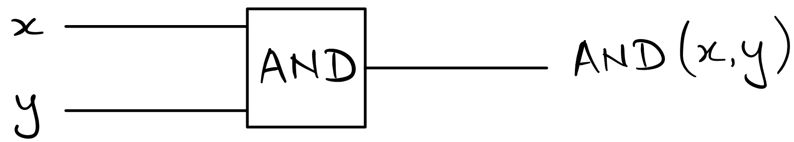
QWORLD

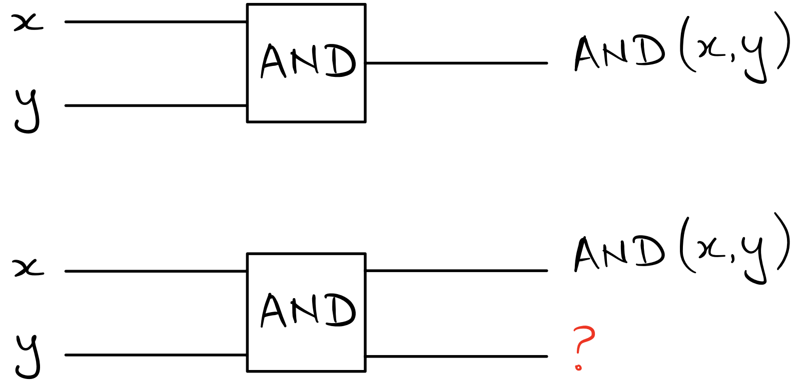
INTRODUCTION TO QUANTUM ALGORITHMS

Jibran Rashid

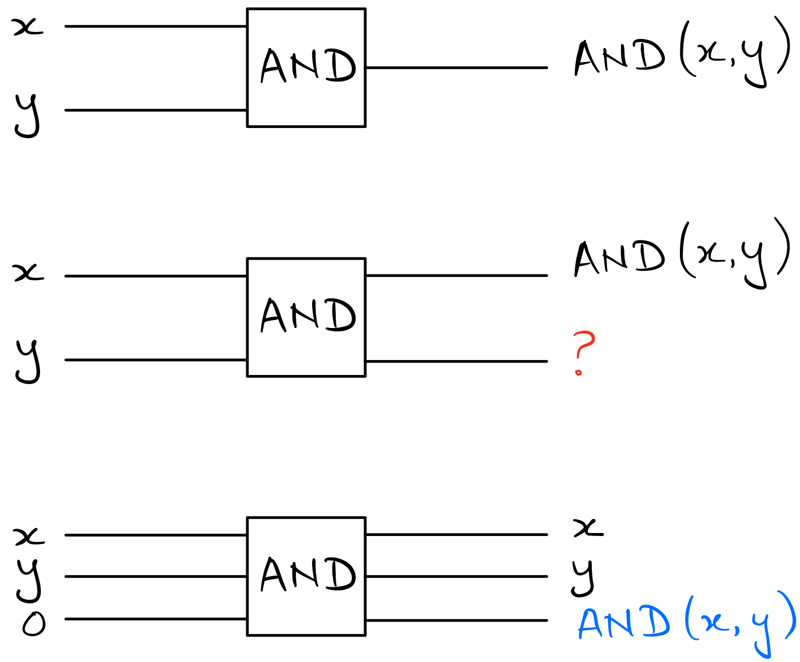
Reversible Transformations



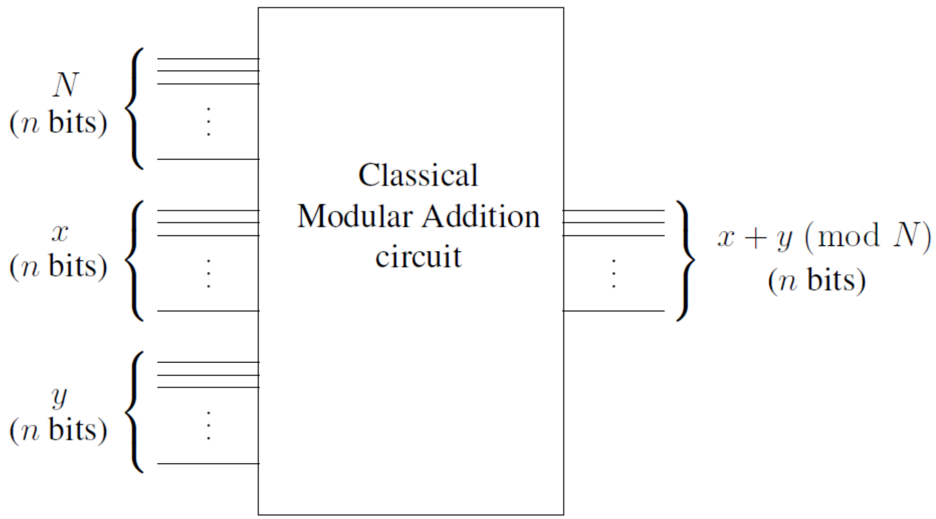
Reversible Transformations

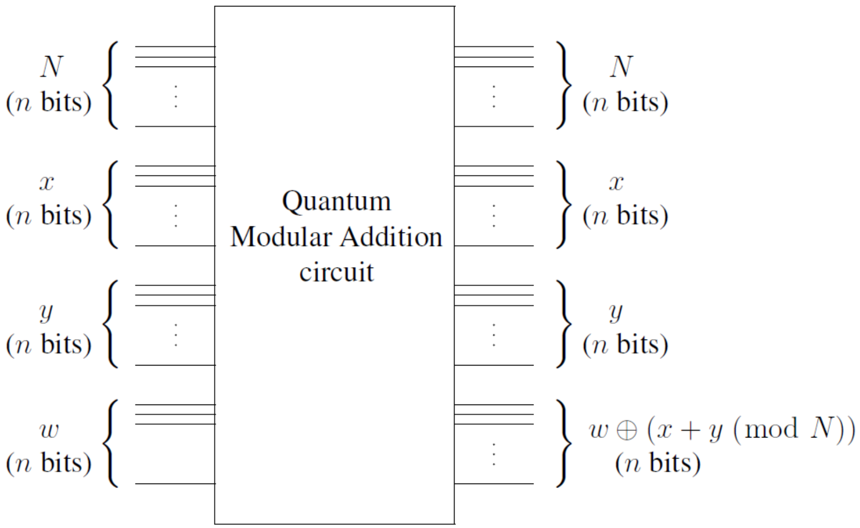


Reversible Transformations



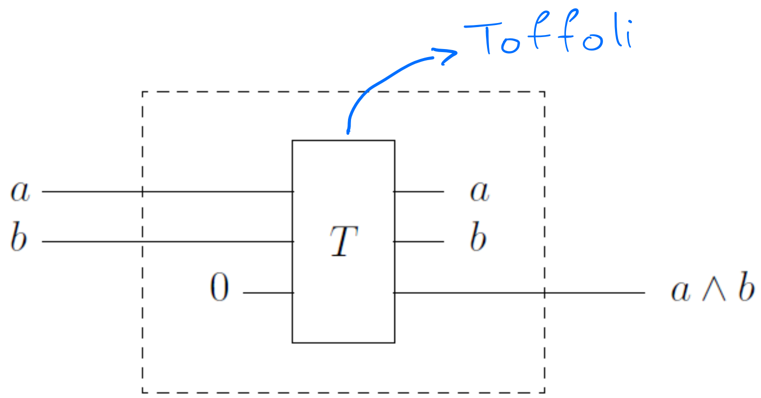
