11 Measuring Indices of Refraction

The passage of 'fringes' in a interferometer depends on changes in the phase shift in one beam of the interferometer relative to the other. One full cycle of some detectable quantity goes by when a 2π-radian phase shift occurs due to some change of an independent variable. In this section you'll see that the measurement of phase shift via a fringe count can be the basis of a method for measuring an important property of materials, the index of refraction. This section is written assuming that it's a Michelson interferometer that's being used to make the measurement; the same techniques can be used in Sagnac or Mach-Zehnder interferometers, provided provision is made in the theoretical treatment for the fact that these latter devices involve a one-way (rather than two-way) passage of the light through a sample.

a. The index of refraction of gases

TeachSpin provides a 'gas cell', an optical sample chamber with two transparent end windows, which can be placed into the beam in one arm of an interferometer. The idea is that as the pressure of gas in the cell varies from near-vacuum to near-atmospheric, the density and the index of refraction of the gas will vary, and that in turn will change the phase shift in that arm, and cause a succession of fringes to appear at the interferometer's output.

It's worth recalling that the index of refraction of a material, n, is defined by $n = c/v_{ph}$, where the phase velocity of the wave is in turn given by $v_{ph} = \omega/k$. The angular frequency ω is connected to more familiar quantities via

$$\frac{\omega}{2\pi} = f = \frac{c}{\lambda_{vac}} \ ,$$

so these combine to give the wave number k as

$$k = \frac{2\pi}{\lambda_{vac}} n \ .$$

The relevance of the quantity k is that a (complex) wave propagates in space according to

$$\exp i(k\,x - \omega\,t) = \exp\left(i \frac{2\pi}{\lambda_{vac}} n\,x\right)\ e^{-i\,\omega\,t} \ .$$

Hence a sample chamber of fixed (internal) length L, under a change of index of refraction from $n = 1$ (in the vacuum condition) to a given n-value, causes a phase shift for one-way travel of light given by

$$\Delta\phi = \frac{2\pi}{\lambda_{vac}} \Delta n\, L = \frac{2\pi}{\lambda_{vac}} (n - 1)\, L \ .$$

In a Michelson interferometer with two-way passage of the light, the phase shift is double this, and the fringe count M between 'vacuum' and 'full' conditions is given by

$$M = \frac{\Delta\phi}{2\pi} = \frac{1}{2\pi}\, 2\, \frac{2\pi}{\lambda_{vac}} (n - 1)\, L \ = \frac{n-1}{\lambda_{vac}} (2\,L) \ .$$

The TeachSpin gas cell has internal length L = (100.0 ± 0.1) mm, and when used with red HeNe laser radiation of vacuum wavelength λ_{vac} = 632.990 nm, it will give about M = 85 fringes even for the small change in index of refraction of air from 1.0000 to about 1.0003 which occurs between vacuum and 1-atmosphere pressure. So if you've been used to assigning "1" to the index of refraction of air, now you'll be able to quantify departures from n=1 with a precision of a part per million!

There's some good physics in the index of refraction of a gas, in its dependence on the pressure and temperature of the gas, and the polarizability of the molecules composing it. The behavior of gases and even liquids is very well described by the Lorenz-Lorentz law, and this in turn allows the measurement of the index of refraction of a gas at a given temperature and pressure to be corrected back to the values expected for STP [standard temperature and pressure] (which in the optical community is chosen to be 15 °C and 760 Torr).

Depending on what gases you have available for study, you might try measuring the index of refraction of air, nitrogen, oxygen, helium, argon, or hydrogen. You might also understand how the electric polarization of a gas *mixture* is related to the polarizability and the number density of the molecules composing it, and learn how a measurement of the index of refraction of a mixed sample can be a simple surrogate for measuring the mixing ratio of its constituents. If you look up quantum-mechanical calculations for the polarizability of the helium atom computed from first principles, you'll be able to compare measured and predicted values for its refractive index.

The TeachSpin apparatus makes the pressure dependence of index of refraction easy to study, since it provides a piezo-resistive differential-pressure transducer that translates gas-pressure difference between its two ports into a real-time analog voltage. It can be used to measure cell pressure relative to ambient atmosphere, or (given a suitable forepump) relative to vacuum. The nominal calibration constant of the transducer is

$$\frac{\delta V_{out}}{\delta(\Delta p)} = 10.\ \frac{\text{Volts}}{\text{atmosphere}},$$

but the actual value can be checked against some local pressure standard such as a U-tube mercury manometer or a Bourdon gauge. Even the local weather report, or airport's value of present atmospheric pressure can be used, provided you recall that these are *not* actual measured surface values of air pressure, but rather ones extrapolated down to sea level.

Your TeachSpin equipment includes a gas manifold, described in Appendix S, with valves that allow the controlled addition, or removal, of gas from your sample cell, and also the establishment of vacuum conditions on one side of your differential-pressure sensor if you wish. There's also a port to permit the controlled venting of the line to your forepump to ambient pressure. You'll want to use the flexible plastic hoses provided to permit the manifold to be mounted *off* of your optical breadboard, so that mechanical manipulations of the valves don't cause irrelevant deformations of your interferometer.

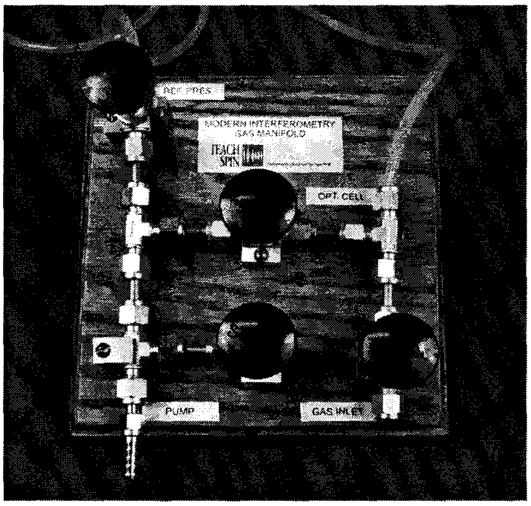

Figure 11-1: The Gas manifold

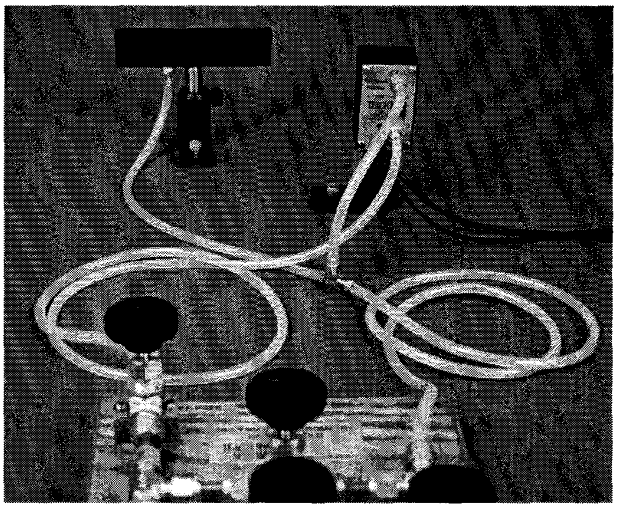

Figure 11-2: Cell and transducer interconnected

Finally it's worth remembering that nominal atmospheric pressure is 101,325 Pascals ≡ 760 Torr, and that a forepump of modest performance can be expected to yield, for present purposes, a very good approximation to a perfect vacuum. The limiting pressure of a forepump might be 10 to 50 mTorr, which is less than 10^{-4} of an atmosphere, and in any case these low pressures can be measured absolutely, if not very accurately, with a thermocouple gauge.

b. The index of refraction of slabs

Your measurement method for index of refraction of gases depends on the ability to count fringes while changing the density of a sample continuously over a range, and this method of course fails to work for solid samples. Interferometry presents another method for measuring the index of refraction of solids, provided only that they are available in the form of a parallel-sided transparent slab, which can be introduced into one arm of an interferometer. Again, you can't count the fringes that go by while you're sliding the sample sideways into the beam, so you need a cleverer way to make the sample's effect change continuously. The technique that's often used is to start with a sample already in the beam, but with its faces perpendicular to the light beam, and then to count fringes as the sample is rotated by a known amount away from perpendicularity.

