Graded Assignment of Applied Financial Econometrics

1) Stock Choice

The Stock I choose from the Dow Jones Industrial Average (DJIA) index for my research is 3M (Ticker: MMM). The reason I choose this stock is that the research period contains the finance crisis in 2008, however, 3M as a multinational conglomerate corporation operating in the fields of the industry is more resistant against the financial crisis.

2) Log return and close price of 3M during 2004 and 2009

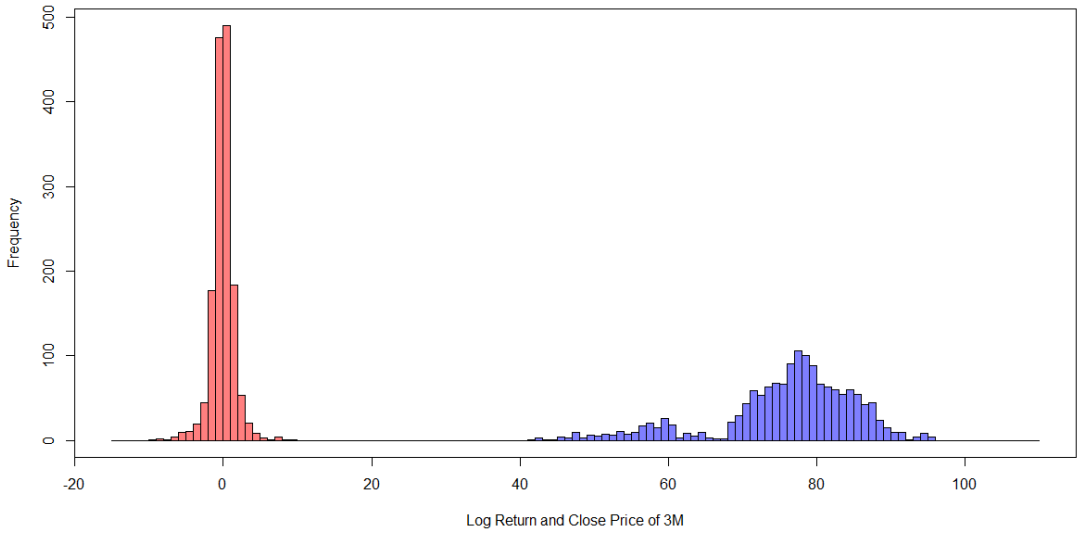

Figure 1: Distribution of Log Return and Close Price of 3M from 2004 to 2009. The blue bins are the close price of 3M in $ and the red bins are the log return of 3M in %.

Figure 1 shows the distribution of 3M of close price and log return from 2004 to 2009. It is clear that the log return of 3M follows more or less a normal distribution, while the close price of 3M has a left-skewed normal distribution or even no clear distribution to recognize. One important stylized fact of the daily log return could be proved in *Figure 1*, that the stock market exhibits occasional very large drops but not equally large up-moves.

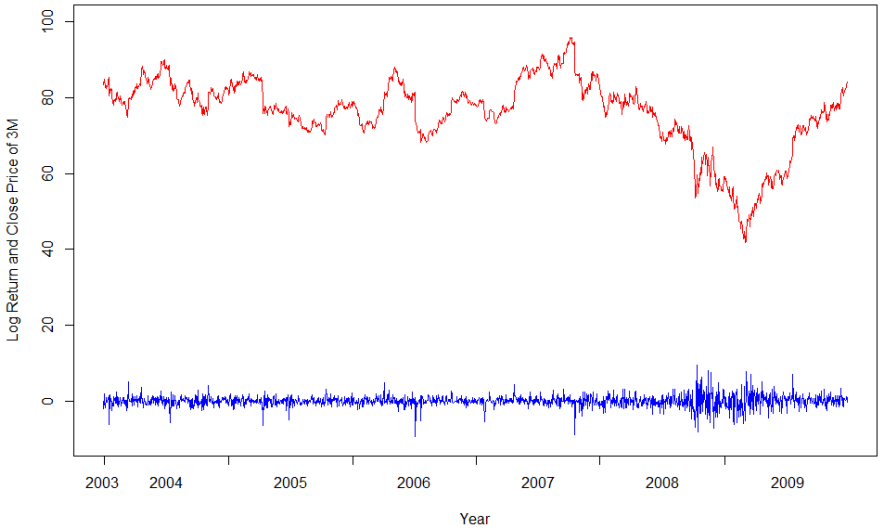

Figure 2: Log Return and Close Price of 3M from 2004 to 2009. The blue curve is the log return of 3M in % and the red curve is the close price of 3M in $.