Reversible Computation

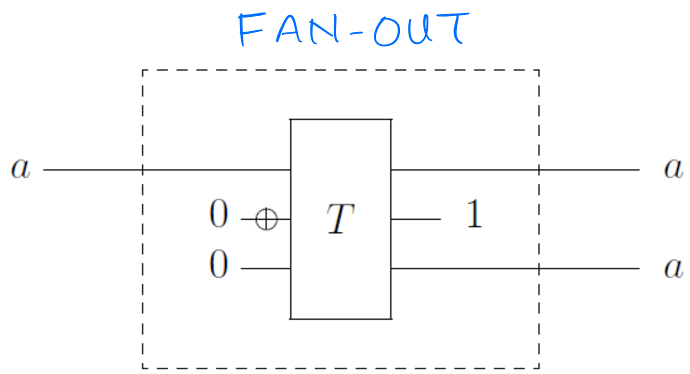




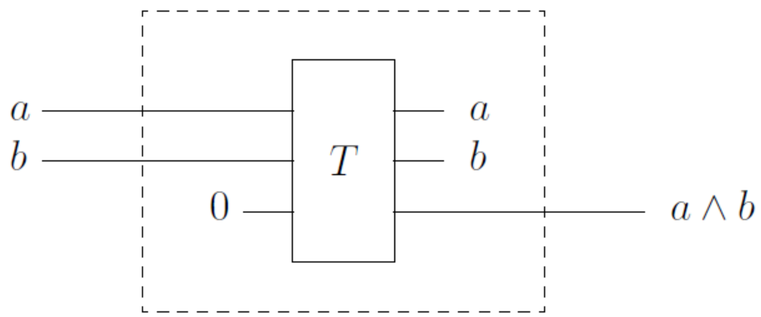
$a \oplus a = 0$

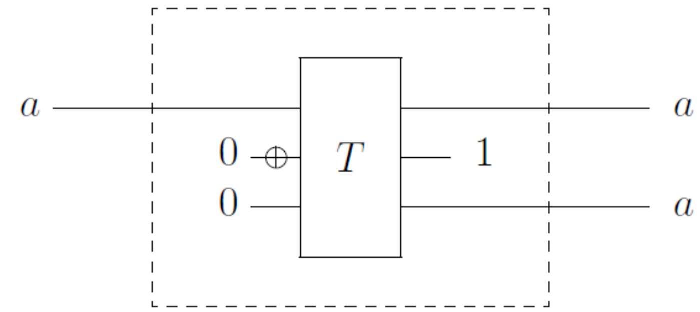
Reversible Computation

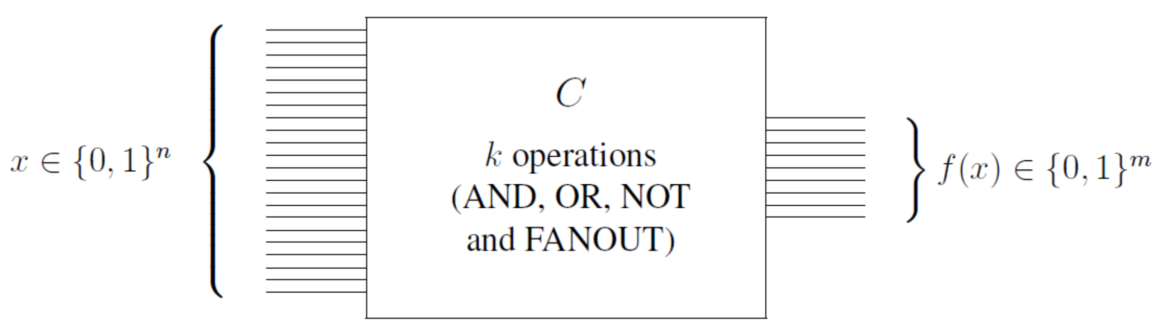




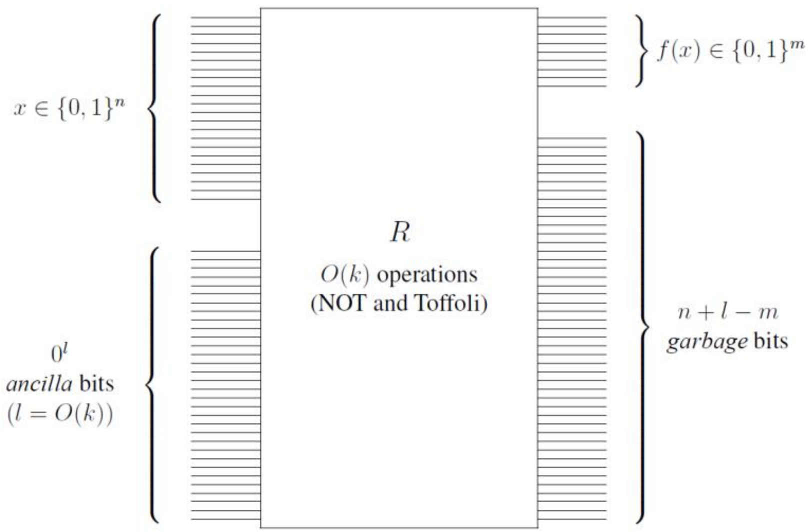
Reversible Computation

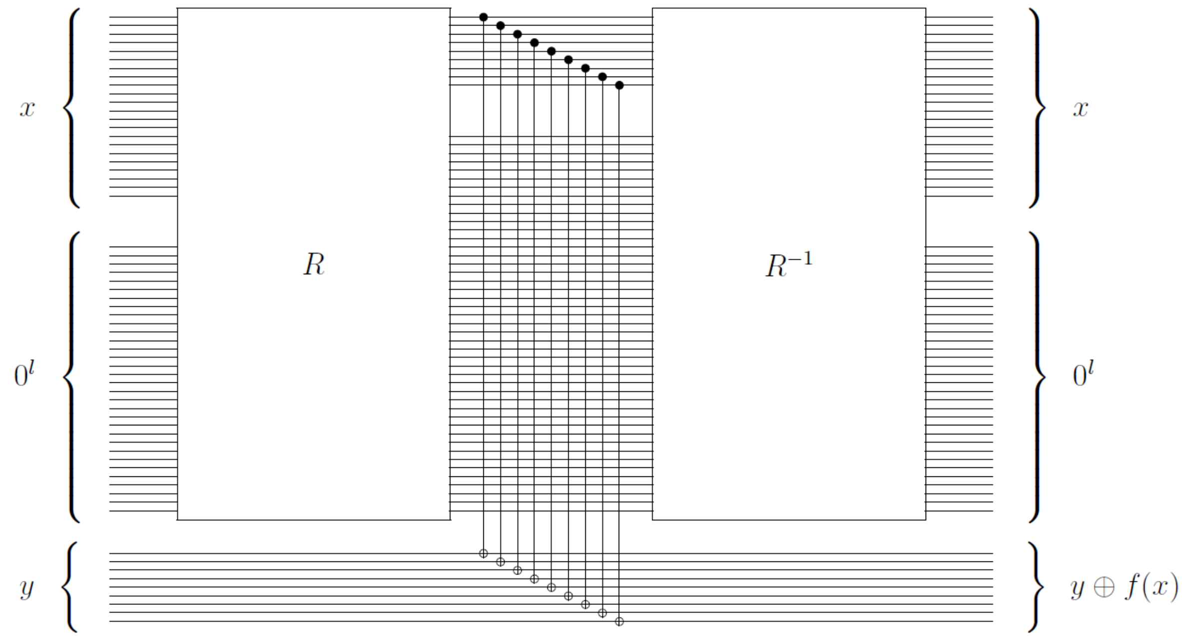




