TRUE, Gener_Mov_Avr determines the window of the moving average of sizes of parallel generated states required to find new best accepted states. A reasonable number for many problems is 3.

If and when OPTIONS->Gener_Mov_Avr is set to 0, then OPTIONS->Gener_Block is not changed thereafter.

9.3.73. OPTIONS->Gener Block

If ASA_PARALLEL is set to TRUE, Gener_Block is an initial block size of parallel generated states to calculate how to determine a new best accepted state.

9.3.74. OPTIONS->Gener_Block_Max

If ASA_PARALLEL is set to TRUE, Gener_Block_Max is an initial maximum block size of parallel generated states to calculate to determine a new best accepted state.

This can be useful if your parallel code assigns new processors "on the fly," to compensate for some cost functions being more CPU intensive, e.g., due to boundary conditions, etc. Then OPTIONS->Gener_Block_Max may be larger than the number of physical processors, e.g., if OPTIONS->Gener_Block would call for such a size.

9.3.75. OPTIONS->Random_Array_Dim

When ASA_SAVE is set to TRUE, OPTIONS->Random_Array_Dim defines the dimension of the array used to hold shuffled random numbers used by the random number generator defined in asa_usr.c. The default is to use SHUFFLE defined in asa_usr.h for Random_Array_Dim.

9.3.76. OPTIONS->Random_Array

When ASA_SAVE is set to TRUE, OPTIONS->Random_Array holds the shuffled random numbers used by the random number generator defined in asa_usr.c The default is to set the pointer of the static array random_array at the top of asa_usr.c.

9.3.77. OPTIONS->Asa Recursive Level

When using ASA recursively, it often is useful to be able to keep track of the level of recursion.

If ASA_SAVE is set to TRUE, set Asa_Recursive_Level to be 0 at the most inner shell and increment at each successive outer shell. Then, ASA_SAVE will take effect at the most outer recursive shell.

10. User Module

This module includes asa_usr.c, asa_usr.h, asa_usr_cst.c, and asa_usr_asa.h. You may wish to combine them into one file, or you may wish to use the ASA module as one component of a library required for a large project.

10.1. int main(int argc, char **argv)

In main (), set up your initializations and calling statements to asa. The files asa_usr.c, asa_usr.h, and asa_usr_cst.c provide a sample () function, as well as a sample cost function for your convenience. If you do not intend to pass parameters into main, then you can just declare it as main () without the argc and argv arguments, deleting other references to argc and argv.

main () returns 0 for a normal exit, -1 if there was a calloc allocation error in asa.c, or -2 if there was a calloc allocation error in asa_usr.c.

If ASA_LIB is set to TRUE, then asa_main () is used as a function call instead of main (). If SELF_OPTIMIZE is set to TRUE, then the first main ()/asa_main () in asa_usr.c is closed off, and a different main ()/asa_main () procedure in asa_usr.c is used.

asa_main () returns 0 for a normal exit, -1 if there was a calloc allocation error in asa.c, or -2 if there was a calloc allocation error in asa_usr.c.

If you require parameters to be passed by asa_main () back to your main program, e.g., cost_value, *cost_parameters, *exit_code, etc., then these can be added as *main_cost_value, *main_cost_parameters, *main_exit_code, with memory allocated in your own main (), etc. Such use is illustrated by ASA_TEMPLATE_LIB.

At the end of the asa_usr.c, part of ASA_TEMPLATE_LIB is an example of a main () program that could call asa_main ().

randflt () calls resettable_randflt () each time to actually implement the RNG. This is to provide the capability of getting the same runs if the same multiple calls to asa () are made, e.g., when using ASA_LIB set to TRUE. To enforce this, asa_main () should call resettable_randflt (rand_seed, 1) at the beginning of each run.

```
double *cost_parameters,
double *parameter_lower_bound,
double *parameter_upper_bound,
```

10.3. int initialize_parameters(

double *parameter_upper_bound, double *cost_tangents, double *cost_curvature,

ALLOC_INT *parameter_dimension, int *parameter int real,

#if OPTIONS FILE DATA

FILE *ptr_options,

#endif

USER_DEFINES * USER_OPTIONS)

Before calling asa, the user must allocate storage and initialize some of the passed parameters. A sample () function is provided as a template. In this procedure the user should allocate storage for the passed arrays and define the minimum and maximum values. Below is detailed all the parameters which must be initialized. If your arrays are of size 1, still use them as arrays as described in asa_usr.c. Alternatively, if you define 'int user_flag', then pass &user_flag.

