MATH500 Big Data and Social Network Visualization

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Some Statistical Tests: t-tests and ANOVA
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Comparing the Underlying Means of Two Groups

Comparing the Underlying Means of More Than Two Groups

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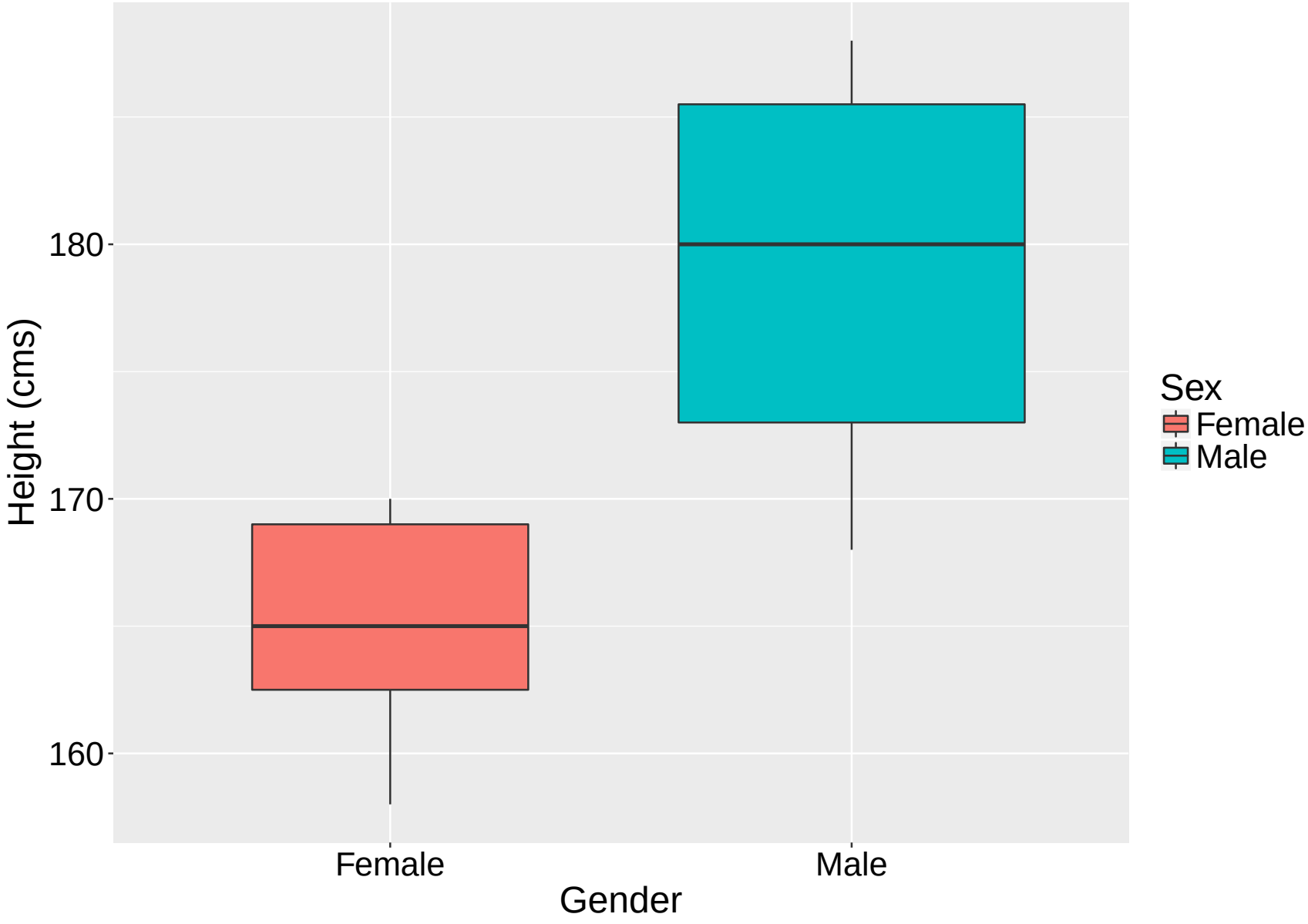


Comparing the Underlying Means of Two Groups, continued

We can ask:

- Is there an underlying difference in mean height between females and males?