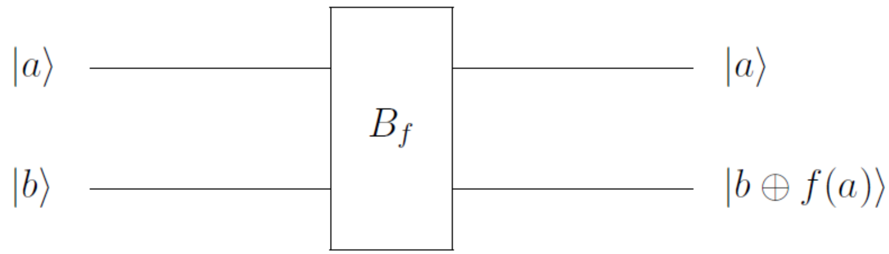


Reversible Computation





Classical Gates Via Unitaries



$a, b \in \{0,1\}$

Oracle

$$B_f : |a\rangle|b\rangle \longrightarrow |a\rangle|b \oplus f(a)\rangle$$

$$f : \{0,1\}^n \longrightarrow \{0,1\}$$

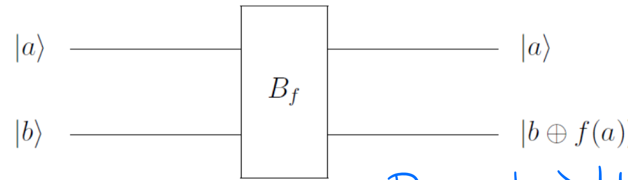
$$f : \{0,1\}^n \longrightarrow \{0,1\}$$

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