As shown in *Figure 2*, the close price (red curve) of 3M in the second half of the year 2007 started to enter in a downward channel. Beginning at the middle of 2008 the log return (blue curve) of 3M began to tabulate strongly until the end of 2009 due to the financial crisis in the US, which also accords with the stylized fact that the variances cluster. Besides, the close price of 3M has an eyeballing recognizable random walk behavior (non-stationary), as the movement of the daily close price of 3M is not around its mean. However, the daily log return of 3M, which is more or less stationary over the period from 2004 to 2009. This is in line with the stylized fact that daily returns have very little autocorrelation, it is impossible to predict the return from their own past.

The possible existing of an unit root in close price time series but not in the log return time series determines the behavior of both time series over the sample period. This guess is confirmed by using AR(1) Model and Augmented Dickey-Fuller Test (ADF). For log return, we can reject the existing of an unit root in both AR(1) and ADF tests, while for the close price we cannot reject the H_0 that there is an unit root in the time series leading to the random walk pattern.[1]

3) Summary Statistic and QQ-Plot of Daily Log Return of 3M

Panel A: General Statistic Features of Daily Log Return of 3M from 2004 to 2009

Mean	Standard Deviation	Min.	1st. Quantile	Median	3st. Quantile	Max.	Skewness	Kurtosis
-0.0009%	0.0155	-9.3837%	-0.6722%	0.0272%	0.7261%	9.4204%	-0.2358	6.4677

Panel B: Quantiles of Daily Log Return of 3M from 2004 to 2009

1% Quantile	5% Quantile	10% Quantile	25% Quantile	50% Quantile	75% Quantile	90% Quantile	95% Quantile	99% Quantile
-5.0400%	-2.2028%	-1.5519%	-0.6722%	0.0272%	0.7261%	1.5187%	2.2903%	4.27775522

Table 1: Descriptive Statistics of Daily Log Return of 3M from 2004 to 2009.

As given in *Table 1 Panel A*, the skewness of the daily log return of 3M is -0.2358 which indicates left skewness. Besides, the kurtosis is 6.4677 (excess kurtosis is 6.4677-3), which means that the distribution is leptokurtic. So in general, the distribution of daily log return of 3M does not have the normality. Another result of the normality test also confirms this shown in *Table 2*, we can reject the H_0 that the data follow a normal distribution. Besides, the results form *Panel A* also indicates a famous stylized fact of the stock return that The standard deviation of returns completely dominates the mean of returns at short horizons such as daily.

X-Squared	df	*p*-Value
2645.90	2.00	0.0000

Table 2: Jarque Bera Test of Daily Log Return of 3M

The quantile test results shown *Table 1 Panel B* also gives an impression of fat tails of the distribution. Next, the QQ-Plot is used to visualize the daily log return of 3M in the context of normality.

[1] The test results of AR(1) and ADF are not shown in this text. If needed, I can send you extra table of the both test resuls.

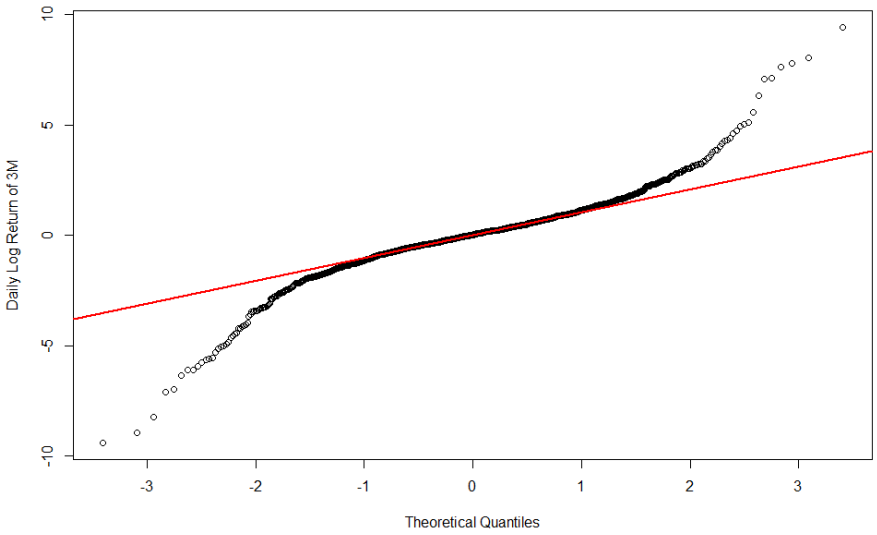

Figure 3: QQ-Plot of Daily Log Return of 3M from 2004 to2009. The theoretical quantile is the red line

