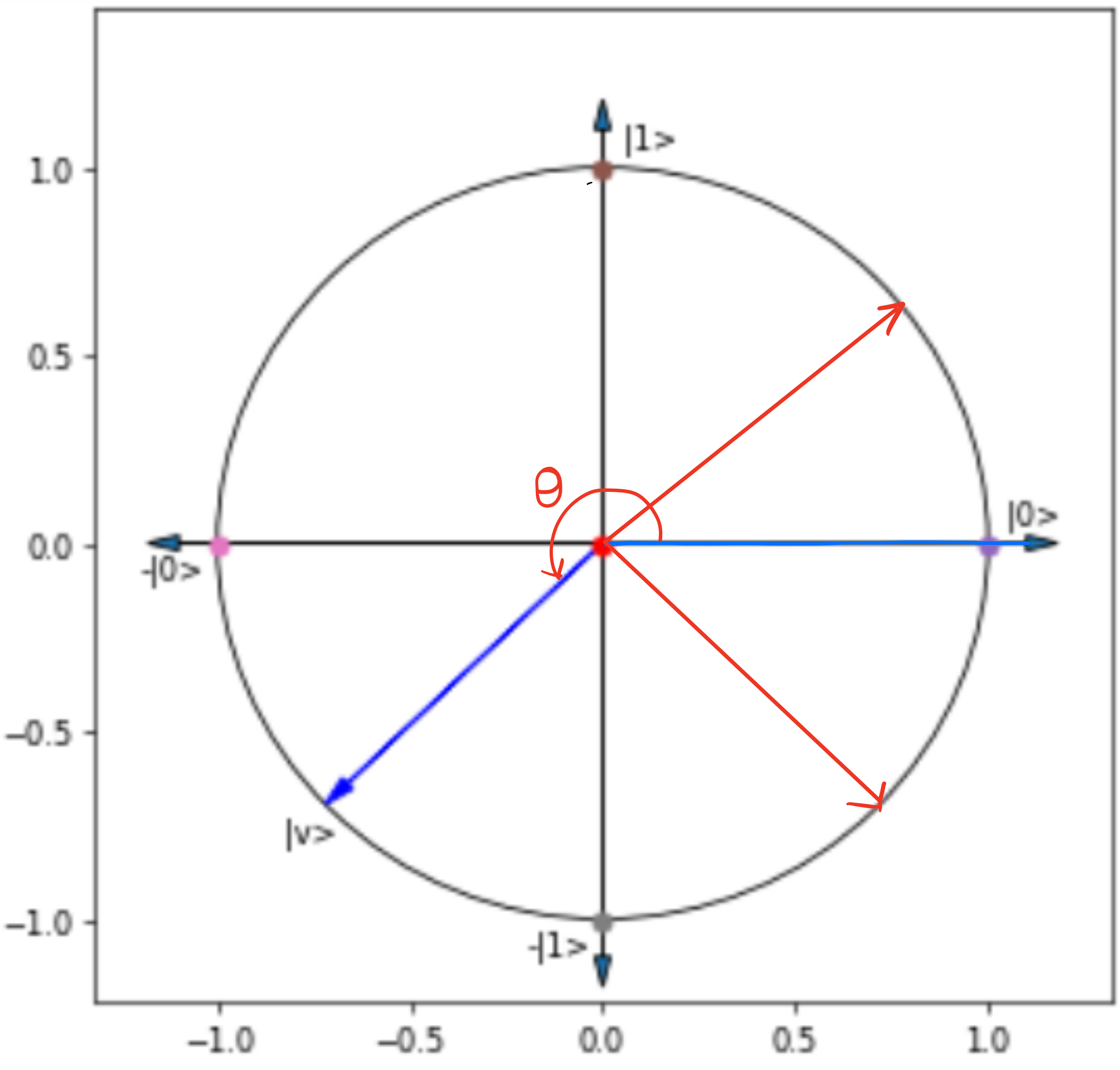
Qubit on a Unit Circle



QBronze Summary

n-qubit Quantum State $|\psi\rangle = \sum_{i=0}^{2^n-1} \alpha_i |i\rangle$, $\sum_{i=0}^{2^n-1} \alpha_i^2 = 1$, $\alpha_i \in \mathbb{R}$