As written above, these are the names used in the user module. All these parameters could be passed globally in the user module, e.g., by defining them in asa_usr.h instead of in main () in asa_usr.c, but since the asa module only passes local parameters to facilitate recursive use, this approach is taken here as well.

initialize_parameters () returns 0 for a normal exit or -2 if there was a calloc allocation.

10.4. int recur_initialize_parameters(

double *recur cost parameters,

double *recur_parameter_lower_bound,

double *recur parameter upper bound,

double *recur_cost_tangents,

double *recur cost curvature,

ALLOC INT *recur parameter dimension,

int *recur parameter int real,

#if OPTIONS FILE DATA

FILE *recur_ptr_options,

#endif

USER DEFINES * RECUR USER OPTIONS)

This procedure is used only if SELF_OPTIMIZE is TRUE, and is constructed similar to initialize_parameters (). recur_initialize_parameters () returns 0 for a normal exit or -2 if there was a calloc allocation.

10.5. double cost_function(

double *x.

double *parameter_lower_bound,

double *parameter_upper_bound,

double *cost tangents,

double *cost curvature,

ALLOC_INT *parameter_dimension,

int *parameter int real,

int *cost_flag,

int *exit code,

USER_DEFINES *USER_OPTIONS)

10.5.1. cost_function

You can give any name to cost_function as long as you pass this name to asa; it is called cost_function in the user module. This function returns a real value which ASA will minimize. In cases where it seems that the ASA default parameters are not very efficient for your system, you might consider modifying the cost function being optimized. For example, if your actual cost function is of the form of an exponential to an exponential, you might do better using the logarithm of this as cost_function.

10.5.2. *x

x (called cost_parameters in the user module) is an array of doubles representing a set of parameters to evaluate.

10.5.3. double *parameter lower bound

See discussion below on parameter upper bound.

10.5.4. double *parameter_upper_bound

These two arrays of doubles are passed. Since ASA works only on bounded search spaces, these arrays should contain the minimum and maximum values each parameter can attain. If you aren't sure, try a factor of 10 or 100 times any reasonable values. The exponential temperature annealing schedule should quickly sharpen the search down to the most important region.

Passing the parameter bounds in the cost function permits some additional adaptive features during the search. For example, setting the lower bound equal to the upper bound will remove a parameter from consideration for sampling, although it still will be printed out with other parameters if OPTIONS permit. Whenever this occurs, within asa() the effective dimension defining the size of the search space is reduced by the number of parameters so removed.

For example, if your parameter constraints are correlated in subsets, you can implement the following in your cost_function () in asa_usr.c or asa_usr_cst.c. Immediately upon entering cost_function () after receiving a full set of new parameters from asa (), check all correlated subsets of points. If some correlated subset is not valid, for all parameters that do satisfy your constraints, save parameter_lower_bound [] and parameter_upper_bound [] in some temporary arrays, set parameter_lower_bound [] equal to parameter_upper_bound [], *cost_flag equal to FALSE, and return to asa (). Have asa () keep recalculating the new subsets of points until all subsets are valid. Then, reset parameter_lower_bound [] and parameter_upper_bound [] from the temporary arrays, and continue on with the rest of cost_function (). You may wish to perform this with the Quenching OPTIONS turned on, so that you also can accordingly adaptively modify the annealing rates using the new effective number of active parameters.

10.5.5. double *cost_tangents

This array of doubles is passed. On return from as a this contains the first derivatives of the cost function with respect to its parameters. These can be useful for determining the value of your fit. In the default implementation of ASA, the tangents are used to determine the relative reannealing among parameters.

10.5.6. double *cost curvature

This array of doubles is passed next. On return from asa, for real parameters, this contains the second derivatives of the cost function with respect to its parameters. These also can be useful for determining the value of your fit, e.g., as a "covariance matrix" for the fitted parameters.

When the DEFINE_OPTIONS Curvature_0 option is set to TRUE the curvature calculations are bypassed. This can be useful for very large spaces.

10.5.7. ALLOC_INT *parameter_dimension

An integer containing the dimensionality of the state space is passed next. The arrays x (representing cost_parameters), parameter_lower_bound, parameter_upper_bound, cost_tangents, and parameter_int_real (below) are to be of the size *number_parameters. The array curvature which may be of size the square of *number parameters.