As shown in *Figure 3*, the daily log return of 3M does not actually follow a normal distribution, as if so, all the points that represent the daily log return would lie on the red line or at least were very close to it. However, this pattern could not be found in *Figure 3*. The strong deviations of the points are at the tails of the distribution in both positive and negative directions signaling the fat tails of the log return distribution. Therefore, the normality of the distribution of daily log return of 3M is not held.

4) Kernel Density Estimation

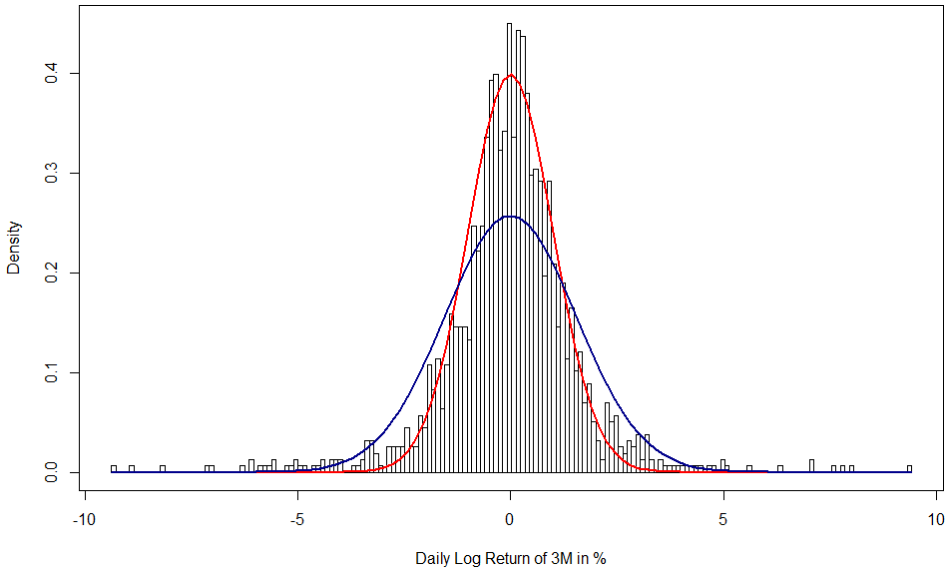

Figure 4: Histogram of Daily Log Return of 3M Comparing to Standard Normal Distribution N(0,1) and Normal Distribution N($\mu_{Daily\ Log\ Return}, \sigma_{Daily\ Log\ Return}$). The red curve is the standard normal distribution N(0,1) and the blue curve is the Normal Distribution N($\mu_{Daily\ Log\ Return}, \sigma_{Daily\ Log\ Return}$)

The histogram of daily log return of 3M has a shape of normal distribution. Two simulations are also superimposed in the graph. It is clear according to the visualization in *Figure 4* that the daily log return of 3M could be better simulated by using standard normal distribution function

N(0,1). Therefore, the choice of μ and σ in normal distribution context for the daily log return of 3M distribution simulation is 0 and 1.