The effect of a plane-parallel slab on the phase of a beam of light is complicated by the fact that there are two geometric effects going on simultaneously; one is a phase change due to index of refraction, and the other is a geometric effect due to refraction. The calculation of the phase shift expected for a slab of thickness T and index n, when tilted by angle θ relative to the face-on condition, is best computed by comparison to a mythical ray of light that misses the sample altogether. The geometry is shown in the figure below:

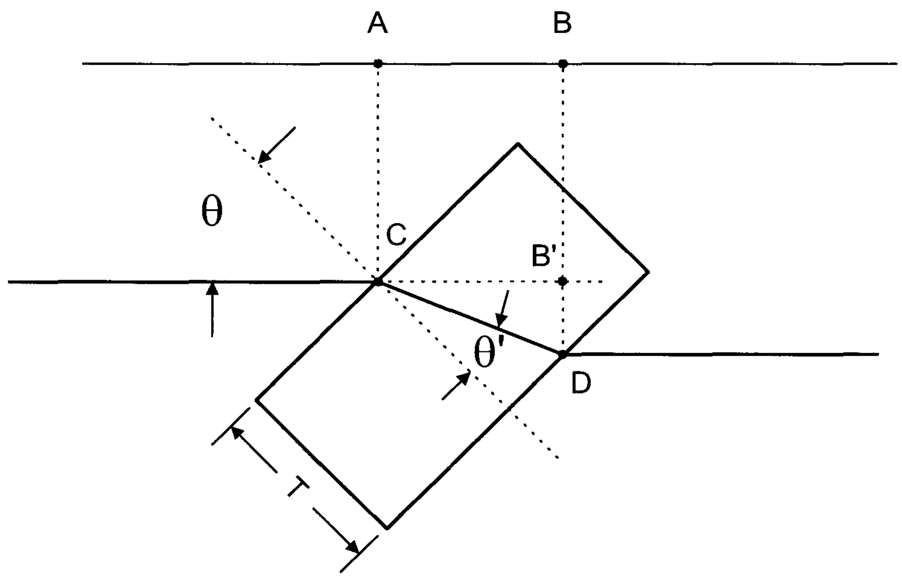

Figure 11-3: The geometry of refraction through a plane-parallel slab of thickness T, tilted by angle θ away from face-on to a beam of light

Recall that the wave number k gives phase accumulation per unit distance, so in physical distance L, the phase accumulation is given by

$$\Delta\phi = k\,L = \frac{2\pi\,(\mathrm{rad})}{\lambda_{vac}}\,n\,L\ ,$$

and that net phase accumulated is the product of this rate times the distance traversed. Hence in the diagram, the difference in phase accumulation can be written as

$$\delta(\Delta\phi) = \frac{2\pi\,(\mathrm{rad})}{\lambda_{vac}}\,(n\cdot\overline{CD} - 1\cdot\overline{AB})\ .$$

Geometry and Snel's Law give the distances involved as

$$\overline{CD} = \frac{T}{\cos\,\theta'}\ \text{ and } \overline{AB} = \overline{CB'} = \overline{CD}\cos(\theta - \theta'),\ \text{where}\ \sin\theta = n\sin\theta'\ ,$$

so finally we can write, for the phase shift due to a slab at angle θ, relative to the same slab face-on, the expression

$$\phi(\theta) = \frac{2\pi}{\lambda_{vac}}\,T\,\{\frac{n - \cos(\theta\text{-}\theta')}{\cos\,\theta'} - \frac{n\text{-}1}{1}\}\ .$$

This is a messy function of the index of refraction n, the more so when the internal angle θ' is written in terms of the measured external angle θ. Using Snel's relation 1.sin θ = n.sin θ' for small values of the angles, and expanding in powers of the angle θ, one can derive the approximate expression

$$\phi(\theta) = \frac{2\pi}{\lambda_{vac}}\,T\,\{\frac{n\text{-}1}{2n}\,\theta^2 + O(\theta^4)\}\ ,$$

which is enough to show that the expected fringe-count signal will be approximately *quadratic* in angle θ. For a two-pass encounter with a typical glass slab having n = 1.5 and T = 1 mm, we find that the fringe count M is given by

$$M = 2\frac{\Delta\phi}{2\pi} \approx 2\frac{T}{\lambda_{vac}}\frac{n-1}{2n}\theta^2 ,$$

and reaches over 5 full fringes already at $\theta = 0.1$ rad = 5.73°.

The complicated phase-shift function above thereby predicts a fringe count M as a function of θ, with a functional shape depending on the index of refraction n of the slab. The value of index n can be extracted by comparing the $M(\theta)$ data with functional shapes computed for a set of candidate n-values. Alternatively, the complicated functional dependence of fringe-count on index n can actually be inverted to give

$$n = \frac{\alpha^2 + 2(1 - \cos\theta)(1 - \alpha)}{2(1 - \cos\theta - \alpha)} ,$$

where $\alpha = M(\theta)\lambda_{vac}/2T$ is directly related to the observed fringe count.

Your TeachSpin equipment includes some slabs of glass and quartz that are mounted on posts in such a way that they can be introduced into a light beam and then rotated about a vertical axis by a controllable amount. There is a holder for a single slab of glass (nominal thickness T = 1.0 mm), another holding a pair of such glass slabs, and a third holding a single slab of quartz (nominal thickness T = 0.50 mm). You also have a compensator plate of 1" diameter and 2.0 mm thickness. Any of these devices can be mounted into the miniature rotation stage, which allows them to be rotated freely, or dialed through a modest range of angle under carefully controlled angular translations. So you can measure the refractive index of any of them, and check for its dependence on polarization. [Appendix P describes the structure and use of this rotation stage.]

Hence the ability to count fringes as a function of angular displacement can be used to deduce index of refraction; alternatively, these slabs and their rotator can be used to make small and very accurately controllable phase shifts in an interferometer.

4 Interlude on *Polarization*

All your work in interferometry to this point has made use of the wave nature of light, but has not really depended on whether light is a transverse or a longitudinal disturbance. But you know that light is, in fact, a transverse electromagnetic disturbance, and hence there remains the optician's choice over one additional degree of freedom, the direction of (say) its electric-field vector. This interlude will introduce you to a set of polarization techniques, valuable in general but also applicable to some really clever techniques in interferometry.

Your tools for this interlude include the HeNe and diode laser sources, the alignment towers as view screens, and (as diagnostic devices) the two Polaroid-type linear polarizers in rotary mounts on rod-and-post holder bases. In addition, you will get acquainted with the remarkably useful polarizing beam-splitter cubes (PBSCs) that you'll be using in other interferometers.

You might start with either of your lasers mounted in its support cradle, and turned on so as to deliver a beam across an otherwise empty optical breadboard. Send its beam to illuminate an alignment tower as view screen, and now interpose in the beam one of your Polaroids. Some light will still come through; show that you can *extinguish* the light by rotating the Polaroid. This is enough to show that your laser is delivering linearly polarized light. [Not all HeNe lasers do so -- generic HeNe deliver 'randomly polarized' light -- but yours does. Check your diode laser, too.] Now back to the extinction criterion -- what happens as you rotate the Polaroid away from that extinction angle? [here, meaning the rotating the Polaroid *in its own plane*, using the ring-mount that makes it easy to do so.] You should see more light, and you should see maximal transmission at a *pass* angle, ±90° away from the extinction angle.

To see that the polarization of the beam is produced at the source, now loosen the mounting screws in the laser holder, and rotate your HeNe laser bodily, by some modest angle, about its own light-output axis. Clamp it back down, and confirm that the extinction and pass angles of the Polaroid have rotated by the same amount.