Setting the lower bound equal to the upper bound will remove a parameter from consideration. Whenever this occurs, within asa() the effective dimension defining the size of the search space is reduced by the number of parameters so removed.

10.5.8. int *parameter_int_real

This integer array is passed next. Each element of this array (each flag) can be: REAL_TYPE (-1) (indicating the parameter is a real value), INTEGER_TYPE (1) (indicating the parameter can take on only integer values), REAL_NO_REANNEAL (-2), or INTEGER_NO_REANNEAL (2). The latter two choices signify that no derivatives are to be taken with respect to these parameters. (Derivatives can be taken with INTEGER_TYPE (1) only if OPTIONS->Include_Integer_Parameters is set to TRUE.) For example, this can be useful to exclude discontinuous functions from being reannealed. Note that the values of the parameters and their ranges are always passed as doubles in the code, but their values will be integral for those parameters which are defined as INTEGER_TYPE or INTEGER_NO_REANNEAL.

If a system parameter is discrete, but not a simple set of sequential integers, then it may be necessary to define a transformation within the cost function in terms of a new parameter which is a set of sequential integers. Also note that ASA_RESOLUTION may be used in some such cases, which overrides the use of parameter_int_real. Then, this new parameter, instead of the original discrete parameter, can be passed between asa () and the cost_function (). The (approximate) range of the transformed parameter must be reflected in the values assigned to parameter_lower_bound [] and parameter_upper_bound [], as discussed above. Of course, this transformation may be supplemented by constraints that can be enforced using the *cost_flag in the user module, as discussed below.

10.5.9. *cost flag

cost_flag is the address of an integer. In cost_function (), *cost_flag should be set to FALSE (0) if the parameters violate a set of user defined constraints (e.g., as defined by a set of boundary conditions) or TRUE (1) if the parameters represent a valid state. If *cost_flag is returned to asa () as FALSE, no acceptance test will be attempted, and a new set of trial parameters will be generated.

If another algorithm suggests a way of incorporating constraints into the cost function, then this modified cost function can be used as well by ASA, or that algorithm might best be used as a front-end to ASA.

If OPTIONS->User_Tangents [FALSE] has been set to TRUE, then asa () expects the user to test the value of *valid_state_generated_flag that enters from asa (). If *cost_flag enters with a value of -1, then the user is expected to calculate the cost_tangents [] array before exiting that particular evaluation of the cost function. An example is provided with the ASA_TEMPLATE_SAMPLE example.

When ASA_QUEUE is TRUE, then the value of *cost_flag is tested for FALSE in previous states in queue and an invalid statement is available in the printout.

10.5.10. int *exit_code

The address of this integer is passed to asa. On return it contains the code for the reason as exited. When ASA_PRINT is TRUE, the value is printed out.

NORMAL_EXIT = 0. Given the criteria set largely by the DEFINE_OPTIONS, the search has run its normal course, e.g., when Limit_Acceptances or Limit_Generated is reached.

P_TEMP_TOO_SMALL = 1. A parameter temperature was too small using the set criteria. Often this is an acceptable status code. You can omit this test by setting NO_PARAM_TEMP_TEST to TRUE as one of your Pre-Compile Options; then values of the parameter temperatures less than EPS_DOUBLE are set to EPS_DOUBLE.

C_TEMP_TOO_SMALL = 2. The cost temperature was too small using the set criteria. Often this is an acceptable status code. You can omit this test by setting NO_COST_TEMP_TEST to TRUE as one of your Pre-Compile Options; then a value of the cost temperature less than EPS_DOUBLE is set to EPS_DOUBLE.

COST_REPEATING = 3. The cost function value repeated a number of times using the set criteria. Often this is an acceptable status code.

TOO_MANY_INVALID_STATES = 4. Too many repetitive generated states were invalid using the set criteria. This is helpful when using *cost_flag, as discussed above, to include constraints. Note that when Limit_Invalid_Generated_States is set to -1, this test is bypassed.

IMMEDIATE_EXIT = 5. The user has set OPTIONS->Immediate_Exit to TRUE, or deleted file asa_exit_anytime when ASA_EXIT_ANYTIME is TRUE.

INVALID_USER_INPUT = 7. The user has introduced invalid input. When entering asa (), a function asa_test_asa_options () checks out many user-defined parameters and OPTIONS, and prints out invalid OPTIONS when ASA_PRINT is set to TRUE.