Unitary Evolution $U|\psi\rangle = |\varphi\rangle = \sum_{i=0}^{2^n-1} \beta_i |i\rangle$, $\sum_{i=0}^{2^n-1} \beta_i^2 = 1$, $\beta_i \in \mathbb{R}$ $U^T U = \mathbb{1}$

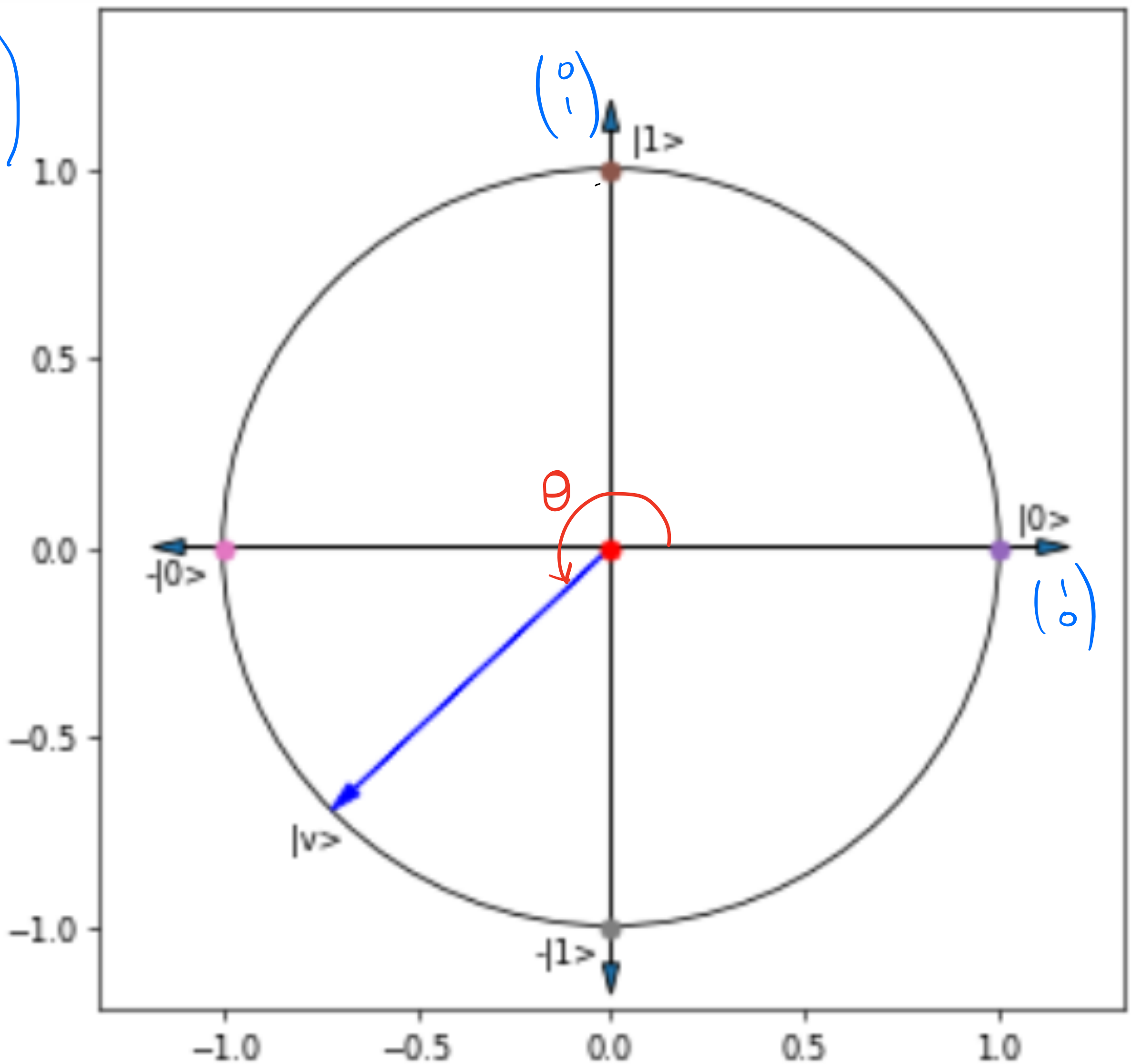
Measurement Probability to observe particular outcome i on measuring $|\psi\rangle$ is given by α_i^2