Phase Kickback

Recall

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$



$$B_f : |a\rangle|b\rangle \longrightarrow |a\rangle\underline{|b \oplus f(a)\rangle}$$

Consider $B_f|a\rangle|-\rangle$

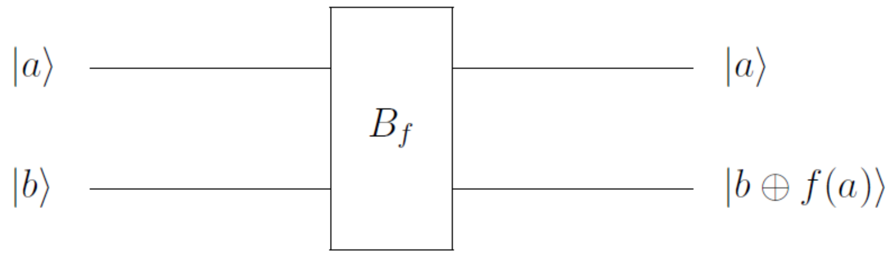
$$= B_f|a\rangle\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \frac{1}{\sqrt{2}}\left(B_f|a\rangle|0\rangle - B_f|a\rangle|1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|a\rangle|0 \oplus f(a)\rangle - |a\rangle|1 \oplus f(a)\rangle\right)$$

$$f(a) = \begin{cases} 0 \longrightarrow |a\rangle\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \\ 1 \longrightarrow |a\rangle\frac{1}{\sqrt{2}}\left(|1\rangle - |0\rangle\right) \end{cases} = (-1)^{f(a)}|a\rangle|-\rangle$$