INVALID_COST_FUNCTION = 8. The user has returned a value of the cost function to asa () which is not a valid number, e.g., not between -MAX_DOUBLE and MAX_DOUBLE. Or, the user has returned a value of a parameter no longer within its proper range (excluding cases where the user has set the lower bound equal to the upper bound to remove a parameter from consideration). Note that to enforce this action, noEXIT INVALID COST FUNCTION must be set to FALSE as discussed above.

INVALID_COST_FUNCTION_DERIV = 9. While calculating numerical cost derivatives, a value of the cost function was returned which is not a valid number, e.g., not between -MAX_DOUBLE and MAX_DOUBLE. Or, while calculating numerical cost derivatives, a value of a parameter no longer within its proper range (excluding cases where the user has set the lower bound equal to the upper bound to remove a parameter from consideration) was set. In such cases, review the bounds of parameters and the OPTIONS used to determine how

derivatives are calculated and used.

When calculating derivatives, an invalid cost function, i.e., returning *cost_flag = FALSE from the cost function, will exit asa() with *exit_code INVALID_COST_FUNCTION_DERIV. Also, when calculating derivatives, no extra test is performed to check that parameters are within their lower and upper bounds (since meshes for derivatives may exceed these bounds). If this is not desired, then within the cost function, within a test for USER_OPTIONS->Locate_Cost == 3, a decision may be made whether to return this *exit_code.

CALLOC_FAILED = -1. Calloc memory allocation has failed in asa.c. This error will call Exit_ASA(), the location will be printed to stdout, and asa () will return the double -1 to the calling program. In asa_usr.c, if asa () returns this *exit_code a warning will be printed both to stdout and to USER_OUT. Note that if a calloc memory allocation fails in asa_usr.c, this error will call Exit_USER() to print the location to stdout and then return -2.

Note that just relying on such a simple summary given by *exit_status can be extremely deceptive, especially in highly nonlinear problems. It is *strongly* suggested that the user set ASA_PRINT=TRUE before any production runs. An examination of some periodic output of ASA can be essential to its proper use.

10.5.11. USER DEFINES *OPTIONS

All Program Options are defined in this structure. Since Program Options are passed to as a and the cost function, these may be changed adaptively.

The Program Options also can be read in from a separate data file, asa_opt, permitting efficient tuning/debugging of these parameters without having to recompile the code. This option has been added to the asa module.

10.6. double recur cost function(

double *recur_cost_parameters,
double *recur_parameter_lower_bound,
double *recur_parameter_upper_bound,
double *recur_cost_tangents,
double *recur_cost_curvature,
int *recur_parameter_dimension,
int *recur_parameter_int_real,
int *recur_cost_flag,
int *recur_exit_code,
USER_DEFINES * RECUR_USER_OPTIONS)

This procedure is used only if SELF_OPTIMIZE is TRUE, and is constructed similar to cost_function ().

10.7. double user_random_generator(LONG_INT *rand_seed)

A random number generator function must be selected. It may be as simple as one of the UNIX® random number generators (e.g. drand48), or may be user defined, but it should return a real value within [0,1) and not take any parameters. A good random number generator, randflt, and its auxiliary routines, including an alternative RNG, are provided with the code in the user module.

The random seed, first defined in the user module, is passed to asa (), where it can be reset. This can be useful for some parallelization algorithms.

Most random number generators should be "warmed-up" by calling a set of dummy random numbers. Here, randflt () does this when it is first called, or when it is fed a negative random seed (which can be a useful flag for asa_seed () below).

randflt () calls resettable_randflt () each time to actually implement the RNG. This is to provide the capability of getting the same runs if the same multiple calls to asa () are made, e.g., when using ASA_LIB set to TRUE. To enforce this, asa_main () should call resettable_randflt (rand_seed, 1) at the beginning of each run.

10.8. LONG_INT asa_seed(LONG_INT seed)

When ASA_LIB is set to TRUE asa_seed () becomes available to set the initial random seed. This can be useful for repeated calls to asa_main () described above. If an absolute value of seed greater than 0 is given to a call of asa_seed, then seed is used as the initial random seed; otherwise asa_main () will by default create its own random seed, and the user need not be concerned with any call to asa_seed (). An example of use is given when ASA_TEMPLATE_LIB is set to true.