Qubit on a Unit Circle

$|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$X|0\rangle = |1\rangle$

$X|1\rangle = |0\rangle$

$X|\psi\rangle = \cos\theta(X|0\rangle) + \sin\theta(X|1\rangle)$

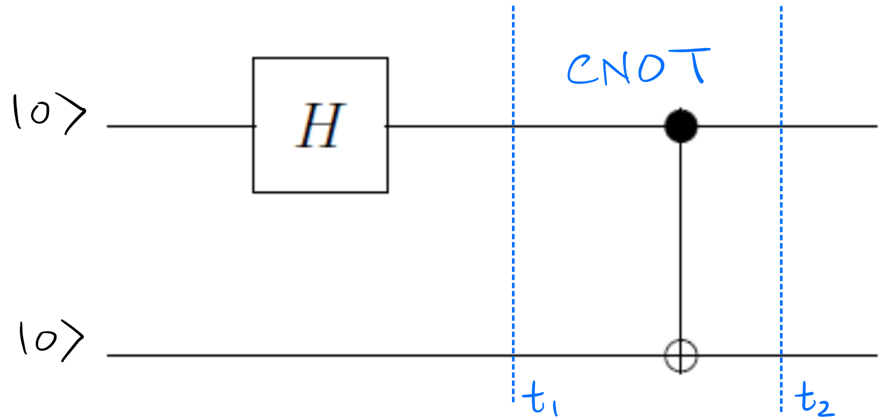
$= \cos\theta|1\rangle + \sin\theta|0\rangle$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$

Preparing a Bell State



$$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$\text{CNOT} \begin{cases} |00\rangle = |00\rangle \\ |01\rangle = |01\rangle \\ |10\rangle = |11\rangle \\ |11\rangle = |10\rangle \end{cases}$$

$$|0\rangle \otimes |0\rangle = |00\rangle \xrightarrow{H \otimes \mathbb{1}} (H \otimes \mathbb{1})|00\rangle = (H|0\rangle) \otimes (\mathbb{1}|0\rangle)$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \otimes |0\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$$

$$\xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}\left(\underbrace{|00\rangle}_{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}} + \underbrace{|11\rangle}_{\begin{pmatrix}0\\0\\0\\1\end{pmatrix}}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

$$\underbrace{\begin{matrix}|00\rangle\\|01\rangle\\|10\rangle\\|11\rangle\end{matrix}}_{\text{Comp. Basis 2 qubits}}$$

Maximally entangled Bell states

$$\underbrace{\begin{matrix}|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right) \\ |\varphi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)\end{matrix}}_{\text{Bell basis}}$$

$$\frac{1}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

Entanglement vs Perfect Correlation

$$\hat{V} = \frac{1}{2}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

perfect
classical
correlation

$$|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

Qtm
Correlation

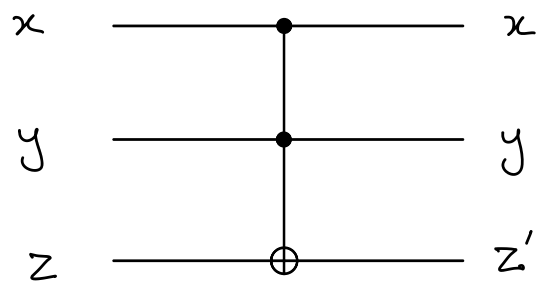
Entanglement

Toffoli Gate

$$\left.\begin{array}{l} |000\rangle \\ |001\rangle \\ |010\rangle \\ |011\rangle \\ |100\rangle \\ |101\rangle \end{array}\right\} \text{identity}$$

