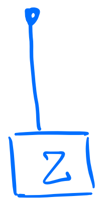
Superdense Coding

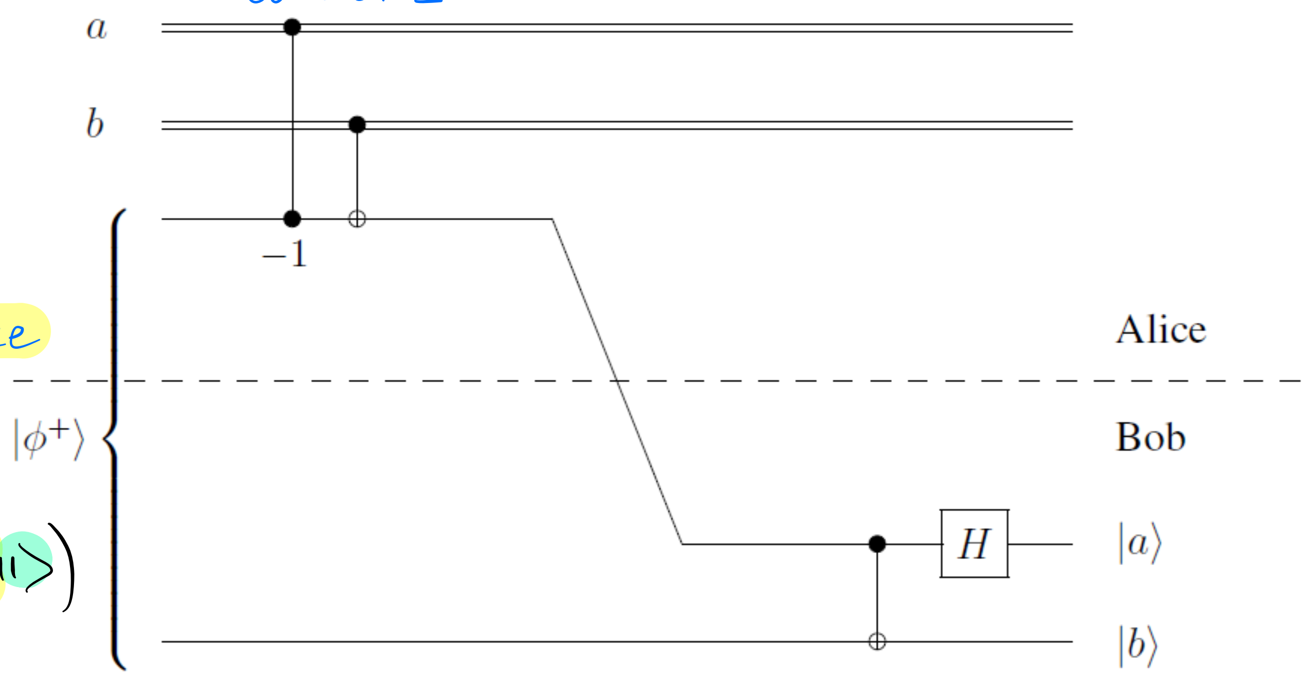
$a, b \in \{0, 1\}$

Holevo's Bound

Ctrl-Z



Control-Z



Alice

Bob

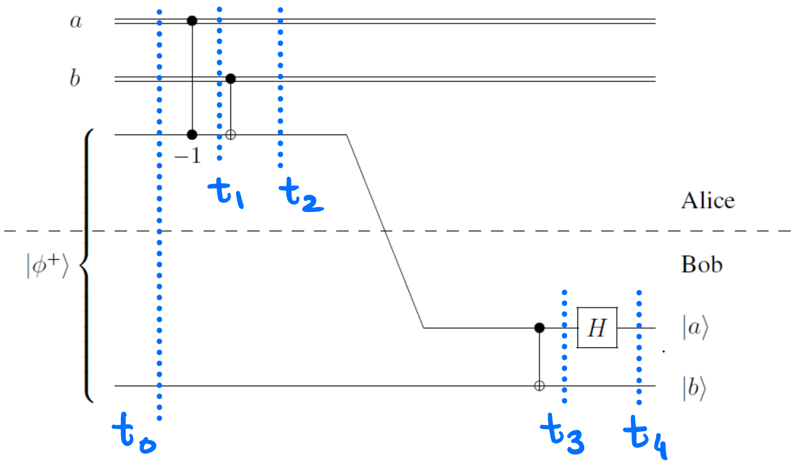
$$|\phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Superdense Coding

