Dear Editor and Referee,

We would like to thank the referee for their careful and insightful feedback. We have updated the manuscript considering their questions and comments and have included our responses below. We have also included a copy of the revised manuscript with line numbers for reference.

General comments:

“A point that might be discussed is the impact of the crystals’ energy measurement saturation on the algorithm performance. I assume the G4 simulation used is not including that effect, i.e. it’s assuming that the dynamic range of the energy measurement fully covers the possible muon dE/dx range of values ? At least from the text that seems to be the case (maybe this point could be clarified a bit better).

But in order to better evaluate the possible impact on the CUORE tracking, given that that detector is used as a benchmark, an estimate of the performance expected with the energy saturating at ˜20 MeV (or at values realistic for that experiment, that I gather from the text are around 20 MeV) might be relevant. The impact of the energy measurement resolution could also be discussed.”

In addition to increasing the statistics used in the simulations, we include two more sets of simulations implementing finite energy resolution and saturation effects to study how they affect performance. Descriptions of the additional simulations may be found at line 255-270, with the (rather minimal) impact of such instrumental effects discussed at lines 306-317, and presented in Table 1 and Figure 5d.

“A couple more details on the LSQ algorithm used for comparison might also be added.”

This has been provided in Eq. 3.1 and text at lines 125-135.

The comments on Section 3:

“- shouldn’t the sign of lambda_2 in the objective function f_2 be positive and not -0.2 mmˆ-1 as in the text ? In fact the function should give penalties close to 1 to trajectories that cross "unlit" crystals.”

This is correct, the typo in the sign of the parameter has been fixed.

“- the statement that more information is used in MO optimization is indeed correct, but it’s maybe not correct to say that a LSQ fitting would not use the geometrical information ? It depends of course on how a LSQ would be implemented, but the geometrical information will be the basis of any implementation I would say. The statement is instead of course fully correct when referred to the to the dE/dX information.”

This wording around line 215-223 has been edited for clarification on the specific additional information used by the MOO algorithm.

“- is the PDF used for the dE/dx taking into account the detector response ? In particular the saturation at about 20 MeV mentioned to some extent later ? Are the effects of the energy measurement resolution negligible ?”

The dE/dx PDF does not consider detector response. Saturated channels are however excluded from the third objective function on account of them providing unreliable energy estimates. Text has been added at line 192 to explain this better. Largely, fluctuations in physical energy deposition are dominant to those

from detector response. The “Saturation” and “Finite Resolution” simulations show that detector effects have a small but non-zero impact.

The comments on Section 4:

“- A few more details on the "naive" LSQ algorithm that is used as a comparison could be given. For example, the exact chi-square function definition and the uncertainties associated to the track position measured in each crystal. Also, whether some fit optimization step has been adopted, or whether the chi-square or chi-square probability obtained are consistent with the expectation for a LSQ fit result, given the conditions ?”

This is a very interesting question, but has a somewhat complicated answer. We perform an unweighted LSQ fit that effectively minimizes the right-hand side of Equation 3.1.
It is actually implemented using a PCA algorithm which finds the direction of maximum variance about the mean position of the hit crystals. These two algorithms can be shown to be equivalent. We have included text to this effect around line 134.

We do not use an energy-weighted chi^2, though this certainly an interesting proposal. Our goal in this paper is not to demonstrate that MOO is categorically better than any possible linear LSQ algorithm. Rather, we use the naive LSQ algorithm as a simple benchmark for comparison. Its simple form makes obvious some of the shortcomings of LSQ approach and the way that MOO avoids them. We do feel that MOO would outperform almost any linear LSQ algorithm, but we also feel that finding an optimal LSQ algorithm to back this claim is beyond the scope of this paper.

To compare the resulting chi^2 to an expected distribution would require re-scaling by an appropriate set of errors or uncertainties. Calculating the appropriate error for the cubic crystal geometries considered is not obvious, and in fact is a further motivation to move to the MOO fit which avoids this problem all together. For this reason, we cannot extract meaningful uncertainties on the resulting track — we hope for this to be the subject of future work in the MOO framework. Further, it is also the case that collapsing crystals to their center positions creates biases that makes the expected distribution of chi^2 values very difficult to predict. For example, if a muon grazes the corners of only two crystals, the MOO algorithm will be able to understand that avoiding other crystals and matching the deposited energy will require careful placement of the reconstructed track, while the LSQ algorithm will draw a line directly through the two crystal centers and return a chi^2 = 0. In fact, there are many situations (with two or more intersected crystals) where the LSQ will return a chi^2=0, but be very far from the true track. Fundamentally, this is because the assumptions of the LSQ algorithm are not justified.

“- Not sure how the improvement factors 1.7 and 4.4 in the MSE are indicated in Figure 5: the figure only shows the reconstructed vs generated correlation plots for the phi and theta angles obtained with the MOO, so no direct comparison with the same plots obtained with the LSQ algorithm. Maybe the LSQ plots corresponding to 5(a) and 5(b) can be added ? Or, the text can be rephrased, stating that this improvement is observed but removing the reference to figure 5 where the improvement can’t be appreciated directly ? It can of course be indirectly appreciated from the comparison of the angular errors in figure 5(d)”

We have removed the reference to Figure 5. As the focus of the paper is on the MOO algorithm, we did not feel it was worthwhile to show too many figures on the LSQ algorithm. However, we have added Table 1, which provides some summary values on reconstruction performance.

“- Figure 6: there seems to be a sharp change in the LSQ performance in reconstructing the track length, when this goes below ˜50 mm. Could this effect be briefly explained ?”

This corresponds to the 50 mm side-length of the crystals for the CUORE geometry considered, we provide some additional discussion in lines 300-305. For true tracks passing close to the crystal center, the LSQ algorithm often does do a good job of reconstructing it, but this is only possible for pathlengths between 50 mm to sqrt(3)*50 mm.

“- As mentioned before, a comparison of the MOO vs LSQ performance could be added, including a realistic response of the energy deposit measurement, i.e. with the saturation at ˜20 MeV as in the real CUORE detector.”

Addressed in previous comments.

Other minor editorial comments:

“- the plots in Figure 4, 5(d) and 7, use colors or symbols that are not distinguishable when printed in black and white. Would it be possible to re-do the plots for example choosing different colors and / or line styles, and different marker styles ?”

Implemented as requested.

“- the "f" in Pareto Front is sometimes capital, sometimes not, maybe it’s better to choose a unique way (lowercase might be better ?)”

Implemented as requested.

Thank you very much for the consideration of our manuscript, and many thanks again to our referee for their time and attention.

Sincerely,

Julian Yocum, Daniel Mayer, Jonathan Ouellet and Lindley Winslow