This is a **profound question**. We are not asking whether there is a difference between the mean height of females and the mean height of males in the data collected.

We are asking, *more generally*, whether there is a difference between the mean height of females and the mean height of males in a much bigger *population*.

t-test: Underlying Difference in Mean

To answer this question, we can use a *t*-test:

```
t.test(Height ~ Sex, data = qd, var.equal = TRUE)
```

```
#
#   Two Sample t-test
#
# data:  Height by Sex
# t = -4.5075, df = 16, p-value = 0.0003579
# alternative hypothesis: true difference in means is not e
# 95 percent confidence interval:
#  -20.374200  -7.340085
# sample estimates:
# mean in group Female   mean in group Male
#             165.1429             179.0000
```

p-value: Underlying Difference in Mean

We need to focus on the *p*-value

```
# [1] 0.0003579246
```

In general, if the *p*-value is less that 0.05, *there is an underlying difference in mean height between females and males*.

In this case, the *p*-value is considerably less than 0.05. Therefore, we conclude that

- **the data supply evidence that there is an underlying difference in mean height between females and males**.

Obtaining the *p*-value using the `lm` Function

We can obtain the above *p*-value 0.0003579246 in a different way

- using the `lm` function for `linear modelling`:

```
m <- lm(Height ~ Sex, data = qd)
summary(m)
```

```
#                Estimate Std. Error  t value     Pr(>|t|)
# (Intercept) 165.14286   2.403229 68.71707 3.326021e-21
# SexMale      13.85714   3.074221  4.50753 3.579246e-04
```

Comparing the Underlying Means of Two Groups

We can also ask:

- Is the mean height of females less than the mean height of males in a wider population of students?

Again, we are not asking whether the mean height of females is less than the mean height of males in the data collected.

We are asking, *more generally*, whether the mean height of females is less than the mean height of males in a much bigger *population*.

t-test: Is the Mean Height of Females Less Than the Mean Height of Males in the Population?

To answer this question, we can use a t-test, with `alternative = "less"`:

```
t.test(Height ~ Sex, data = qd, var.equal = TRUE,
       alternative = "less")
```

```
#
#   Two Sample t-test
#
# data:  Height by Sex
# t = -4.5075, df = 16, p-value = 0.000179
# alternative hypothesis: true difference in means is less than 0
# 95 percent confidence interval:
#       -Inf -8.489911
# sample estimates:
# mean in group Female   mean in group Male
#             165.1429             179.0000
```

p-value: Is the Mean Height of Females Less Than the Mean Height of Males in the Population?

We need to focus on the *p*-value

```
# [1] 0.0001789623
```

If the *p*-value is less that 0.05, we conclude that

- **the mean height of females is less than the mean height of males in a wider population of students**.

p-value: Is the Mean Height of Females Less Than the Mean Height of Males in the Population?

We need to focus on the *p*-value

```
# [1] 0.0001789623
```

If the *p*-value is less that 0.05, we conclude that

- **the mean height of females is less than the mean height of males in a wider population of students**.
- What do you conclude?

Comparing the Underlying Means of More Than Two Groups

Comparing the Underlying Means of More Than Two Groups: Some Data

Consider these data on the number of hours worked in a week by randomly selected accountants, GPs, lecturers and plumbers:

Accountants	45	38	40	42	48	37	44	40	39	42	41
	40	36	40	48							
GPs	60	57	44	52	57	45	42	56	53	42	44
	54	51	58								
Lecturers	52	45	40	48	36	50	56	42	37	43	47
Plumbers	44	39	50	37	45	39	52	45	39	48	44
	43	53									