Now set your lasers to a nominal condition of this rotation: with the HeNe, rotate until the power cord at the back end of the laser lies at the lowest position (6 o'clock on the dial) possible; with the diode laser, rotate so that the ceramic-faced thermoelectric modules contact the aluminum support below the laser. I claim that each beam now has the electric-field vector lying vertically in space -- but you need to confirm that. The method of choice is NOT to rely on the angular numbers printed on the Polaroid's mount; you can't know for sure if the optical pass direction of the Polaroid material has been aligned mechanically along the mounting ring, nor can you be sure whether it's the 0° vs. the 90° marking on its scale that's intended to mark the pass condition. Instead, it's time to perform the unambiguous 'Brewster test' to confirm the direction of your laser-beam's E-field.

To make this test, you need no Polaroids, but only a plain and uncoated slab of dielectric that you can handle; a microscope slide is conventional, but any small slab of window glass or even clear plastic will work fine. What you want to try is shown in the two diagrams below -- you want the beam to pass through the slab, and you want to identify the beams that originate from partial reflection of the beam from the faces of the slab. In Fig. 4-1 below, <u>a view from vertically above</u>, the reflected beam is being deflected by about 60° from its original direction, and is falling on a alignment tower for you to view. The diagnostic is to rotate the slab, about a vertical axis, so that the reflected beam swings through an arc of ±20° or so from this condition. What you're looking to see is if there is an orientation of the slab that causes the reflected beam's intensity to drop to nearly zero.

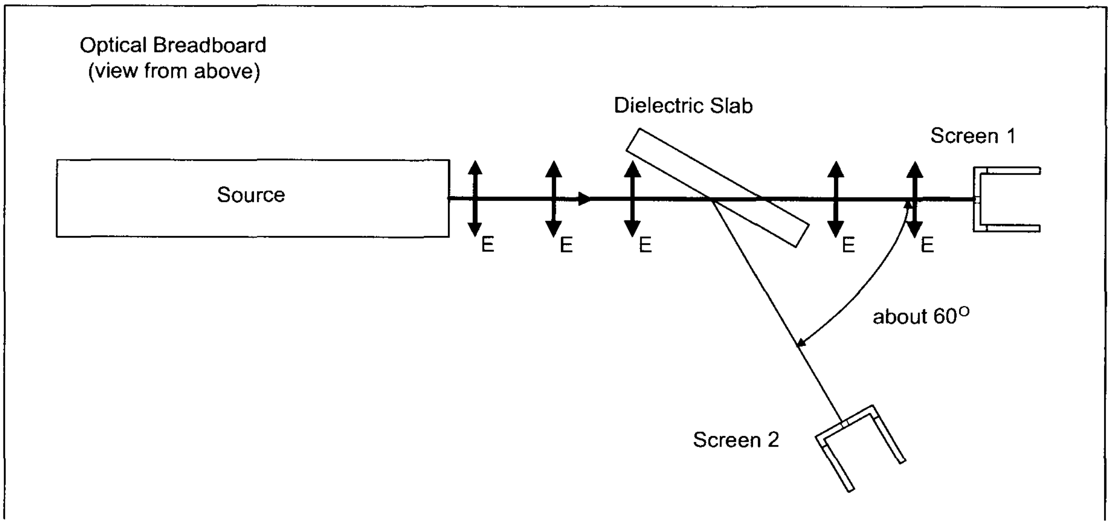

Figure 4-1: One way to try the 'Brewster test' for polarization – a view from **above**

In Fig. 4-2 below, you have <u>a side-on view</u> of another use of the same slab. Here the beam is being deflected downwards, through about 60°, and falling onto a white card that's been laid flat onto the optical breadboard. Now the diagnostic is to rotate the slab, about a horizontal axis, so that the reflected beam swings through an arc of ±20° or so from this condition. Again you're looking to see if there's an orientation of the slab that causes the reflected beam's intensity to drop to nearly zero. In *this* orientation of the slab, you should be able to see the deep minimum in spot intensity that develops when you've achieved the Brewster condition at the slab. If you can achieve a *zero* in reflected-beam intensity, then you have the slab oriented at the Brewster angle, *and* the laser beam with a vertical polarization.

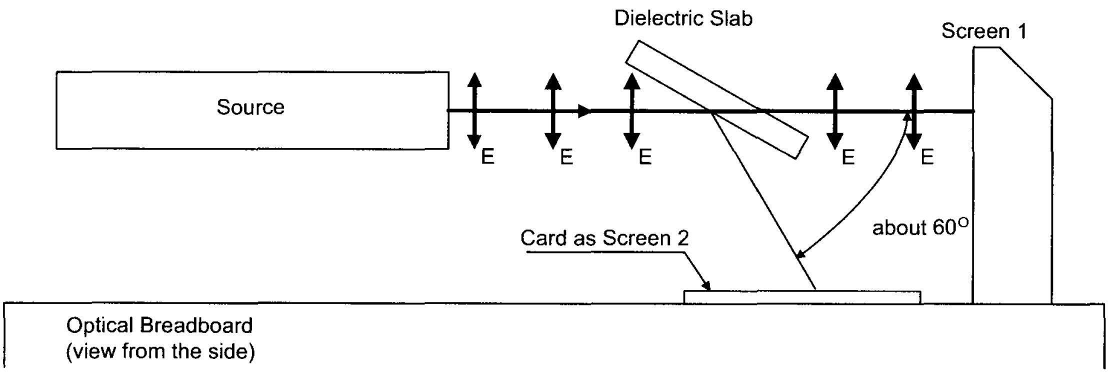

Figure 4-2: Another way to try the 'Brewster test' for polarization – a view from the **side**

[Rotate the HeNe laser source about its output-beam axis by perhaps 5-10° away from the previous orientation, and repeat the test in Fig. 4-2; you'll see a Brewster minimum again, but this time *not* getting right down to zero intensity. Exploit your understanding to get the laser oriented so as to deliver its E-field exactly vertically.]

Once you've accomplished this test, you can deliver a beam of confirmed vertical polarization (meaning, in modern optical usage, having a vertically-directed E-field), and now you can use it to test the meaning of the angular scale on your Polaroids. Be sure you write down some established fact, such as "For a vertically polarized beam, this Polaroid, set so its angular scale reads 1° or 181°, will pass the beam; but when set to 91° or 271°, will block the beam." Hereafter, your Polaroid will be the easy diagnostic for testing the polarization of optical beams.

Now rotate your HeNe laser by a full 45° about its long axis and confirm (by the extinction test) that it's delivering a linearly polarized beam (of tilted polarization, of course). Downstream, put a first Polaroid into position, set for now to its 0° setting. Now use a second Polaroid, downstream from the first, to find out

- whether the beam emerging from the first Polaroid is linearly polarized,
- and if so, what is the *direction* of its polarization.

Repeat these two tests with the upstream Polaroid set to its 90° setting.

You'll need to form a mental picture of what a Polaroid does: it does *not* merely change the intensity of a light beam passing through it, but *also* changes the light's polarization. In particular, by fully absorbing the light component with E-field perpendicular to its pass direction, it delivers an output beam whose E-field is wholly along its pass direction.

Now with your HeNe laser's E-field axis tilted to the 45° position, and a single downstream Polaroid available to rotate through its setting range of 0 - 90°, you can get a beam (admittedly of somewhat variable intensity) which is always linearly polarized, but whose polarization direction you can swing smoothly through the whole 0-90° range. Now let that beam fall onto one of your polarizing beam-splitter cubes (PBSCs) held in its upright holder and mounted on one of your beamsplitter bases. The optical element you see looks like a solid cube of glass, 1" on an edge, but it is in fact made of two 45-45-90° prisms glued together on a common hypotenuse. [Carefully remove the four mounting screws in the top plastic retaining plate to get a view of this common hypotenuse. While you're at it, note the two shallow grooves, one above and one below the cube, which hold the cube in place. Note also that the cube's hypotenuse is intended to lie parallel to the long axis of the rectangular top retaining plate, so you'll always know which orientation it has.]