$Z|0\rangle = |0\rangle$

$Z|1\rangle = -|1\rangle$

$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$



$H|0\rangle = |+\rangle$

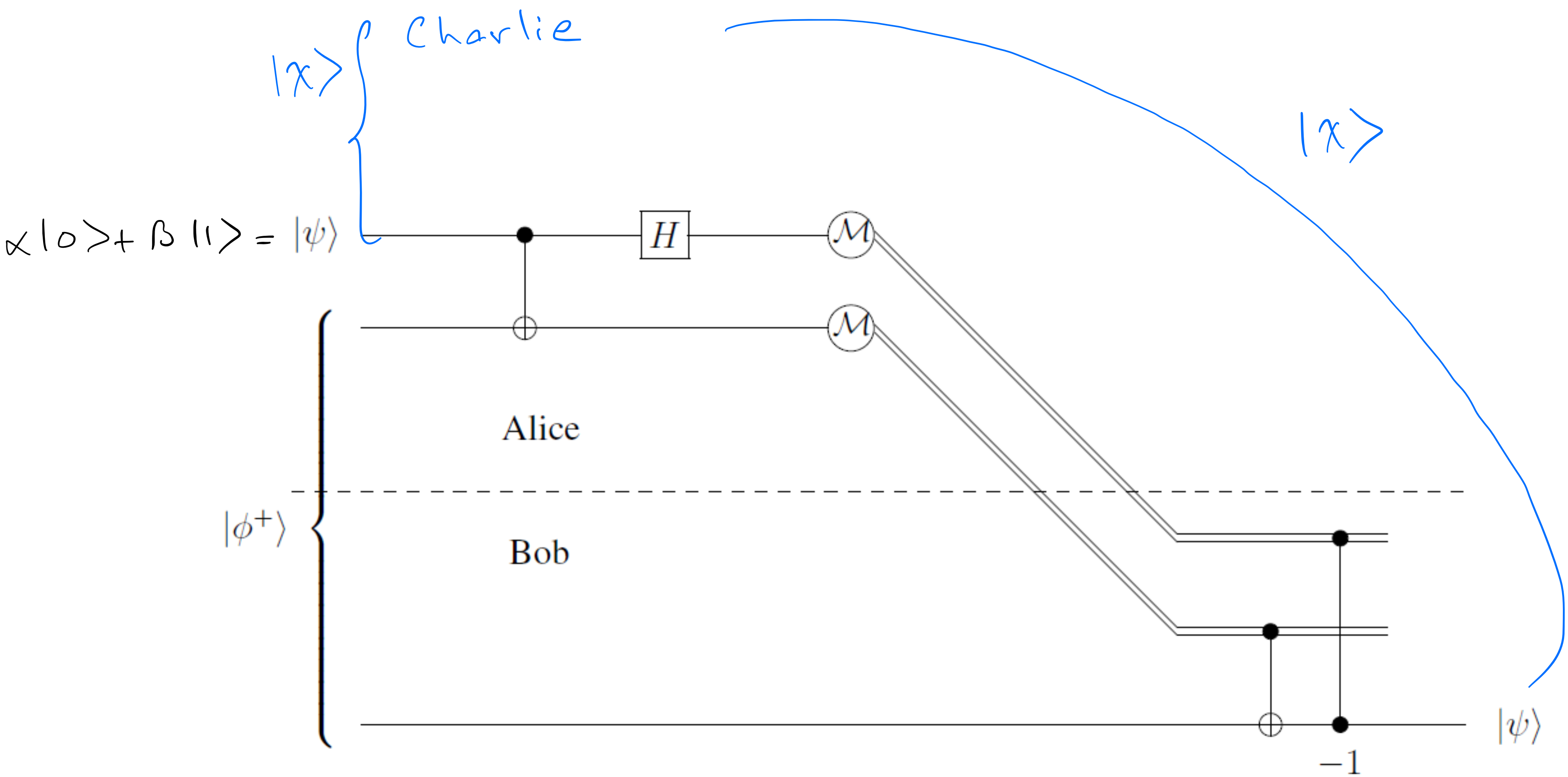
$H|+\rangle = |0\rangle$

$H|1\rangle = |-\rangle$

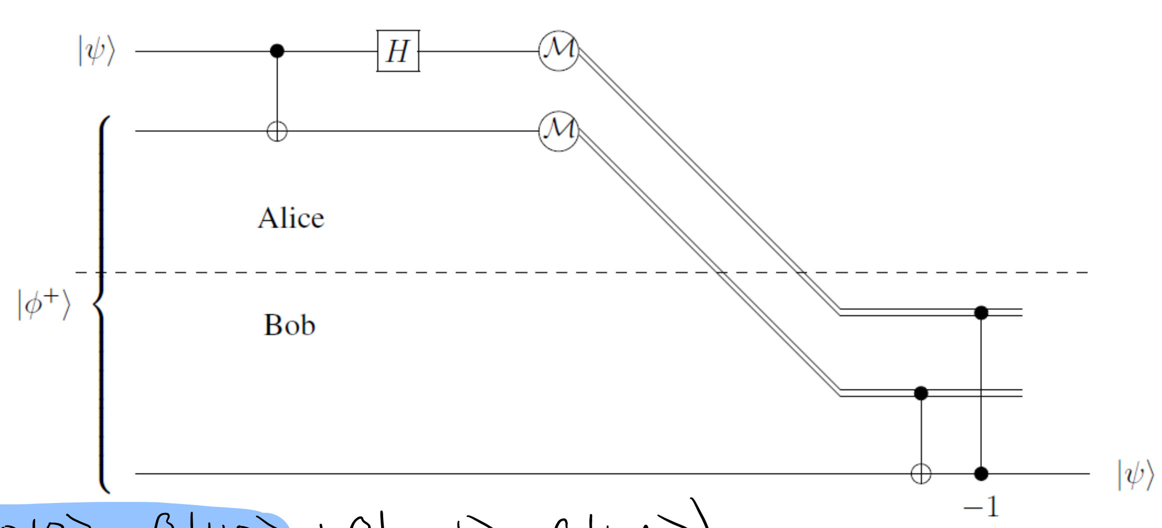
$H|-\rangle = |1\rangle$

ab	State at t_1	State at t_2	State at t_3	State at t_4
00	$\lvert\phi^+\rangle$	$\lvert\phi^+\rangle$	$\frac{1}{\sqrt{2}}(\lvert 00\rangle + \lvert 10\rangle) = \lvert +\rangle\lvert 0\rangle$	$\lvert 00\rangle$
01	$\lvert\phi^+\rangle$	$\frac{1}{\sqrt{2}}(\lvert 10\rangle + \lvert 01\rangle) = \lvert\psi^+\rangle$	$\frac{1}{\sqrt{2}}(\lvert 11\rangle + \lvert 01\rangle) = \lvert +\rangle\lvert 1\rangle$	$\lvert 01\rangle$
10	$\frac{1}{\sqrt{2}}(\lvert 00\rangle - \lvert 11\rangle) = \lvert\phi^-\rangle$	$\lvert\phi^-\rangle$	$\frac{1}{\sqrt{2}}(\lvert 00\rangle - \lvert 10\rangle) = \lvert -\rangle\lvert 0\rangle$	$\lvert 10\rangle$
11	$\lvert\phi^-\rangle$	$\frac{1}{\sqrt{2}}(\lvert 10\rangle - \lvert 01\rangle) = -\lvert\psi^-\rangle$	$\frac{1}{\sqrt{2}}(\lvert 11\rangle - \lvert 01\rangle) = -\lvert -\rangle\lvert 1\rangle$	$-\lvert 11\rangle$

Quantum Teleportation