Comparing the Underlying Means of More Than Two Groups: Some Data, continued

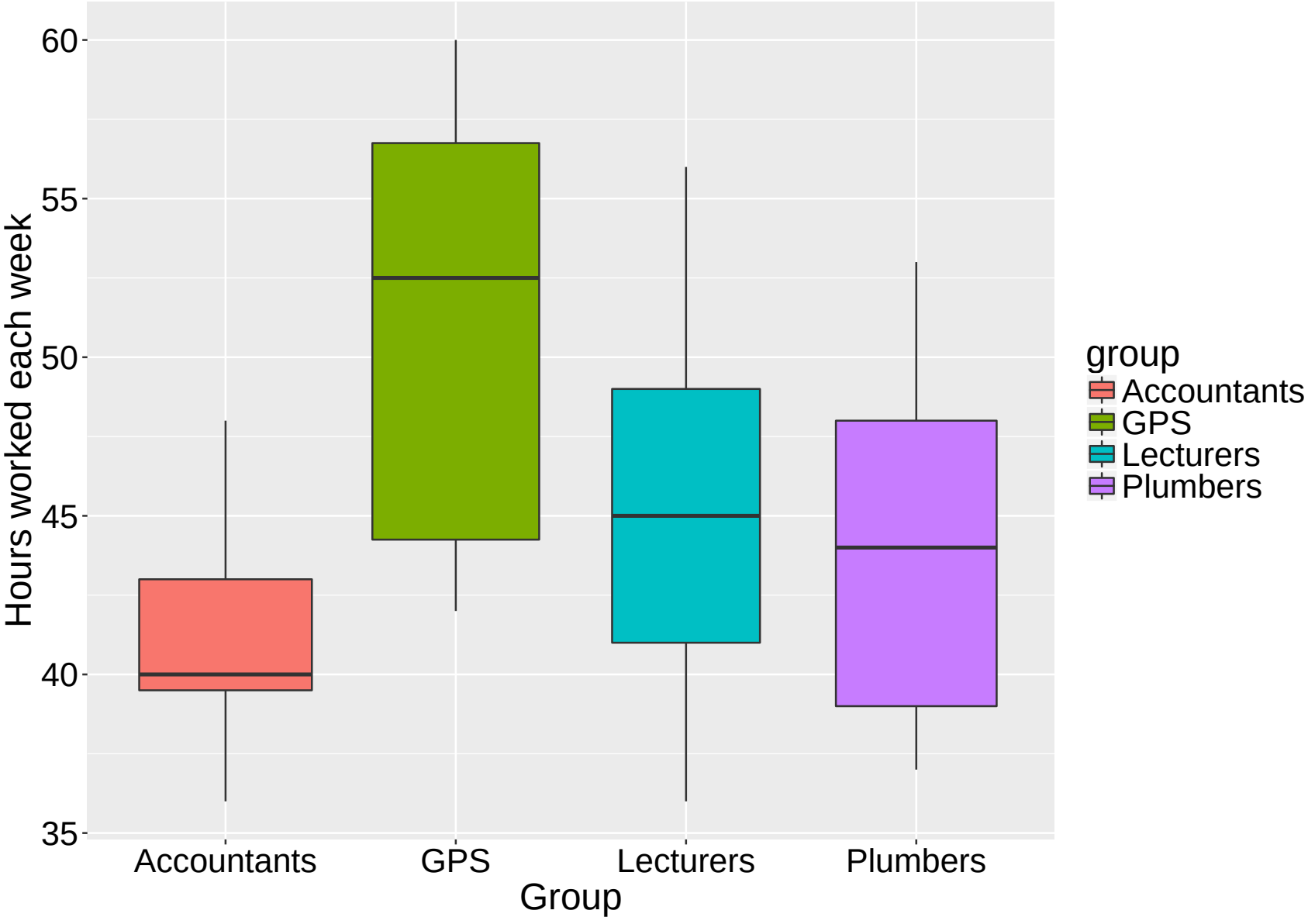
A good way of storing these data in **R** is to use a data frame with one variable for the hours worked and another for the corresponding employment group (accountants, GPs, lecturers or plumbers)

Here are some selected rows:

```
hw_g[c(1, 4, 17, 22, 33, 39, 42, 48), ]
```

```
#    hours_worked       group
# 1            45 Accountants
# 4            42 Accountants
# 17           57         GPS
# 22           42         GPS
# 33           48   Lecturers
# 39           43   Lecturers
# 42           39    Plumbers
# 48           45    Plumbers
```

Comparing the Underlying Means of More Than Two Groups: Plotting the Data



One-way Analysis of Variance

- The plot displays the weekly hours worked by accountants, GPs, lecturers and plumbers.