A PBSC is intended for use with light falling nearly perpendicularly on its faces, and for such light, the internal hypotenuse face acts like a beam-splitter set at 45°. The result is that you might expect some of the incident light to emerge undeflected from the face opposite the entry face, and some of the light to emerge from a side face, partially reflected off the internal hypotenuse face. Set your PBSC in its mount on your breadboard so that your laser beam passes through it, and set up alignment towers to serve as viewing screens for these two beams. [Why don't you get any light emerging from the *fourth* face of the cube?].

Now look at the intensity of these two beams, as a function of the angular orientation of your upstream Polaroid. What does the cube do to an input beam that is vertically polarized? What does the cube do to an input beam that is horizontally polarized? What does it do to a generic beam?

You can do more than this. With the input beam at a generic angle of linear polarization, so that both output beams are in view, use your second Polaroid to diagnose the polarization of both output beams, the straight-through 'passed' beam and the deflected-by-90° 'bent' beam. You should find the 'passed' beam always has one state of polarization, whatever its intensity and whatever the state of the input polarization. You should also find the 'bent' beam always has another state of polarization, whatever its intensity and whatever the state of the input polarization. You should also know that the common-hypotenuse face internal to the cube is coated with a multilayer dielectric that absorbs a negligible amount of light, so that (*un*like a Polaroid) this PBSC device only redirects light, according to its polarization. You may think of it as 'sorting' light, which enters going one direction but with two polarization components, into two separate output beams, each of one single polarization (orthogonal to the other's).

If you use a z-axis to designate the vertical direction in your lab, and take as x-axis the direction of propagation of your input laser beam, then in this picture you can describe the general state of input light as

$$\mathbf{E}_{in} = (E_1\, \hat{\mathbf{y}} + E_2\, \hat{\mathbf{z}}) \exp i(k\, x - \omega\, t)$$

while the two output beams (at least for an ideal PBSC) can be written as

$$\mathbf{E}_{pass} = (E_1\, \hat{\mathbf{y}}) \exp i(k\, x - \omega\, t)\ ,\ \ \mathbf{E}_{bent} = (E_2\, \hat{\mathbf{z}}) \exp i(k\, y - \omega\, t)$$

Note that the 'bent' beam is propagating in a new direction, and note that each 'piece' of the input beam has been preserved in coefficient. In fact, with E_1 and E_2 standing for generic complex coefficients, the input beam need not even be restricted to a state of linear polarization. By virtue of the superposition principle for light, an input beam of *any* polarization has been disassembled into two beams, each preserving the amplitude of the original beam's components along **y** and **z**.

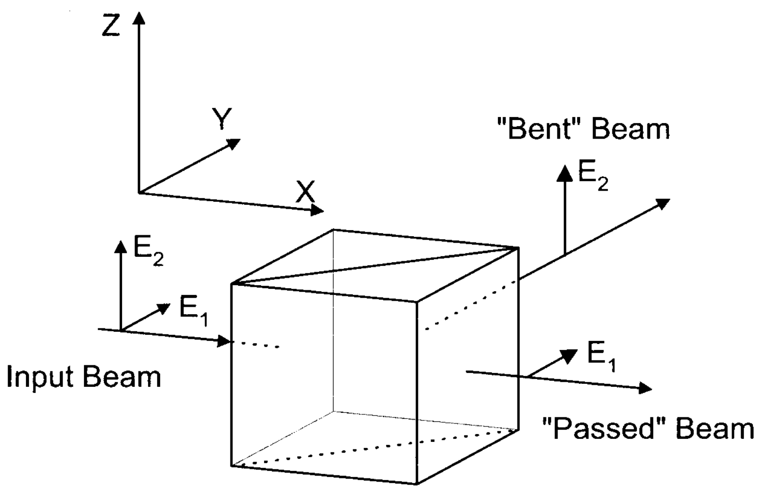

Figure 4-3: One choice of axes for understanding the polarizing beam-splitting cube (PBSC)

5 The Sagnac Interferometer

The first thing you need to know about a Sagnac interferometer is that the name is French, and the pronunciation is approximately 'Sahn-yock'. You'll build one, and find that it is not very well suited to measuring lengths, or length changes. You will find that it is an interferometer of unparalleled fringe stability, and you'll learn why. You will also see that it can be built in such a way as to exploit polarization of light in some truly clever ways, both in the two separated beams and in the detection of the light. Finally, you'll see that it's an interferometer of optimum 'photon efficiency'; in principle, every photon leaving the laser ends up on one of two detectors, and the output depends directly on the difference of these two detector signals. As a result, you'll get to experience an interferometer in which fringe stability well under 10^{-3} *of a fringe* is readily attained.

a. The Sagnac topology

How is this all achieved? The first and characteristic difference in a Sagnac interferometer is that both of the two beams emerging from the beamsplitter end up going around all four sides of a rectangle (in opposite directions simultaneously), each returning to the original beamsplitter to re-combine there. The result is that any mechanical motion of any of the optical components is common to both beams, and (to first order) does not show up in the path-length difference at all. You'll find that it's actually possible to have the two beams traverse (in opposite directions) the very same path through the air, and contact the same points on the mirrors, further improving this 'common-mode rejection' of influences like mirror vibration and air turbulence.

The layout of a Sagnac interferometer that you can build with your TeachSpin components is shown in Fig. 5-1 below. There are a few special features of the components that are now worth describing.

a) The easiest source on which to learn the ideas is the HeNe laser, chosen for its good beam quality; because of the use made of polarization tricks, the HeNe is best rotated in its mount so as to deliver its linearly polarized output at an angle of 45° away from the vertical.

b) The beam reaches the interferometer proper via the two steering mirrors, making a near-90° bend at each mirror as usual. The second steering mirror is the one mounted on the one-dimensional rack-and-pinion slide, and that slide is oriented so that the beam leaving the second steering mirror can readily be translated sideways, remaining parallel to its original direction.

c) The interferometer starts with a polarizing beam-splitter cube (PBSC) serving as beamsplitter, and (given the state of input polarization) the beams emerging from it are approximately equal in power, orthogonal in direction, and also orthogonal in polarization.

d) Note that there are three mirrors that serve to define the other corners of a rectangle; these are mirrors of the sort used as end mirrors in the Michelson interferometer, but here are used at 45° angle of incidence. See Appendix L for the mounting holes to use in the mirror bases for this new angle of incidence.

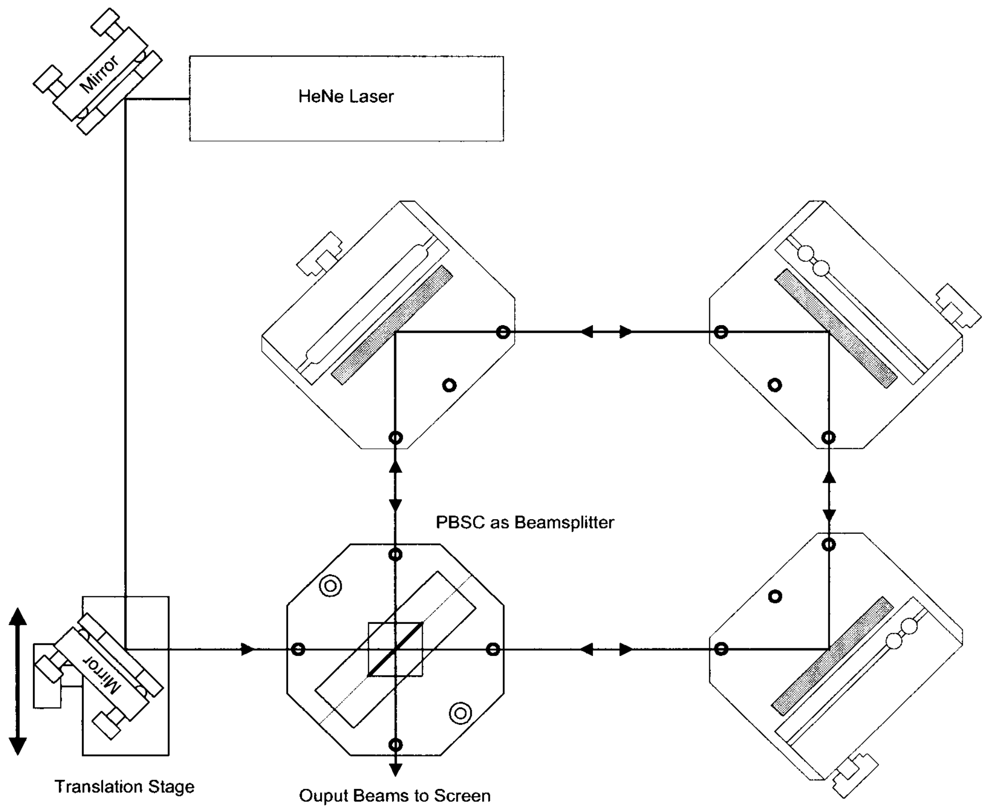

Figure 5-1: The topology of a Sagnac interferometer (not to scale)