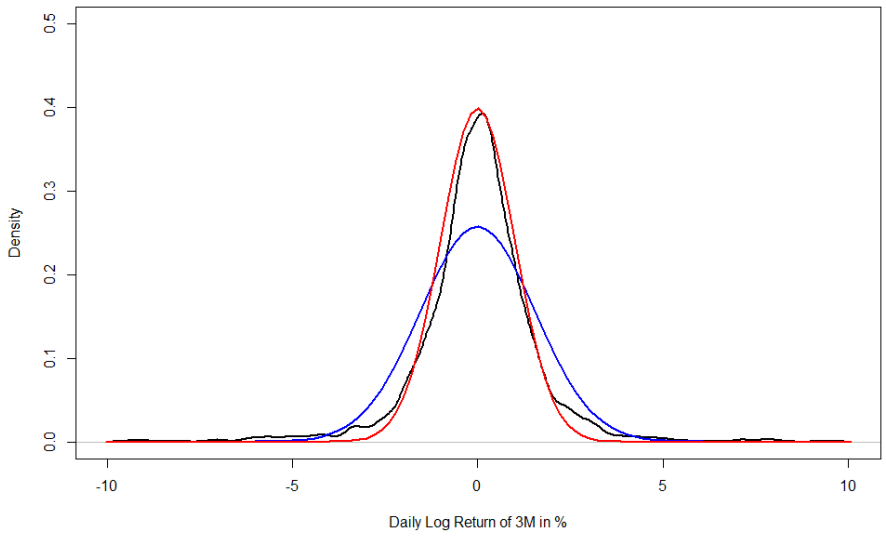

Figure 5: Kernel Density Estimation of Daily Log Return of 3M Comparing to Standard Normal Distribution N(0,1) and Normal Distribution N($\mu_{Daily\ Log\ Return}, \sigma_{Daily\ Log\ Return}$). The red curve is the standard normal distribution N(0,1) and the blue curve is the Normal Distribution N($\mu_{Daily\ Log\ Return}, \sigma_{Daily\ Log\ Return}$)

Figure 5 shows the kernel density estimation of the daily log return taking the Gaussian shape comparing to standard normal distribution N(0,1) and normal distribution N($\mu_{Daily\ Log\ Return}, \sigma_{Daily\ Log\ Return}$). As shown in the figure, the shape of the standard normal distribution is mostly close to the shape of kernel density estimation of the daily log return. However, the tails of kernel density estimation of the daily log return are fatter than the tails of the standard normal distribution, which indicates the extreme negative and positive returns in time series. In terms of kurtosis, kernel density estimation of the daily log return has a kurtosis very close to 3.

5) t-Distribution for Data Fit

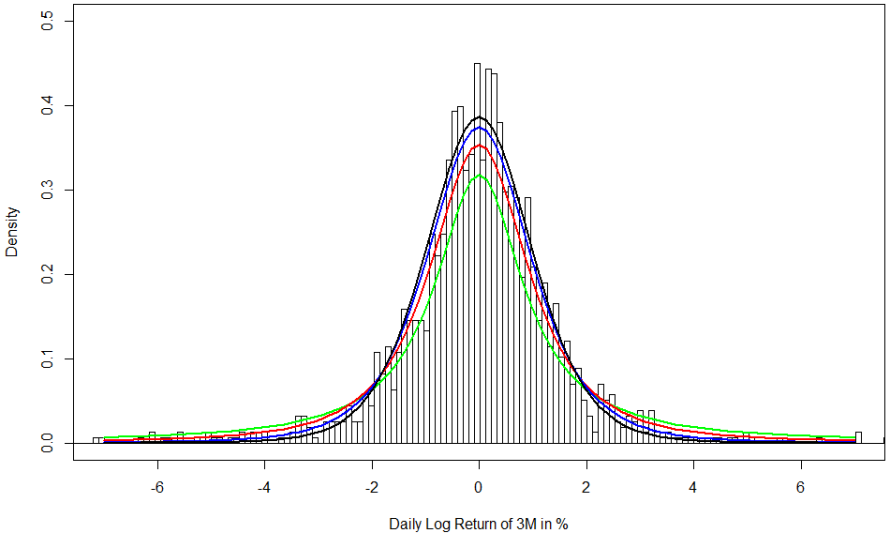

Figure 6: Daily Log Return of 3M Comparing with t-Distribution with Different Degrees of Freedom. The green curve is the t-distribution with 1 degree of freedom, the red curve is the t-distribution with 2 degree of freedom, the blue curve is the t-distribution with 4 degree of freedom and the black curve is the t-distribution with 8 degree of freedom.

The results of the fit goodness test of t-distribution with different degrees of freedom (DOF=1, 2, 4 and 8) to the daily log return of 3M are shown in *Figure 6*. It is difficult to say that t-distribution with 8 degrees of freedom has a clear better fit than others. As t-distribution with DOF=8 catches the peak in the density but cannot cove the fat tails of the daily log return of 3M compared to other simulations. But in general, a higher degree of freedom leads to higher goodness of fit of the t-distribution to the daily log return data, the problem is the estimation of the parameters.