If the value of seed is less than 0, this signals the default randflt () to initialize the array of seeds to be shuffled. This is especially useful when using repeated calls to asa_main () when ASA_LIB is TRUE.

10.9. double user_cost_schedule(

double test_temperature,
const void *OPTIONS_TMP)

If USER_COST_SCHEDULE [FALSE] is set to TRUE, then this function must define how the new cost temperature is calculated during the acceptance test. The default is to return test_temperature. For example, if you sense that the search is spending too much time in local minima at some stage of search, e.g., dependent on information gathered in USER_OPTIONS, then you might return the square root of test_temperature, or some other function, to slow down the sharpening of the cost function acceptance test

10.10. double recur_user_cost_schedule(

double test_temperature,
const void *OPTIONS TMP)

If USER_COST_SCHEDULE [FALSE] and SELF_OPTIMIZE [FALSE] both are set to TRUE, then this function must define how the new cost temperature is calculated during the acceptance test. As discussed above for user_cost_schedule (), you may modify the default value of test_temperature returned by this function, e.g., dependent on information gathered in RECUR_USER_OPTIONS.

10.11. void user_acceptance_test(

double *uniform_test, double current_cost, double *parameter_lower_bound, double *parameter_upper_bound, ALLOC_INT *number_parameters, const void *OPTIONS TMP)

If USER_ACCEPTANCE_TEST [FALSE] is set to TRUE, then this function must determine the acceptance test, e.g., as an alternate to the Boltzmann test. USER_ACCEPT_ASYMP_EXP [FALSE] is an example of a class of such modifications.

10.12. void recur_user_acceptance_test(

double *uniform_test, double recur_current_cost, double *recur_parameter_lower_bound, double *recur_parameter_upper_bound, ALLOC_INT *recur_number_parameters, const void *OPTIONS_TMP)

If USER_ACCEPTANCE_TEST [FALSE] and SELF_OPTIMIZE [FALSE] both are set to TRUE, then this function must determine the acceptance test, e.g., as an alternate to the Boltzmann test.

10.13. double user_generating_distrib(

LONG_INT *seed,
ALLOC_INT *number_parameters,
ALLOC_INT index_v,
double temperature_v,
double init_param_temp_v,
double temp_scale_params_v,
double parameter_v,
double parameter_range_v,
double *last_saved_parameter,
const void *OPTIONS_TMP)

If USER_GENERATING_FUNCTION [FALSE] is set to TRUE, then this function (referred by USER_OPTIONS->Generating_Distrib in the user module) must define the probability distribution (or whatever algorithm is required) to use for generating new states, e.g., as an alternate to the ASA distribution.

Even mild modifications to the ASA distribution can be useful, e.g., slowing down the annealing schedule by taking a fractional root of the current temperature.

The passed array last_saved_parameter [] contains all the last saved parameters, which are sometimes required for algorithms requiring decisions based on all current parameters. This permits the use of the ASA code for heuristic algorithms that may violate its sampling proof, but nevertheless are useful to process some complex systems within a SA framework.

10.14. double recur_user_generating_distrib(

LONG_INT *seed,
ALLOC_INT *number_parameters,
ALLOC_INT index_v,
double temperature_v,
double init_param_temp_v,
double temp_scale_params_v,
double parameter_v,
double parameter_range_v,
double *last_saved_parameter,
const void *OPTIONS_TMP)

If USER_GENERATING_FUNCTION [FALSE] and SELF_OPTIMIZE [FALSE] both are set to TRUE, then this function (referred by RECUR_USER_OPTIONS—>Generating_Distrib in the user module) must define the probability distribution (or whatever algorithm is required) to use for generating new states, e.g., as an alternate to the ASA distribution.

The passed array last_saved_parameter [] contains all the last saved parameters, which are sometimes required for algorithms requiring decisions based on all current parameters. This permits the use of the ASA code for heuristic algorithms that may violate its sampling proof, but nevertheless are useful to process some complex systems within a SA framework.

10.15. int user_reanneal_cost(

double *cost_best, double *cost_last, double *initial_cost_temperature, const void *OPTIONS TMP)

If USER_REANNEAL_COST [FALSE] is set to TRUE, then this function must define how the new cost temperature is calculated during reannealing.

If USER_REANNEAL_COST [FALSE] and SELF_OPTIMIZE [FALSE] both are set to TRUE, then this function must define how the new cost temperature is calculated during reannealing.