$$B_f : |a\rangle|-\rangle \longrightarrow (-1)^{f(a)}|a\rangle|-\rangle$$

Classical Gates Via Unitaries



$$B_f : |a\rangle|-\rangle \longrightarrow (-1)^{f(a)} |a\rangle|-\rangle$$

QWORLD

QUANTUM QUERY ALGORITHMS

Jibran Rashid

Hadamard on n Qubits

$H|0\rangle = |+\rangle$ $\quad$ $H|1\rangle = |-\rangle$

$$H \otimes H\,|00\rangle = |+\rangle|+\rangle$$

$$= \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

Simple Search

f_{00}

input	output
00	1
01	0
10	0
11	0

f_{01}

input	output
00	0
01	1
10	0
11	0

f_{10}

input	output
00	0
01	0
10	1
11	0

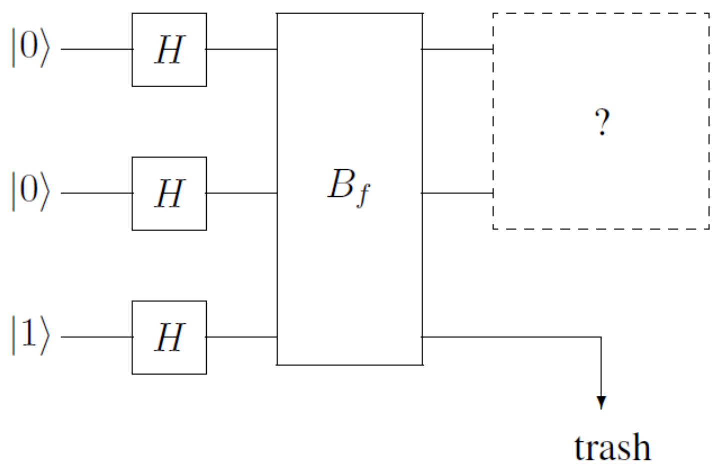
f_{11}

input	output
00	0
01	0
10	0
11	1

$f : \{0,1\}^2 \to \{0,1\}$

2^{2^n}, $n = 2$

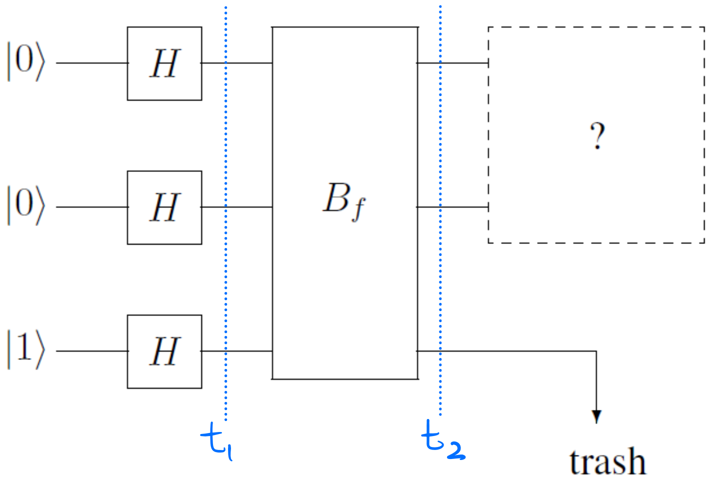
$2^4 = 16$



Simple Search

$$|\psi_1\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Determine $|\psi_2\rangle$ via phase kickback

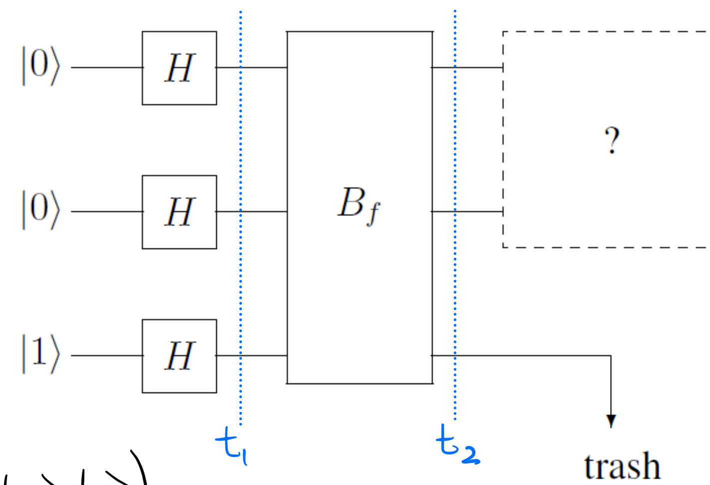


Simple Search

$$|\psi_1\rangle = \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Determine $|\psi_2\rangle$ via phase kickback

$$B_f : |a\rangle|-\rangle \longrightarrow (-1)^{f(a)}|a\rangle|-\rangle$$



$$B_f|\psi_1\rangle = \frac{1}{2}\Big(B_f|00\rangle|-\rangle + B_f|01\rangle|-\rangle + B_f|10\rangle|-\rangle + B_f|11\rangle|-\rangle\Big)$$

$$= \frac{1}{2}\Big((-1)^{f(00)}|00\rangle + (-1)^{f(01)}|01\rangle + (-1)^{f(10)}|10\rangle + (-1)^{f(11)}|11\rangle\Big)|-\rangle$$