$|110\rangle \rightarrow |111\rangle$

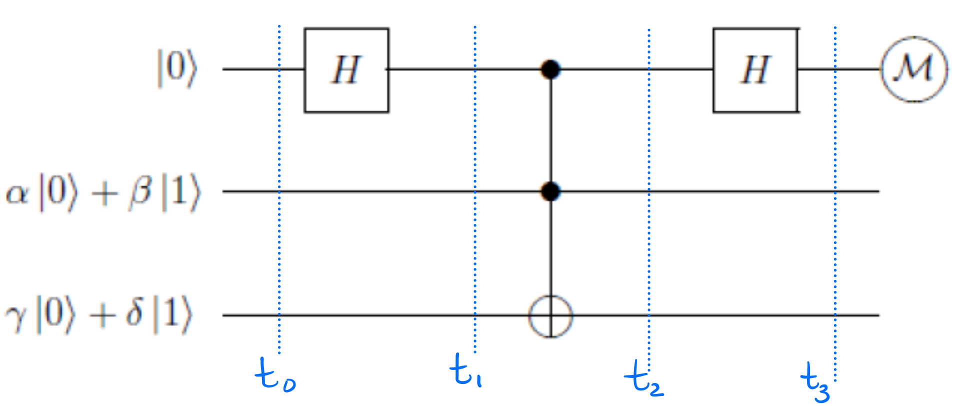
$|111\rangle \rightarrow |110\rangle$



CCNOT

Toffoli

Circuit Evaluation



$$|0\rangle(\alpha|0\rangle+\beta|1\rangle)(\gamma|0\rangle+\delta|1\rangle)$$

$$\xrightarrow{H\otimes 1\otimes 1} |+\rangle(\alpha|0\rangle+\beta|1\rangle)(\gamma|0\rangle+\delta|1\rangle)$$

$$= \frac{1}{\sqrt{2}}(\alpha|00\rangle+\beta|01\rangle+\alpha|10\rangle)(\gamma|0\rangle+\delta|1\rangle) + \frac{\beta}{\sqrt{2}}|11\rangle(\gamma|0\rangle+\delta|1\rangle)$$

$$\xrightarrow{CCNOT} \frac{1}{\sqrt{2}}(\alpha|00\rangle+\beta|01\rangle+\alpha|10\rangle)(\gamma|0\rangle+\delta|1\rangle) + \frac{\beta}{\sqrt{2}}|11\rangle(\gamma|1\rangle+\delta|0\rangle)$$

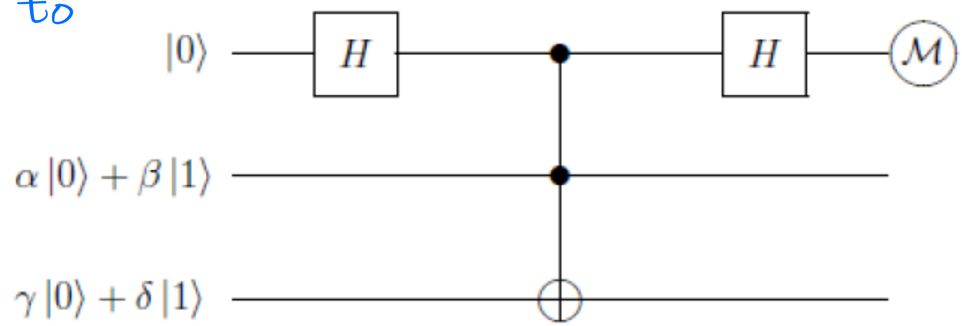
$$\xrightarrow{H\otimes 1\otimes 1} \frac{1}{2}(\alpha|00\rangle+\alpha|10\rangle+\beta|01\rangle+\beta|11\rangle+\alpha|00\rangle-\alpha|10\rangle)(\gamma|0\rangle+\delta|1\rangle) + \frac{1}{2}(\beta|01\rangle-\beta|11\rangle)(\gamma|1\rangle+\delta|0\rangle)$$

$$= \frac{1}{2}\left[2\alpha\gamma|000\rangle+2\alpha\delta|001\rangle+\beta(\gamma+\delta)|010\rangle+\beta(\gamma+\delta)|011\rangle+\beta(\gamma-\delta)|110\rangle+\beta(\delta-\gamma)|111\rangle\right]$$

Circuit Evaluation

What is probability for first qubit to be in state $|1\rangle$?

$$\frac{1}{2}\left(\beta^2\left(\delta-\gamma\right)^2\right)$$



Given that first qubit is measured in state $|1\rangle$, what is the probability distribution for second qubit?

with prob. 1, 2^{nd} qubit is in state $|1\rangle$.

Does there exist a choice for α, β, γ & δ for which first qubit is measured in state $|1\rangle$ with probability 1?

$$\alpha=0, \quad \beta=1, \quad \delta=\frac{1}{\sqrt{2}}, \quad \gamma=-\frac{1}{\sqrt{2}}$$

$$\frac{1}{2}\Big[2\alpha\gamma|000\rangle+2\alpha\delta|001\rangle+\beta(\gamma+\delta)|010\rangle+\beta(\gamma+\delta)|011\rangle+\beta(\gamma-\delta)|110\rangle+\beta(\delta-\gamma)|111\rangle\Big]$$