6) Series Correlation in Close Prices and Returns of 3M

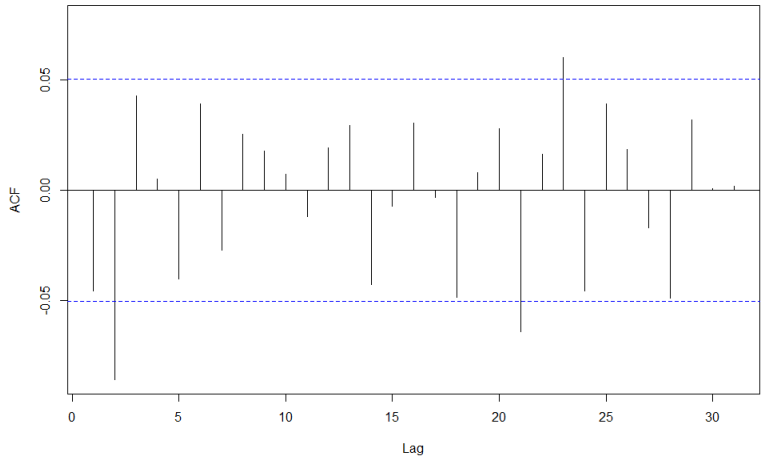

Figure 7: Series Correlation in Daily Log Return of 3M

The result of the series correlation in log return of 3M printed in *Figure 7* test clearly indicates that the log return of 3M does not have a significant series correlation with its past. The correlation coefficients of some lags, e.g. lag=3, 21 and etc., may have statistical significance, but do not have any economic significance. Therefore, the daily log return of 3M does not have random walk pattern indicating a stationary process.

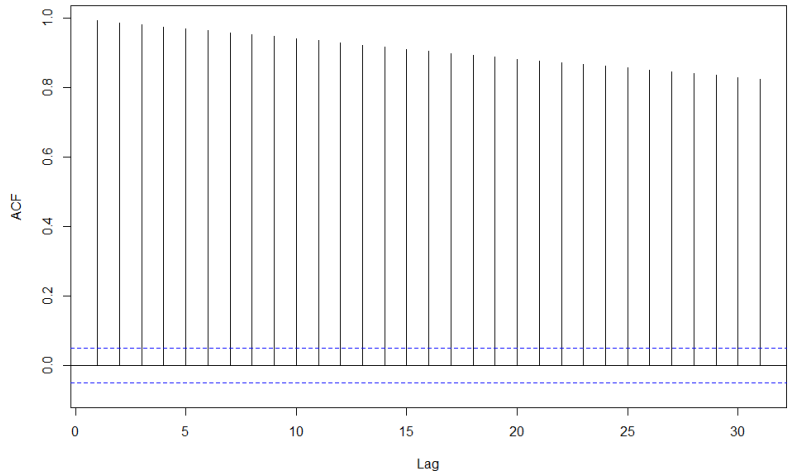

Figure 8: Series Correlation in Close Price of 3M

Comparing to *Figure 7*, the close price of 3M shows clear series correlation pattern in *Figure 8*, as the autocorrelations are all significant and very high cross lags. Therefore, the conclusion could be drawn that the close price of 3M has a random walk pattern in its time series.

7) Portmanteau-White-Noise-Test (Box-Ljung)

Test Types	Box-Pierce	Box-Ljung
p-Value of Daily Log Return of 3M	0.07537	0.07509
p-Value of Close Price of 3M	0.0000	0.0000

Note: all the tests with lag=1 (1 degree of freedom)

Table 3: Portmanteau-White-Noise-Test of Daily Log Return and Close Price of 3M

As expected, the results of Portmanteau-white-noise-test (Box-Ljung) presented in *Table 3* indicate that the daily log return of 3M does not have random walk pattern, as the p-value is higher than 0.05. Therefore, at 5% level, we cannot reject the H_0 that the daily log returns of 3M are independently distributed meaning that there is no serial correlation between log returns. However, the close price test results are highly significant, which gives the indication of the serial correlation between prices. So there is a random walk pattern in the close price of 3M from 2004 to 2009.

Conclusion

In general, the daily log return of 3M from 2004 to 2009 does not actually show a random walk pattern but is a stationary process. The log return does not follow a standard normal distribution and even not normal distribution due to its fat tails in both the. All of these truths are in line with the stylized facts of the returns. The best simulation of the log return is the t-distribution with a high degree of freedom and standard normal distribution N(0,1), however, they are both not perfect.

In terms of the close price of 3M time series, the process is clearly a random walk with the existing unit root. Therefore, the best prediction of the close price of 3M tomorrow is today's close price. The way to get rid of the unit root is to turn the close price to the return to make the process stationary.