PREVIEW

- 1.3: PROPOSITIONAL EQUIVALENCES
  - Tautology/Contradiction
  - Establishing logical equivalences using truth tables
  - Using established identities to simplify logical formulas, and to prove logical equivalences

HONOUR HOMEWORK: To be done before next week's tutorials. Numbers are from the 7th edition, with 6th edition numbers in brackets, red, WHEN DIFFERENT.

- Section 1.2: #31 (63 in section 1.1).
- Section 1.3 (1.2 in 6th ed) : #1, 5, 7, 9, 15, 31.

1. LOGIC AND PROOFS

1.2. **Applications of Propositional Logic.** Read in text page 16-22.

- System Specifications
- Boolean Searches
- Logic Circuits

1.3. **Propositional Equivalences.**

Definition **1.1.** *A* ***tautology*** *is a logical formula that is always true (i.e. true under all possible truth value assignments of propositional variables). A* ***contradiction*** *is a logical formula that is always false. A* ***contingency*** *is a logical formula that is neither a tautology nor a contradiction.*

Example **1.** *Consider the following compound propositions:*

1. $p \wedge \neg p$
2. $p \vee \neg p$
3. $p \wedge q$

Definition **1.2.** *Two logical formulas are said to be* ***logically equivalent*** *provided they have the same truth values under all assignments of propositional variables (i.e. if they have the same truth table).*

Notation: we write $p \equiv q$ for "p is logically equivalent to q"

Example **2.** $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Example 3. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

We have just proved a couple of rules that we can now use to simplify logical formulas (they are called **DeMorgan's laws**)

Other elementary logical identities can be established the same way (using truth tables). Here is a list of common ones:

$p \wedge T \equiv p$ $p \vee T \equiv p$	Identity Laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination Laws
$p \vee p \equiv p \wedge p \equiv p$	Idempotent Laws
$\neg\neg p \equiv p$	Double Negation
$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutativity Laws
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associativity Laws
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributivity Laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	DeMorgan Laws
$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	Absorption Laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation Laws

Example 4. *Simplify the following formulas*

$$\neg(\neg p \wedge \neg q) \equiv$$

$$(p \wedge q) \vee (p \wedge \neg q) \equiv$$

$$\neg(p \wedge \neg q) \vee (\neg p \wedge q) \equiv$$

Application 1. *Logic Circuits - consider the last formula in example 4*

An important logical equivalence involving conditionals

$$\boxed{p \longrightarrow q \equiv \neg p \vee q}$$

This is useful for proving equivalences involving the conditional connective.

An important property of the contrapositive: $\boxed{\neg q \longrightarrow \neg p \equiv p \longrightarrow q}$

Example 5.

$p \to q$: "If $n^2 + 2n - 1$ is an odd integer, then n is an even integer."

Proof:

$\neg q \to \neg p \equiv$

COMMON MISTAKES:

- Thinking an implication and its converse are logically equivalent, but

$$p \longrightarrow q \not\equiv q \longrightarrow p$$

- Thinking an implication and its inverse are logically equivalent, but

$$p \longrightarrow q \not\equiv \neg p \longrightarrow \neg q$$

Time Permitting, Some exercises (from Rosen Textbook):

(1) Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

(2) Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

(3) How many different truth tables of compound propositions are there that involve the propositional variables p and q?

(4) Determine if the following proposition is *satisfiable*
$(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

(5) Find a compound proposition involving the propositional variables p, q, and r that is true when exactly two of p, q, and r are true and is false otherwise. [HINT: form a disjunction of conjunctions.]