5.4.3 Ejercicios

Calcular las siguientes integrales utilizando los métodos vistos.

1. $\int (3x^4+5x^2+8)^4(12x^3+10x)\,dx$

2. $\int x\cos(x)\,dx$

3. $\int x^3\ln(x)\,dx$

4. $\int \cos(5x)\,5\,dx$

5. $\int \frac{2+e^x}{e^x+2x}\,dx$

1) POR SUSTITUCIÓN

$u = 3x^4+5x^2+8$

$du = 12^3+10x\ dx$

$\therefore 1 = \int u^4 du = \frac{u^5}{5} + C =$

$\therefore \frac{(3x^4+5x^2+8)^5}{5} + C =$

6. $\int x\sqrt{x-1}\,dx$

7. $\int_0^8 \frac{1}{\sqrt{x+1}}\,dx$

8. $\int_0^{2\pi} x\,sen(x)\,dx$

2. Por Partes

$u = x \quad du = 1\,dx$

$dv = \cos(x)dx \quad v = sen(x)$

$\therefore \int u\,dv = uv - \int v\,du$

$\therefore \int x\cos(x)dx = x\,sen(x) - \int sen(x)\,dx$

$\therefore = x\,sen(x) - (-\cos(x)) + c$

$\therefore = x\,sen(x) + \cos(x) + c$

3. $\int x^3 \ln(x)dx$

Por Partes

$u = x^3 \quad dv = \ln(x)dx$

$du = 3x^2\,dx \quad v = \int \ln(x)dx =$

NO

$\boxed{\int u\,dv = uv - \int v\,du}$

$u = \ln(x) \quad dv = x^3\,dx$

$du = \frac{1}{x}\,dx \quad V = \int x^3\,dx = \frac{x^4}{4} + c$

$\int \ln(x)\,x^3\,dx = \ln(x)\frac{x^4}{4} + c - \int \frac{x^4}{4}\frac{1}{x}\,dx$

$\therefore = \ln(x)\frac{x^4}{4} - \int \frac{x^4}{4x}\,dx + c = \ln(x)\frac{x^4}{4} - \int \frac{x^3}{4}\,dx + c$

$= \ln(x)\frac{x^4}{4} - \frac{1}{4}\frac{x^4}{4} + c = \ln(x)\frac{x^4}{4} - \frac{x^4}{16} + c = \frac{4\ln(x)\,x^4}{16} - \frac{x^4}{16} + c$

$\therefore = \frac{4\ln(x)x^4 - x^4}{16} + c = \boxed{\frac{x^4(4\ln(x)-1)}{16} + c}$

④ $\int \cos(5x)\, 5\, dx$

POR SUSTITUCIÓN

si $f(x) = \cos(x)$

y $g(x) = 5x$

$\therefore \rightarrow$ ④ $= \int f(g(x))\, g'(x)\, dx$ $\quad \therefore u = g(x)$

$\therefore u = 5x$ $\quad$ y $du = g'(x) dx$

y $du = 5dx$

y $\therefore$ ④ $= \int f(u)\, du = \int F'(u)\, du = F(u) + c$

$\therefore \int f(u)\, du = \int \cos(u)\, du = \text{sen}(u) + c =$

$\therefore = \boxed{\text{sen}(5x) + c}$

⑤ $\int \frac{2 + e^x}{e^x + 2x}\, dx = \int \frac{e^x + 2}{e^x + 2x}\, dx$

POR SUSTITUCIÓN

si $f(x) = \frac{1}{x}$ $\quad \therefore$ ⑤ $= \int f(g(x))\, g'(x)\, dx$

y $g(x) = e^x + 2x$ $\quad \therefore \rightarrow u = e^x + 2x$

$\therefore \rightarrow g'(x) = e^x + 2$ $\quad$ y $du = e^x + 2\, dx$

$\therefore$ 5 $= \int f(u)\,du = \int \frac{1}{u}\,du = \ln(|u|) + c$

$\therefore = \boxed{\ln(|e^x + 2x|) + c}$

6 $\int \underbrace{x}\,\underbrace{\sqrt{x-1}}\,dx$ SOLUCIÓN PROF

(~~ni idea como se razonó~~)

SUSTITUCIÓN

$\hookrightarrow$ si $u = x-1$

$du = 1\,dx$ $u = x-1 \therefore u+1 = x-1+1 = x$

si $u = x-1 \therefore \Rightarrow x = u+1$

distributiva

$\therefore = \int \underbrace{(u+1)}_{x}\,\underbrace{u^{1/2}}_{\sqrt{x-1}}\,\underbrace{du}_{dx} = \int (u^{3/2} + u^{1/2})\,du$

$\therefore = \int u^{3/2}\,du + \int u^{1/2}\,du = \therefore \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} + c$

$= \frac{1}{5} 2u^{5/2} + \frac{1}{3} 2u^{3/2} + c$

$\therefore = 2\left(\frac{1}{5}u^{5/2} + \frac{1}{3}u^{3/2}\right) + c = 2\left(\left(\frac{1}{5}(x-1)^{5/2}\right) + \left(\frac{1}{3}(x-1)^{3/2}\right)\right) + c$

$= 2\left(\frac{3}{15}(x-1)^{5/2} + \frac{5}{15}(x-1)^{3/2}\right) + c$

$\therefore = \frac{6}{15}(x-1)^{5/2} + \frac{10}{15}(x-1)^{3/2} + c = \frac{6(x-1)^{5/2} + 10(x-1)^{3/2}}{15} + c$

POR SUSTITUCIÓN

(7) $\int_0^8 \frac{1}{\sqrt{x-1}}\,dx$ $\quad u = x-1 \quad du = 1dx$

$\therefore (7) = \int_0^8 \frac{1}{u^{1/2}}\,du$

$\therefore = \int_0^8 u^{-1/2}\,du$

$= \left(\frac{u^{1/2}}{1/2}\right)\Big|_0^8 = \left(2u^{1/2}\right)\Big|_0^8$

ley barrow

$= \left(2(x-1)^{1/2}\right)\Big|_0^8 = \left(2(8-1)^{1/2}\right) - \left(2(0-1)^{1/2}\right)$

$\therefore = \left(2(7)^{1/2}\right) - \left(2(-1)^{1/2}\right)$

$= 5{,}2915026 + 2 = 7{,}2915026$

(8) $\int_0^{2\pi} x\,\mathrm{sen}(x)\,dx$ POR PARTES

$\int u\,dv = uv - \int v\,du$

si

$u = \mathrm{sen}(x) \quad dv = x\,dx$

$du = \cos(x)\,dx \quad V = \int x\,dx = \frac{x^2}{2}$

$\therefore = \mathrm{sen}(x)\frac{x^2}{2} - \int \frac{x^2}{2}\cos(x)\,dx$

$\therefore = \mathrm{sen}(x)\frac{x^2}{2} -$?? A PROBAR CON LA OTRA FORMA

$$\int_0^{2\pi} x\,sen(x)\,dx$$

$SI\quad u = x \qquad dv = sen(x)dx$

$du = 1dx \qquad V = \int sen(x)dx$

$V = -\cos(x) + c$

PARA $\int x\,sen(x)\,dx$

$\therefore = x(-\cos(x)) + c - \int(-\cos(x))\,dx$

$\therefore = x(-\cos(x)) + sen(x) + c = F(x)$

$F(x)\Big|_0^{2\pi} = (2\pi(-\cos(2\pi)) + sen(2\pi)) - (0.(-\cos(0)) + sen(0)) =$

$\therefore (2\pi(-1) + 0) - (0 + 0) = \boxed{-2\pi}$