e) After finishing a trip around the whole of the rectangle, each beam encounters the PBSC again, re-entering the cube at a different face from where it left the cube. Precisely because of the polarization-separating property of the PBSC, the input beam component which is 'bent' by the PBSC at its first encounter is also 'bent' by the PBSC at its return; and the input beam component which is 'passed' by the PBSC at its first encounter is also 'passed' by the PBSC at its return. The result is that *both* beams emerge from their second encounter with the PBSC at the heretofore-unused fourth face, and they head away from the PBSC in a conveniently accessible way.

f) To align the interferometer is to get the two counter-propagating beams to overlay each other in space all the way around the rectangular path, and also to overlay each other at the output. You'll soon see how this alignment can be achieved.

g) The two beams emerging from the fourth face of the PBSC are still perpendicularly polarized, so they will *fail* to show interference fringes; that's the motivation for a clever use of polarization optics in diagnosing the output of the interferometer.

b. Aligning a Sagnac interferometer

Now that you know the topology and the components required, here's a suggested procedure for setting up and aligning the interferometer. You'll need the two beam paddles, and you'll be mounting them at a variety of the convenient alignment holes that are found on the top face of the bases of the PBSC-holder and the mirror holders -- see Fig. 5-2 for reference to locations.

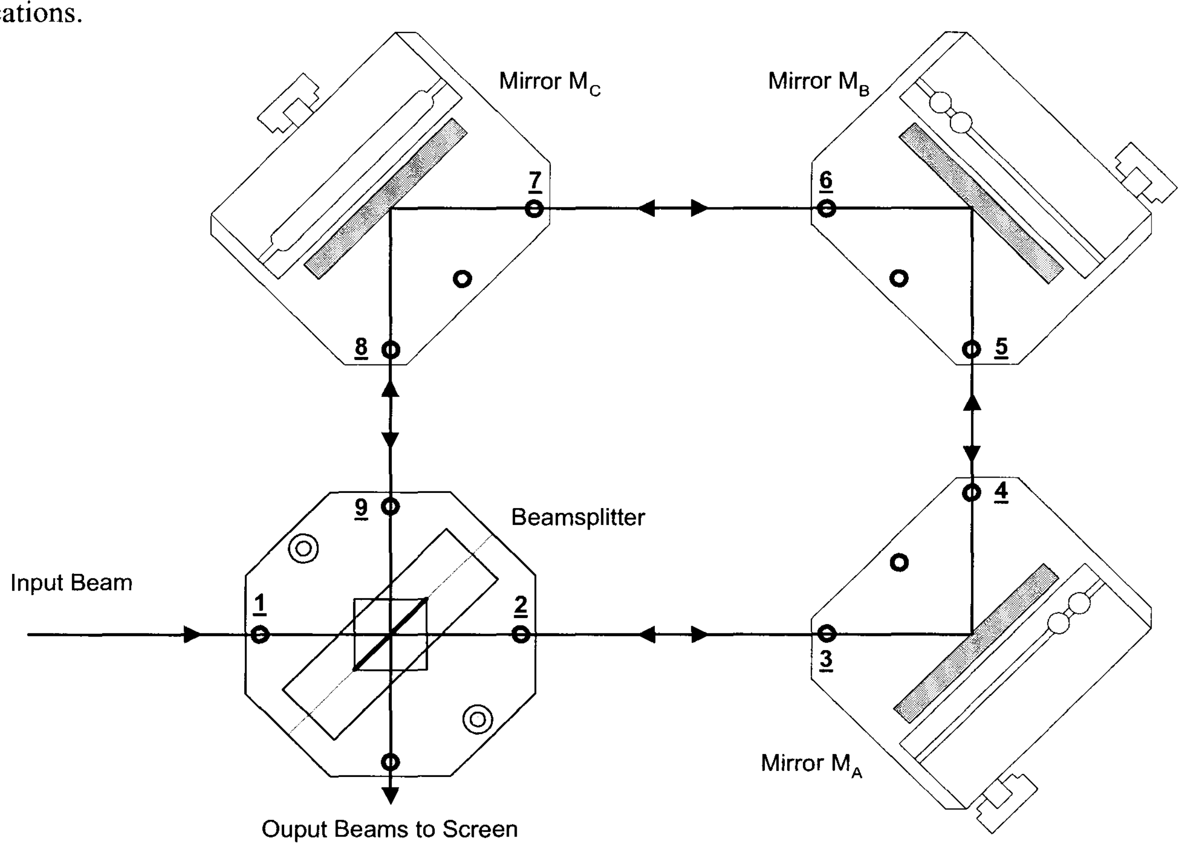

Figure 5-2: Labeling alignment-paddle locations in the Sagnac interferometer

1) Start with a HeNe laser and the two steering mirrors as introduced above, and then put down onto your optical breadboard the PBSC on its base, and mirror M_A as shown. Note that M_A gets the 'passed' beam from the PBSC, and note that M_A is mounted (now using different screw holes in its base) so as to reflect the laser beam through a 90° angle. For now, it would be well to block the 'bent' beam from the PBSC with an alignment tower, since your first goal is to use the steering mirrors M_1 and M_2 to get the 'passed' beam to pass through beam-paddles placed at locations #2 and #3 as shown in the Figure.

2) Now remove the paddles, block the 'passed' beam with an alignment tower, and let the 'bent' beam reach mirror M_C, also mounted so as to deflect a beam through 90°. [It is important that mirrors M_A and M_C have orthogonal directions of the 'hinge lines' in their flexure hinges.] Put beam paddles in place at locations #8 and #9, since you'll want the 'bent' beam to pass through holes in them. But don't use the steering mirrors M_1 and M_2 to

achieve this; instead, achieve the aiming you require by loosening the bolts holding the PBSC-holder to the optical breadboard, and using the 'slop' in its mounting holes. A rotation about a vertical axis of the whole PBSC base will give you one degree of freedom for centering the beam on the beam paddle at location #8; the other perpendicular degree of freedom needed can be had from shimming under one corner of the PBSC mount's baseplate. [See Appendix G for what 'shimming' means in this context.]

3) Now you have beams reaching mirrors M_A and M_C, and you have those mirrors in the right positions and approximately at the right angles. You should now put mirror M_B in position to complete the rectangular perimeter of the interferometer, and you should see two beams hitting its front face. Bolt it loosely to the breadboard, also at the 45° orientation, and mount beam paddles on its base at locations #5 and #6. Now loosen the mounting of M_A and rotate its base on the breadboard until it sends a beam through the paddle at location #5; similarly loosen the mounting of M_C and rotate its base on the breadboard until it sends a beam through the paddle at location #6. Now if you remove the two paddles, you should see one *single* spot on mirror M_B that is illuminated by two separate beams from the PBSC. You will be able to use a fine-adjustment screw on the back of either M_A or M_C to help with the vertical degree of freedom in this overlap. Once you've achieved this, you can tighten down mirrors M_A and M_C.

