PREVIEW

- 1.1: PROPOSITIONAL LOGIC
  - Propositions
  - Logical connectives
  - Truth tables
  - Compound propositions
  - Translating English sentences to logical formulas
  - Bit strings and bit operations
  - Truth tables for compound propositions

HONOUR HOMEWORK: To be done before next week's tutorials. Numbers are from the 7th edition, with 6th edition numbers in brackets, red, WHEN DIFFERENT.

- Section 1.1: #1, 3, 9 (5), 11 (7), 31 (27), 43 (37).
- Section 1.2: #31 (63 in section 1.1).

1. LOGIC AND PROOFS

1.1. Propositional Logic.

Definition **1.1.** *A* ***proposition*** *is a declarative sentence that is either true or false.*

Example **1.** *Which of the following are logical propositions?*

1. Toronto is the capital of Canada.
2. $1+1=2$.
3. Three is greater than five.
4. What time is it?
5. $x+1=2$.

We can encode propositions by letters (normally use lower case $p, q, r, s, \ldots$). This allows us to represent complicated combinations of logical propositions in concise formulas.
We also encode True and False as T and F respectively (alternatively, we can encode these as 1 and 0 respectively).

LOGICAL CONNECTIVES:

Negation	(not)	$\neg$
Conjuction	(and)	$\wedge$
Disjunction	(or)	$\vee$
Exclusive or		$\oplus$
Conditional	(if then)	$\longrightarrow$
Biconditional	(if and only if)	$\longleftrightarrow$

Logical connectives are used to build **compound propositions**.

Let p denote “I just won the lottery" q denote “I am buying everyone a pony"

(1) Negation $\neg p$

(2) Conjunction $p \wedge q$

(3) Disjunction (inclusive or) $p \vee q$

(4) Exclusive Or $p \oplus q$

Recall p denotes "I just won the lottery" q denotes "I am buying everyone a pony"

(5) Conditional (implication) $p \longrightarrow q$

TERMINOLOGY: Given conditional $p \longrightarrow q$

- CONVERSE $q \longrightarrow p$
- CONTRAPOSITIVE $\neg q \longrightarrow \neg p$
- INVERSE $\neg p \longrightarrow \neg q$

(6) Biconditional $p \longleftrightarrow q$

TRANSLATING ENGLISH SENTENCES TO LOGICAL FORMULAS

Example 2. *Write the following as logical formulas, and if applicable, state the corresponding converse and contrapositive:*

1. "If Bob goes to the party but John does not, then Mary goes to the party too."
   - Set up:
   - Logical Formula:
   - Converse:
   - Contrapositive:
2. "Whenever it is raining, it is windy and it is not sunny."
   - Set up:
   - Logical Formula:
   - Converse:
   - Contrapositive:

NOTE OPERATOR PRECEDENCE: $\neg$, then $\wedge$, then $\vee$, then $\rightarrow$

Example 3. *Consider the following logical formulas:*

1. $p \rightarrow \neg q \wedge r$ is same as
2. $p \rightarrow (\neg q \wedge r)$ and $(p \rightarrow \neg q) \wedge r$ are Not the same

BIT STRINGS AND BIT OPERATIONS

Definition 1.2. *A **bit string** is a finite sequence of zeros and ones; each zero or one is called a **bit** (from **b**inary dig**it**).*

Interpreting 1 as true and 0 as false, we can apply logical connectives to bit strings of equal length. We use AND, OR and XOR to denote the bit operations corresponding to $\wedge$, $\vee$, $\oplus$ respectively.

Example 4. *Apply bitwise AND, OR, and XOR to the strings $x = 10110$, $y = 11101$*

TRUTH TABLES OF COMPOUND PROPOSITIONS

Good bookkeeping practices:

- if proposition consists of n variables, table should have 2^n rows
- table should start with one column for each variable, list variables in alphabetic order
- have i^{th} variable's column filled by alternating 2^{n-i} T entries and 2^{n-i} F entries
- for example, a truth table for a compound proposition involving p, q, r should have $2^3 = 8$ rows; column for p should have 4 Ts, followed by 4 Fs, column for q should have alternating 2 Ts, then 2 Fs, column for r should have alternating T and F.
- correctly identifying order of operations is key to labeling columns

Example 5. *Build the truth table of*

$$(p \vee \neg q) \longrightarrow (p \wedge q)$$