If USER_REANNEAL_PARAMETERS [FALSE] is set to TRUE, then this function must define how the new temperature is calculated during reannealing.

If USER_REANNEAL_PARAMETERS [FALSE] and SELF_OPTIMIZE [FALSE] both are set to TRUE, then this function must define how the new parameter temperatures are calculated during reannealing.

This is the form of the call to as a from as a_usr.c. A double is returned to the calling program as whatever it is named by the user, e.g., final_cost. It will be the minimum cost value found by as a.

10.20. double asa(

This is how as a is defined in the ASA module, contained in as a.c and as a_usr_as a.h. All but the user_cost_function, user_random_generator, and parameter_initial_final parameters have been described above as they also are passed by user_cost_function ().

10.20.1. double (*user_cost_function) ()

The parameter (*user_cost_function*) () is a pointer to the cost function that you defined in your user module.

10.20.2. double (*user_random_generator) ()

A pointer to the random number generator function, defined in the user module, must be passed next.

10.20.3. LONG_INT *rand_seed

A pointer to the random seed, defined in the user module, must be passed next.

10.20.4. double *parameter_initial_final

An array of doubles is passed (passed as cost_parameters in the user module). Initially, this array holds the set of starting parameters which should satisfy any constraints or boundary conditions. Upon return from the asa procedure, the array will contain the best set of parameters found by asa to minimize the user's cost function. Experience shows that any guesses within the acceptable ranges should suffice, since initially the system is at a high annealing temperature and ASA samples the breadth of the ranges. The default is to have asa generate a set of initial parameters satisfying the user's constraints. This can be overridden using User_Initial_Parameters=TRUE, to have the user's initial guess be the first generated set of parameters.

10.21. void print_time(char *message, FILE * ptr_out)

As a convenience, this subroutine and its auxiliary routine aux_print_time are provided in asa.c to keep track of the time spent during optimization. Templates in the code are provided to use these routines to print to output from both the asa and user modules. These routines can give some compilation problems on some platforms, and may be bypassed using one of the DEFINE_OPTIONS. It takes as its parameters a string which will be printed and the pointer to file to where the printout is directed. An example is given in user_cost_function to illustrate how print_time may be called periodically every set number of calls by using PRINT_FREQUENCY and RECUR_PRINT_FREQUENCY in asa_usr.c. See the ASA-NOTES file for changes in these routines that may be required on some particular systems.

10.22. void sample(FILE * ptr_out, FILE * ptr_asa)

When ASA_TEMPLATE_SAMPLE is set to TRUE, using data collected in the ASA_OUT file, this routine illustrates how to extract the data stored in the ASA_OUT file and print it to the user module.

10.23. void adaptive_options(USER_DEFINES * USER_OPTIONS)

When ADAPTIVE_OPTIONS is set to TRUE, it is possible to change relevant OPTIONS in the USER_DEFINES struct in asa_usr_asa.h during run time, by including an asa_adaptive_options file. This can be very useful to efficiently determine how to tune long runs.

The function call to adaptive_options(USER_OPTIONS) should be placed at the top of your cost function. Some examples of the format of lines in the comma-delimited asa_adaptive_options file are given just above the adaptive_options function in asa_usr.c.

11. Bug Reports and Help With ASA

Please read this ASA-README.[] file and the ASA-NOTES file before seeking help or reporting bugs.

I make every reasonable effort to maintain only current versions of the asa module, to permit the code to compile without "error," not necessarily without compiler "warnings." The user module is offered only as a guide to accessing the asa module. The ASA–NOTES file will contain updates for some standard machines. I welcome your bug reports and constructive critiques regarding this code.

Without seeing any specific output from your system, of course I can't say anything specific. While I cannot promise that I can spend the time to join your quest to the very end to insure you get the global optimal point for your system, I can state that I will at least get back to you after seeing some print out. Many times, it is useful to add ASA_PRINT_MORE=TRUE to your compile DEFINE_OPTIONS to get more info, and enclose relevant portions your ASA_OUT file with your query. (See the section Use of Documentation for Tuning above.)

My policy, as stated in the ASA-NOTES file, is to keep all my help as well as my commercial work with ASA on a confidential basis. I do not divulge any names of people or information about (legal!) projects unless I am given specific permission to do so or unless the work is published. This policy promotes feedback on ASA which benefits all users as well as those individuals seeking help.

"Flames" will be rapidly quenched.

12. References

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