4) Before tightening down mirror M_B, use a beam paddle inserted at location #4 or #7 to identify *beams reaching it from both sides*. You might temporarily use a sheet of paper instead, since with such a semi-transparent beam indicator you get a good idea of the relative position of two beams reaching the paper from opposite sides. Your goal is to rotate mirror M_B by the 'slop' available in its breadboard-mounting holes until you've achieved overlap of the two beams hitting the opposite sides of the paper sheet; you may also use the thumbscrew adjustment on the back of mirror M_B. Don't forget at this stage to tighten down the mounting screws of M_B, and indeed any others that may still be loose.

5) Now put an alignment tower in position to see the *two* beams that emerge from the fourth face of the PBSC. They will probably not yet be overlapping, but they should both be emerging from near the center of the face of the PBSC. If you now adjust the thumbscrew on the back of M_A or M_C, you will find that turning *either* thumbscrew affects the position of *both* spots on your alignment tower. No matter; you can still use those two thumbscrews to give a good overlap. [You should not need to use the thumbscrew on M_B to achieve this overlap.]

6) But when you've achieved overlap, you *won't* see any fringes on your screen. To see why not, readjust the spots so they're a bit away from overlapping, and interpose a Polaroid between the PBSC and your view screen to diagnose the polarization of the two output beams. You should find that the two beams are perpendicularly polarized, and you should mentally trace each beam backwards until you can understand where it got its polarization. You should also understand the theory of why two orthogonally polarized light beams do not show interference phenomena. Finally, you should orient your Polaroid at 45°, and see that both beams now are transmitted (in part) by this Polaroid.

7) Now repeat the adjustments that bring the two spots into overlap, and you should see fringes. They'll be *great* fringes, of superb contrast and stability. The contrast is high because the two 'arms' in this interferometer have nearly identical lengths, so you're optimally set for mutual coherence; they're stable for the reasons described under 'common mode' above, and the fringes' geometrical stability keeps the contrast high in your view on the screen. The final stage of adjustment is not to perfect the overlap of the beam spots, but instead to lower (to zero) the 'spatial frequency' of the fringes. See Appendix M for a discussion of what gives rise to these straight-line fringes and their periodicity on the screen.

8) The very stability of the Sagnac interferometer makes it hard to know where on the interferometer you can 'push' to move through a succession of fringes. So now you can exploit a wonderful feature of this interferometer, by moving *away* from the present configuration that has the two beams nearly perfectly overlapping in space all the way around the interferometer. Go ahead and put beam paddles back in at locations #2 and #3, and now go back to the steering mirror M_2 and (finally) exercise its rack-and-pinion slide capability. You'll want to translate the beam now traversing the *central* holes in paddles at #2 and #3 until it passes instead through the *side* holes in the same paddles. [It doesn't matter to which side you go, but do make it the *same* side on both paddles.] You may need to trim up the adjustments on steering mirrors M_1 and M_2 to align the beam with the paddles; it might help to block the beams between locations #8 and #9 to isolate the beam you are using for alignment.

9) Once you have the input beam thus translated, you should remove all the paddles, and you should immediately see the two output beams, passing through the Polaroid, overlapping on the screen. Again you can touch up the thumbscrews on M_A and M_C for optimal fringes, and again you'll have the Sagnac interferometer working. The difference is that now at any point around the interferometer, the two counter-propagating beams are horizontally separated in space, like the two lanes of traffic on a two-way street. Confirm this by using a sheet of paper as a viewing screen; confirm that you understand the direction of propagation of both of the beams you see; also diagnose the *polarization* of the two beams you see.

10) The value of having the two beams separated in space is that you can now do something to one beam, separate from what you do to the other beam. The TeachSpin gas cell is designed so that it can be mounted with one of the two separated beams passing through it, and the other passing beside it. So now use the hose and syringe to make small pressure changes to the air inside the cell, and see what that does to the intensity of the spot on your view screen. You should see a sinusoidal variation in brightness.

11) Now that you're persuaded that you can make fringes go by, it's time to use a more clever and efficient detection scheme than the Polaroid-and-view screen combination. You want to remove the Polaroid, and to use instead the second PBSC in the curiously tilted arrangement shown in Fig. 5-3; here the overlapped output beam from the Sagnac encounters a PBSC oriented at 45° relative to the horizontal and vertical. [See Appendix N for the procedure required for moving a PBSC from one of your upright beamsplitter mounts into this tilted-cube mount.] For starters, it's well to leave the photodetectors out of the holders that will soon accommodate them; instead, use a large white card that will give you a view of

the beam 'passed' by the second PBSC. Clearly, this beam, like the beam that passed through the Polaroid, represents the overlap and interference of the two beams going around the interferometer, so you should see fringes at this spot. Now note that the beam 'bent' by the second PBSC is further deflected through 90° by an ordinary mirror, so that it too comes out to your viewing card, propagating parallel to your 'passed' beam but displaced 'up and over'.

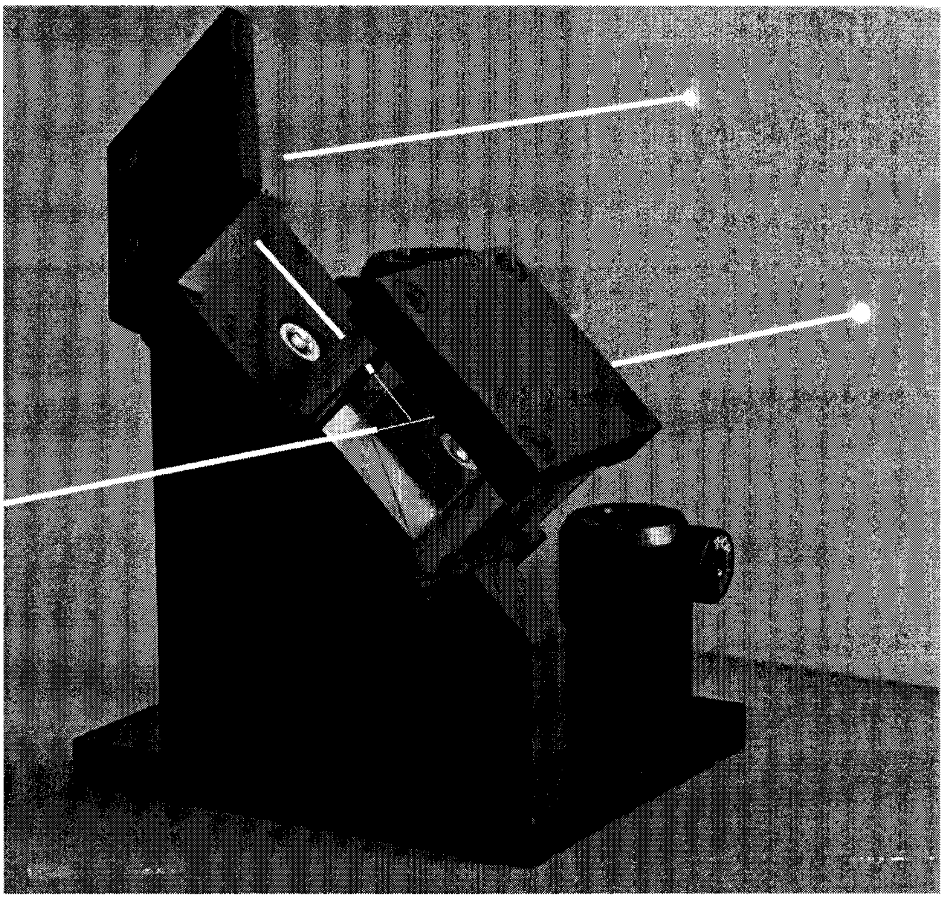

Figure 5-3: Using the tilted PSBC as a polarization separator

12) Your goal is now to adjust your interferometer for zero spatial frequency at both spots, and then to notice that the light intensities in the two spots on your card are complementary -- that one is brightest when the other is dimmest. The reason for this is laid out mathematically below, but it is also obvious on physical grounds: all the light from the laser enters the first PBSC; none of it is absorbed there, so all of it is sent around the interferometer. That light, reaching the first PBSC again as beam recombiner, emerges from the 'fourth face' of the first PBSC, and heads to the second PBSC. That element is also lossless, so all the light from the laser has to emerge into one of the two spots on your card. [Naturally there are losses in the interferometer, due chiefly to reflectivity less than 100% at the metal front-surface mirrors in the interferometer; but in principle this interferometer and

this polarimetric detection scheme can be made to be 100%-efficient: all the light leaving the laser would end up on one detector or the other.]