Quantum Teleportation



$$|\psi\rangle \otimes |\varphi^+\rangle = (\alpha|0\rangle + \beta|1\rangle)\left(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle\right)$$

$$\xrightarrow{CNOT \otimes \mathbb{1}} \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle\right)$$

$$\xrightarrow{H \otimes \mathbb{1} \otimes \mathbb{1}} \frac{1}{2}\left(\alpha|000\rangle + \alpha|100\rangle + \alpha|011\rangle + \alpha|111\rangle + \beta|010\rangle - \beta|110\rangle + \beta|001\rangle - \beta|101\rangle\right)$$

$$= \frac{1}{2}\Big(|00\rangle\underbrace{(\alpha|0\rangle + \beta|1\rangle)}_{|\psi\rangle} + |01\rangle\underbrace{(\alpha|1\rangle + \beta|0\rangle)}_{\begin{aligned}&X(\alpha|1\rangle+\beta|0\rangle)\\&=\alpha|0\rangle+\beta|1\rangle\\&=|\psi\rangle\end{aligned}} + |10\rangle\underbrace{(\alpha|0\rangle - \beta|1\rangle)}_{\begin{aligned}&Z(\alpha|0\rangle-\beta|1\rangle)\\&=\alpha|0\rangle+\beta|1\rangle\\&=|\psi\rangle\end{aligned}} + |11\rangle(\alpha|1\rangle - \beta|0\rangle)\Big)$$

$$ZX(\alpha|1\rangle - \beta|0\rangle)$$
$$= Z(\alpha|0\rangle - \beta|1\rangle)$$
$$= \alpha|0\rangle + \beta|1\rangle$$
$$= |\psi\rangle$$

Comparison