Simple Search

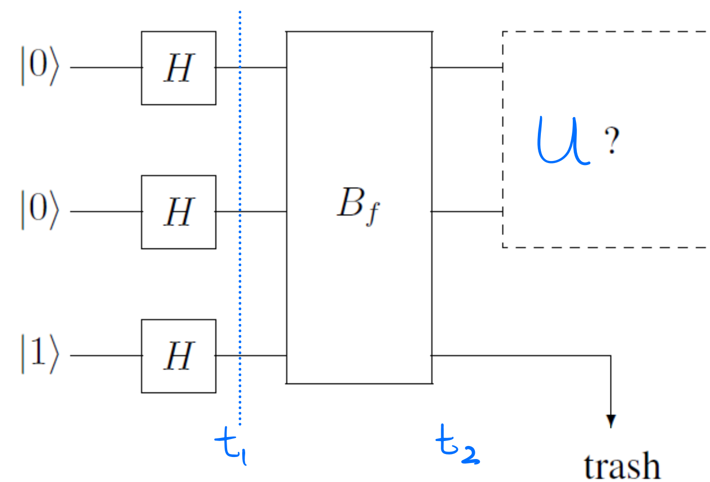
$$\frac{1}{2}\left((-1)^{f(00)}|00\rangle + (-1)^{f(01)}|01\rangle + (-1)^{f(10)}|10\rangle + (-1)^{f(11)}|11\rangle \right)$$

$$f = f_{00} \Rightarrow \frac{1}{2}\left(-|00\rangle + |01\rangle + |10\rangle + |11\rangle \right) \longrightarrow |00\rangle$$

$$f = f_{01} \Rightarrow \frac{1}{2}\left(+|00\rangle - |01\rangle + |10\rangle + |11\rangle \right) \longrightarrow |01\rangle$$

$$f = f_{10} \Rightarrow \frac{1}{2}\left(+|00\rangle + |01\rangle - |10\rangle + |11\rangle \right) \longrightarrow |10\rangle$$

$$f = f_{11} \Rightarrow \frac{1}{2}\left(+|00\rangle + |01\rangle + |10\rangle - |11\rangle \right) \longrightarrow |11\rangle$$



Hadamard on n Qubits

$a \in \{0, 1\}$

$$H|a\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^a |1\rangle\right)$$

$$H\left(H|a\rangle\right) = H\left(\underbrace{\frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^a |1\rangle\right)}_{H\left(H|a\rangle\right) = |a\rangle}\right)$$

Deutsch's Algorithm

$|\psi_0\rangle = |01\rangle$

$|\psi_1\rangle = |+ -\rangle = |+\rangle|-\rangle$

$|\psi_2\rangle = B_f\,|\psi\rangle|-\rangle$

$= \frac{1}{\sqrt{2}}\left(B_f|0\rangle|-\rangle + B_f|1\rangle|-\rangle\right)$

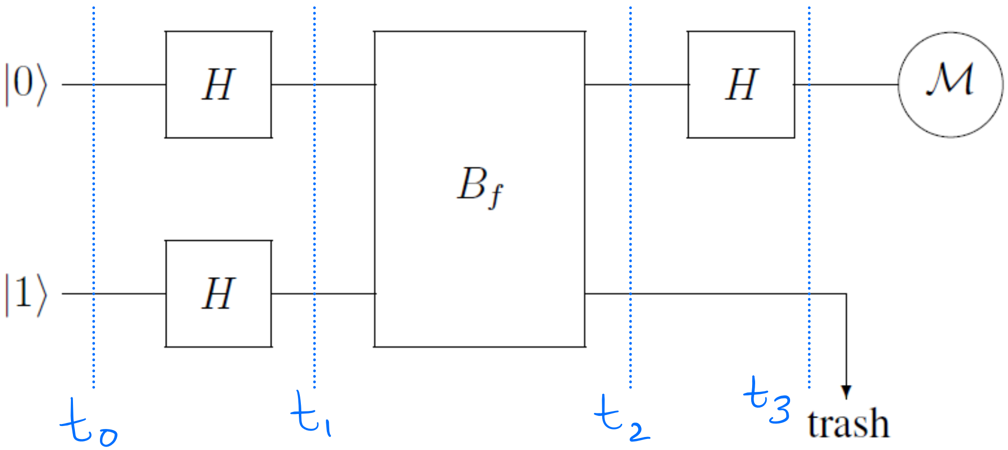
$= \frac{1}{\sqrt{2}}\left((-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right)|-\rangle$