13) Now it's time to put photodetectors in place to quantify these two output beams. Use the rod-and-post holder arrangements to get the two detectors at the right heights; slide the whole polarimetric-detector baseplate laterally to get the horizontal adjustment you need for the 'passed' beam; and use the lateral slide adjustment of the taller post's base to get the horizontal adjustment you need for the 'bent' beam. When you have both beams hitting near the center of their detectors, power up both detectors, and set them to equal gains that also keep their readings on scale. Set up a dual-trace oscilloscope to view the two intensity reading simultaneously, and sweep at perhaps 1 s/division to see what happens to the two signals as you vary the optical phase difference (via the gas pressure in the cell). You should see the complementarity of the two signals; you should also note that the amplitudes of the two sinusoids are very nearly equal.

14) Given two such analog electrical signals, it's a very clever technique to subtract one from the other. That will give a signal which varies on both sides of zero, rather than lying always on one side of zero. The difference signal will then have 'zero crossings', arising when the two beam spots on a card would be of equal intensity. One of the many attractions of such a 'zero crossing' signal is that its location is first-order independent of any power fluctuations in the laser. You can play with the 'subtract' function you might have on your 'scope, but you can also use the signal-flow schematic on the front face of the Modern Interferometry controller to perform this analog subtraction. There is provision to match the amplitude of the two sinusoids, in case detector sensitivities are not quite matched; there is also provision to filter out high-frequency noise in the signals, leaving the steady optical-phase-difference signal controlled by the gas cell. You should begin to see that the interferometric difference signal has *amazingly* low noise, even after you use the further amplification of the difference signal that your controller box permits.

15) Since the two beams in the interferometer are separated in space, there is some non-zero sensitivity to air-density fluctuations inside the interferometer. So this might be the time to re-install the draft shield over the rectangular perimeter of the interferometer. Now you can bring the diagnostic difference signal down to a zero-crossing, park it there, raise the gain on your 'scope detection of the result, and begin to see just how stable a signal you can get from an interferometer. Go ahead and tap the optical breadboard to get an idea of how well vibration is an effect common to both 'arms' of this interferometer, and hence cancelled out. See if you can tell what limits the stability in time of your signal; you may find that time-varying pressure in the gas cell is the biggest problem.

16) There are many ways (other than the use of the gas cell) to vary the optical phase difference in the two arms of the interferometer. One of them is based on the index of refraction of thin slabs; find the holder which carries *two* glass slabs of 1-mm thickness and 1-cm^2 area, and find the angular rotation stage which can rotate its mounting post about a vertical axis. The glass slabs are laid out so that each is canted, or tilted, by 10.0° (±0.5°) away from face-on in their holder. They are also arranged such that you should be able to

position the whole unit (glass slabs, post and rotator) so as to have the two separated beams in the interferometer pass through the two slabs of glass.

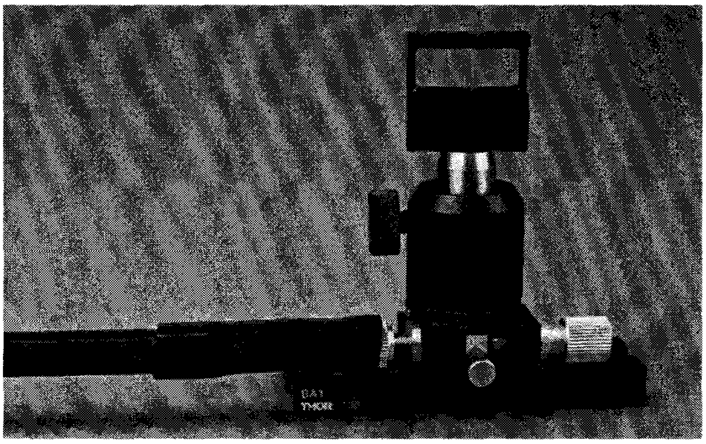

Figure 5-4: The rotation stage, bearing a two-plate tuner for the Sagnac interferometer

When you've gotten this unit into position, clamp it down to the breadboard, and then use thumbscrews at M_A and M_C to trim up the fringe patterns in the two output beams. Now you'll find that as you rotate the glass-slabs' holder, you get a steady succession of fringes, but one which you can 'park' at any desired point with superb stability. The full description of why this occurs is left for later, but note that if the two slabs start with each one illuminated by light making a 10° angle of incidence, then turning the rotator by (say) 1° changes the angle of incidence for one slab to 11°, and the other slab to 9°. Clearly this increases the effective thickness of the slab in one arm, and decreases it in the other, giving rise to the phase difference you can detect.

17) The separated-beam Sagnac interferometer offers access to the two individual beams in it, but it thereby becomes somewhat sensitive again to two noise-contributing effects. For one, the air density will inevitably be somewhat different at the two distinct beam locations. For another, the two beams are now interacting with different places on the three bending mirrors, so that vibrations of the mirrors can affect one beam differently than the other. So you may want to remove the 'two-plate glass tuner' and use the rack-and-pinion adjuster at steering mirror M_2 to take you back to the fully overlapped-beam condition; you might temporarily put paddles back in at locations #1 and #2 to align the beam 'passed' by the PBSC with your chosen axis, and then trim up the fringe pattern by adjusting corner mirrors M_A and M_C. [Having translated the input beam sideways, you'll also need to translate bodily sideways the whole polarimetric detection unit, by its base, to get it centered on the new location of the output beam.] You're back to having the two beams in your interferometer (nominally) overlapping each other all the way around the perimeter of the interferometer, which helps to defeat both noise sources just mentioned. But if also defeats the use of the two-plate tilter to tune the phase difference between the two beams. You might think of trying the one-plate glass-slab tilter instead, but that plate will cause a phase shift that should be common to both beams, so it still won't work as a phase adjuster.

So here's the trick: you also have a one-plate tuner, but this one made of a slab (0.5 mm thick) of crystalline quartz. It is a plate with two distinct indices of refraction, which we may label n_h (for horizontally polarized light) and n_v (for vertically polarized light). The value of this plate is that the two beams traversing your interferometer are not only going in opposite directions; they are also of perpendicular in polarization, and so the two beams will sample the two distinct indices of refraction and thus encounter effectively different plates. So set up the quartz one-plate tilter unit somewhere inside the interferometer, and confirm that rotating it will shift the electronic difference signal you've been detecting. The goal is not to scan through a huge range of fringes, but rather to be able to bring the interferometer from any given starting condition to one of the zero-crossing points of the difference signal.

After confirming that it works, you can take the quartz tilter plate out of the interferometer, and try re-installing it between steering mirror M_2 and the beam-splitter PBSC, or between the PBSC and polarimetric detection unit, and confirm that it still does its job, and try to explain why.

Once you're able to put a draft cover over an overlapped-beams Sagnac interferometer, and once you're able to use the quartz tilter to reach a zero-crossing signal, you can use electronic gain as desired to look at the stability of your interferometric signal. How noisy is it in the short term? How stable is it in time? If there were to be a square-wave modulation superimposed on the signal you see, how large would it have to be to be distinctly detectable? How large a square-wave phase difference would produce this square-wave difference signal? You are now ready for Section 15, in which your newfound capabilities are used to detect the electro-optic effect in a solid material.