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One-way Analysis of Variance

- The plot displays the weekly hours worked by accountants, GPs, lecturers and plumbers.
- The question of interest is: Is the underlying mean number of hours different between these **four** groups?
- To answer this question we perform a **one-way analysis of variance**.
- **An**alysis **o**f **Va**riance is often abbreviated **ANOVA**.

One-way Analysis of Variance using the `lm` Function

```r
m <- lm(hours_worked ~ group, data = hw_g)
  # ANOVA is a linear model
anova(m)
```

```
# Analysis of Variance Table
#
# Response: hours_worked
#           Df  Sum Sq Mean Sq F value    Pr(>F)
# group      3  710.73 236.910  8.1156 0.0001731 ***
# Residuals 49 1430.40  29.192
# ---
# Signif. codes:
# 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

p-value

Here, the *p*-value is

```
# [1] 0.0001731113
```

If the *p*-value is less that 0.05, which it is here, we conclude that

- **there is a difference between the underlying mean number of hours worked in these four groups**.

One-way Analysis of Variance using the aov Function

A **One-way Analysis of Variance** can also be performed using the aov function

```
m_2 <- aov(hours_worked ~ group, data = hw_g)
summary(m_2)
```

```
#              Df Sum Sq Mean Sq F value   Pr(>F)
# group         3  710.7  236.91   8.116 0.000173 ***
# Residuals    49 1430.4   29.19
# ---
# Signif. codes:
# 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Follow-up Analysis

If we find that there is an difference between the underlying means of the groups, we should proceed by performing a **follow-up analysis** to see where the group differences are:

```
TukeyHSD(m_2) # Follow-up: pair-wise comparisons
```

```
#   Tukey multiple comparisons of means
#     95% family-wise confidence level
#
# Fit: aov(formula = hours_worked ~ group, data = hw_g)
#
# $group
#                             diff        lwr        upr
# GPS-Accountants         9.7380952   4.398473 15.0777170
# Lecturers-Accountants   3.7575758  -1.946245  9.4613962
# Plumbers-Accountants    3.1282051  -2.316606  8.5730167
# Lecturers-GPS          -5.9805195 -11.769883 -0.1911564
# Plumbers-GPS           -6.6098901 -12.144249 -1.0755310
# Plumbers-Lecturers     -0.6293706  -6.515892  5.2571510
#                             p adj
# GPS-Accountants        0.0000745
# Lecturers-Accountants  0.3086324
```

Follow-up Analysis, continued

```
TukeyHSD(m_2)$group[,4]
```

```
#       GPS-Accountants Lecturers-Accountants
#          0.00007449852          0.30863235604
#  Plumbers-Accountants          Lecturers-GPS
#          0.42899202976          0.04044799824
#           Plumbers-GPS    Plumbers-Lecturers
#          0.01332125052          0.99188174801
```

- We should look at the p-values in the `p adj` column.

Follow-up Analysis, continued

```
TukeyHSD(m_2)$group[,4]
```

```
#       GPS-Accountants Lecturers-Accountants
#          0.00007449852          0.30863235604
#  Plumbers-Accountants          Lecturers-GPS
#          0.42899202976          0.04044799824
#           Plumbers-GPS    Plumbers-Lecturers
#          0.01332125052          0.99188174801
```

- We should look at the p-values in the `p adj` column.
- When the p-value is less that 0.05, we should conclude that there is an underlying difference.

Follow-up Analysis, continued

```
TukeyHSD(m_2)$group[,4]
```

```
#       GPS-Accountants Lecturers-Accountants
#          0.00007449852          0.30863235604
#  Plumbers-Accountants          Lecturers-GPS
#          0.42899202976          0.04044799824
#           Plumbers-GPS    Plumbers-Lecturers
#          0.01332125052          0.99188174801
```

- We should look at the p-values in the `p adj` column.
- When the p-value is less that 0.05, we should conclude that there is an underlying difference.
- So here there are differences between GPs and accountants, between lecturers and GPs and between plumbers and GPs. GPs work a lot!!!