$= \frac{(-1)^{f(0)}}{\sqrt{2}}\left(|0\rangle + (-1)^{f(0)\oplus f(1)}|1\rangle\right)|-\rangle$

verify

$\frac{1}{\sqrt{2}}\left((-1)^{f(0)}|0\rangle + (-1)^{f(0)\oplus f(1)}(-1)^{f(0)}|1\rangle\right)|-\rangle$

$f(0)\oplus f(1)\oplus f(0) = f(1)$



Input	f_0	f_1	f_2	f_3
0	0	0	1	1
1	0	1	0	1

Constant vs. Balanced

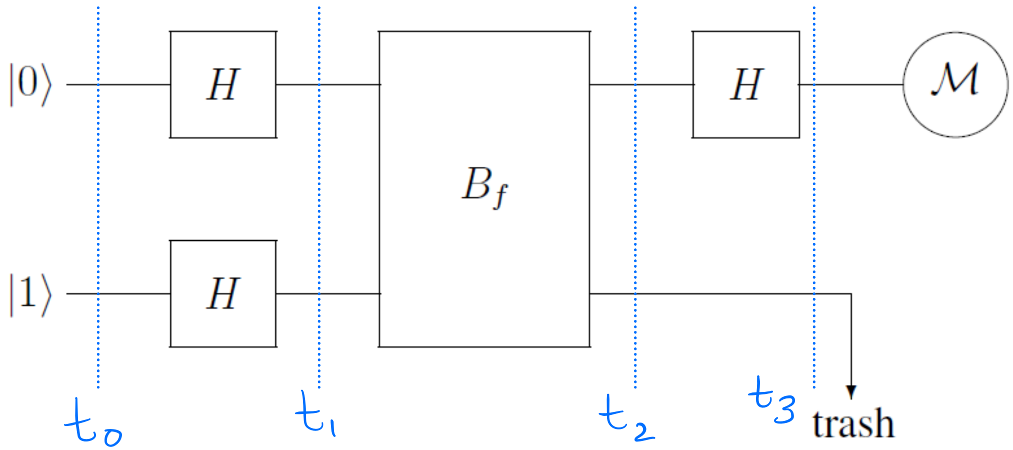
f_0 & f_3 are const.

f_1 & f_2 are balanced

Classically need 2 oracle calls

Deutsch's Algorithm

$$H\left(\frac{1}{\sqrt{2}}\left(|0\rangle + (-1)^a|1\rangle\right)\right) = |a\rangle$$



$$|\psi_2\rangle = \frac{(-1)^{f(0)}}{\sqrt{2}}\left(|0\rangle + (-1)^{f(0)\oplus f(1)}|1\rangle\right)|-\rangle$$

Let $a = f(0) \oplus f(1)$

$$|\psi_2\rangle = \frac{(-1)^{f(0)}}{\sqrt{2}}\left(|0\rangle + (-1)^{a}|1\rangle\right)|-\rangle$$

		f_0	f_1	f_2	f_3
Input	0	0	0	1	1
	1	0	1	0	1

$$|\psi_3\rangle = H \otimes 1\,|\psi_2\rangle = |a\rangle|-\rangle$$

$$= |f(0) \oplus f(1)\rangle|-\rangle$$

Const $\longrightarrow$ $f(0) \oplus f(1) = 0$

Bal $\longrightarrow$ $f(0) \oplus f(1) = 1$