18) You might now check the degree to which you have the counter-propagating beams in your interferometer actually overlapping in space. Use a thin-paper viewing screen to get a view of beams impinging on both sides simultaneously, and test the beams near the PBSC (where the overlap should be near-perfect) and near bending mirror M_B (where the overlap might be at its worst). If the two beams are separated, or of visibly imperfect overlap, near M_B, then you have a diagnostic against which you can improve. The payoff will be lower sensitivity to air density and vibration-induced noise in your output signal; the method is to pay more attention to aligning out the imperfections of the various mechanical and optical components in your interferometer.

c. Understanding polarimetric detection

Here's a computation intended to show how the output of the Sagnac interferometer can be represented mathematically, and how the tilted PBSC detector unit works with that optical output to generate the electrical signals you have seen. We imagine that the input of the Sagnac interferometer is linearly polarized light with direction of polarization tilted at 45°, so that there are vertical and horizontal electric-field components of equal magnitude and in phase:

$$\mathbf{E}_{in} = \frac{E_0}{\sqrt{2}}\,\hat{\mathbf{y}}\exp(ikx - i\omega t) + \frac{E_0}{\sqrt{2}}\,\hat{\mathbf{z}}\exp(ikx - i\omega t) \ .$$

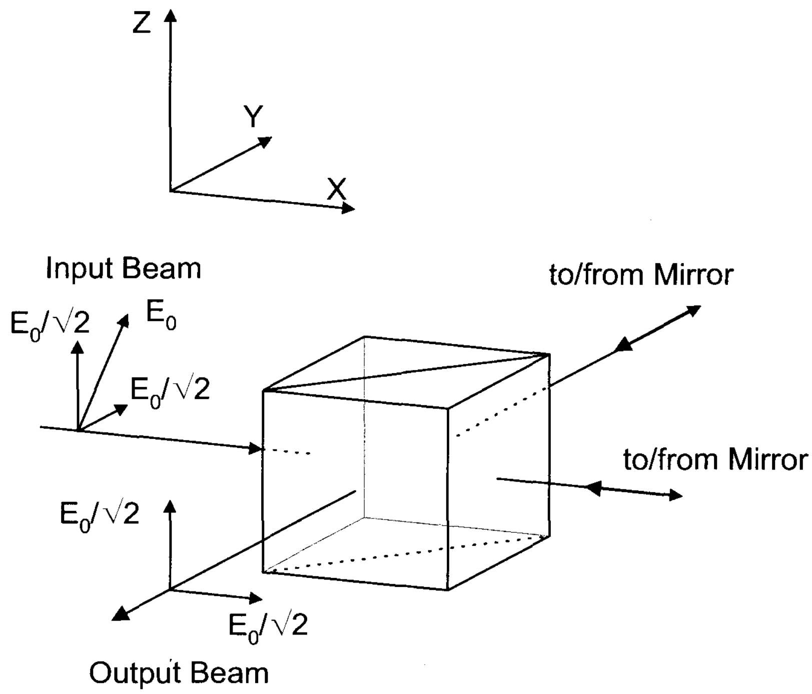

Figure 5-5: Input and output states of polarization for a PSBC

Now the beam emerging from the beamsplitter PBSC is propagating in a different direction; we'll assume that the beam's passage through the interferometer has not changed the amplitude of either component, but has given two distinct phase shifts (to the clockwise- and counterclockwise propagating beams):

$$\mathbf{E}_{\text{out}} = \frac{E_0}{\sqrt{2}}\,\hat{\mathbf{y}}\,\exp(i\,\phi_{\text{ccw}})\,\exp(-iky - i\omega t) + \frac{E_0}{\sqrt{2}}\,\hat{\mathbf{z}}\,\exp(i\,\phi_{\text{cw}})\,\exp(-iky - i\omega t)\ .$$

This beam is in a complicated state of polarization (elliptical in general), but it's not too hard to model what will happen to it upon encountering the second, tilted, PBSC. The projection of $\mathbf{E}_{\text{out}}$ along the vector direction $\mathbf{n}_1$ will be the part of $\mathbf{E}_{\text{out}}$ that will emerge from one of the faces of the second PBSC, while the projection of $\mathbf{E}_{\text{out}}$ along the direction $\mathbf{n}_2$ will emerge from the other face of this PBSC:

$$\mathbf{E}_{\#1} = \mathbf{E}_{\text{out}} \cdot \hat{n}_1 = \mathbf{E}_{\text{out}} \cdot \frac{\hat{\mathbf{x}} + \hat{\mathbf{z}}}{\sqrt{2}}\ ;\ \mathbf{E}_{\#2} = \mathbf{E}_{\text{out}} \cdot \hat{n}_2 = \mathbf{E}_{\text{out}} \cdot \frac{-\hat{\mathbf{x}} + \hat{\mathbf{z}}}{\sqrt{2}}\ .$$

Since these describe the electric fields in two separate beams that are then carried to the two distinct detectors, we want a model for the power in the two beams. In the complex representation of an electric field, that is given by

$$P = \frac{1}{2}|\mathbf{E}|^2\ ,$$

and the signal that we ultimately observe electrically is given by

$$P_1 = \frac{1}{2}|\mathbf{E}_{\#1}|^2\ ;\ P_2 = \frac{1}{2}|\mathbf{E}_{\#2}|^2\ ;\ P = P_1 - P_2\ .$$

There is a fair amount of algebra required to compute the details of these signals, but the results are

$$P_1 = \frac{1}{2}\frac{E_0^2}{4}(2 + 2\cos\Delta\phi) \;;\; P_2 = \frac{1}{2}\frac{E_0^2}{4}(2 - 2\cos\Delta\phi) \;,$$

where $\Delta\phi \equiv \phi_{ccw} - \phi_{cw}$ gives the phase difference accumulated by the two beams making their way, in opposite directions, around the ring of the Sagnac interferometer. The significance of the leading coefficients is best seen by noting that

$$P_1 + P_2 = \frac{1}{2}\frac{E_0^2}{4}(4 + 0) = \frac{1}{2}E_0^2 = P_{in} \;,$$

so sure enough, the two beams headed toward the detectors carry collectively all the power in the beam sent into the interferometer. To put it another way, we can see that the quantity P_{in} also represents the maximum power, alternately occurring in beam #1 and beam #2. Hence if we change from optical powers P to electrical signals S, we can predict that the signals on the two electrical outputs will behave according to

$$S_1 = \frac{S_{max}}{4}(2 + 2\cos\Delta\phi) \;;\; S_2 = \frac{S_{max}}{4}(2 - 2\cos\Delta\phi) \;.$$

But in actual use, we view not the sum, but the difference, of the two signals, which gives a signal taking on both signs,

$$S(\Delta\phi) \equiv S_1 - S_2 = \frac{S_{max}}{4}(4\cos\Delta\phi) = S_{max}\cos\Delta\phi \;.$$

Here the quantity S_{max} has a very readily understood empirical meaning, since S_{max} gives the maxima, and $-S_{max}$ gives the minima, of the electrical signal you've directly monitored.

This also displays the sensitivity of the interferometer to changes in the phase difference $\Delta\phi \equiv \phi_{ccw} - \phi_{cw}$. We need only compute the derivative

$$\frac{\partial S}{\partial(\Delta\phi)} = S_{max} \cdot - \sin\Delta\phi \;,$$

which in turn tells us that the output signal S will change by amount

$$\delta S = \pm S_{max}\,\delta(\Delta\phi)$$

if we choose to operate near the zero-crossing points of the signal. To be concrete, we suppose that S_{max} = 8 Volts, and that we change the phase difference by one one-*thousandth* of a full fringe, ie. $\Delta\phi = 0.001 \cdot 2\pi = 0.00628$ rad. Then this equation predicts a change in the output signal of size

$$\delta S = (8 \text{ Volts})\,(0.00628 \text{ rad}) = 0.050 \text{ Volts} = 50.\text{ mV}.$$

Whether this sort of 'one milli-fringe' change is detectable depends on the noise level of the interferometer's output signal; but some attention to vibration control, and the use of the draft shield and the fully-overlapped beam mode of the interferometer, ought to give you a noise level of only a few mV. Hence you might be in a position to detect a one milli-fringe signal with a signal-to-noise ratio better than ten!