Table 1: Replication of variance decomposition results corresponding to Table 3 of Diebold&Yilmaz.

	US	UK	FRA	GER	HKG	JPN	AUS	IDN	KOR	MYS	PHL	SGP	TAI	THA	ARG	BRA	CHL	MEX	TUR	CFO
US	93.6	1.6	1.5	0.0	0.3	0.2	0.1	0.1	0.2	0.3	0.2	0.2	0.3	0.2	0.1	0.1	0.0	0.5	0.3	6.0
UK	40.3	55.7	0.7	0.4	0.1	0.5	0.1	0.2	0.2	0.3	0.2	0.0	0.1	0.1	0.1	0.1	0.0	0.4	0.5	44.0
FRA	38.3	21.7	37.2	0.1	0.0	0.2	0.3	0.3	0.3	0.2	0.2	0.1	0.1	0.3	0.1	0.1	0.1	0.1	0.3	63.0
GER	40.8	15.9	13.0	27.6	0.1	0.1	0.3	0.4	0.6	0.1	0.3	0.3	0.0	0.2	0.0	0.1	0.0	0.1	0.1	72.0
HKG	15.3	8.7	1.7	1.4	69.9	0.3	0.0	0.1	0.0	0.3	0.1	0.0	0.2	0.9	0.3	0.0	0.1	0.3	0.4	30.0
JPN	12.1	3.1	1.8	0.9	2.3	77.7	0.2	0.3	0.3	0.1	0.2	0.3	0.3	0.1	0.1	0.0	0.0	0.1	0.1	22.0
AUS	23.2	6.0	1.3	0.2	6.4	2.3	56.8	0.1	0.4	0.2	0.2	0.2	0.4	0.5	0.1	0.3	0.1	0.6	0.7	43.0
IDN	6.0	1.6	1.2	0.7	6.4	1.6	0.4	77.0	0.7	0.4	0.1	0.9	0.2	1.0	0.7	0.1	0.3	0.1	0.4	23.0
KOR	8.3	2.6	1.3	0.7	5.6	3.7	1.0	1.2	72.8	0.0	0.0	0.1	0.1	1.3	0.2	0.2	0.1	0.1	0.7	27.0
MYS	4.1	2.2	0.6	1.3	10.5	1.5	0.4	6.6	0.5	69.2	0.1	0.1	0.2	1.1	0.1	0.6	0.4	0.2	0.3	31.0
PHL	11.1	1.6	0.3	0.2	8.1	0.4	0.9	7.2	0.1	2.9	62.9	0.3	0.4	1.5	1.6	0.1	0.0	0.1	0.2	37.0
SGP	16.8	4.8	0.6	0.9	18.5	1.3	0.4	3.2	1.6	3.6	1.7	43.1	0.3	1.1	0.8	0.5	0.1	0.3	0.4	57.0
TAI	6.4	1.3	1.2	1.8	5.3	2.8	0.4	0.4	2.0	1.0	1.0	0.9	73.6	0.4	0.8	0.3	0.1	0.3	0.0	26.0
THA	6.3	2.4	1.0	0.7	7.8	0.2	0.8	7.6	4.6	4.0	2.3	2.2	0.3	58.2	0.5	0.2	0.1	0.4	0.3	42.0
ARG	11.9	2.1	1.6	0.1	1.3	0.8	1.3	0.4	0.4	0.6	0.4	0.6	1.1	0.2	75.3	0.1	0.1	1.4	0.3	25.0
BRA	14.1	1.3	1.0	0.7	1.3	1.4	1.6	0.5	0.5	0.7	1.0	0.8	0.1	0.7	7.1	65.8	0.1	0.6	0.7	34.0
CHL	11.8	1.1	1.0	0.0	3.2	0.6	1.4	2.3	0.3	0.3	0.1	0.9	0.3	0.8	2.9	4.0	65.8	2.7	0.4	34.0
MEX	22.2	3.5	1.2	0.4	3.0	0.3	1.2	0.2	0.3	0.9	1.0	0.1	0.3	0.5	5.4	1.6	0.3	56.9	0.6	43.0
TUR	3.0	2.5	0.2	0.7	0.6	0.9	0.6	0.1	0.6	0.3	0.6	0.1	0.9	0.8	0.5	1.1	0.6	0.2	85.8	14.0

1. Diebold-Yilmaz Spillover Index

a)

In their paper Diebold and Yilmaz provided an approach for the measurement of connections of stock prices between countries. The authors propose an index to study spillover effects (of returns and volatility). This index is based on the method of variance decomposition. Besides the impulse response analysis and Granger causality tests, variance decomposition is a method to understand the interrelation among variables in a vector autoregressive system. Variance decomposition allows the authors to determine how much a shock to a specific variable influences the k-step ahead forecasting error variance (FEV) of a variable of interest. To derive the FEV we need to "eliminate" the common component of a shock. This makes clear, that the variance decompostion method is based on impulse response functions (IRF). The IRF is the standard deviation of a orthogonalized impulse response at a given point in time. Therefore the whole methodology is also based on the Cholesky decomposition ($\Rightarrow$ ordering matters). If we apply this theoretical background for example to a $VAR(1)$ with $N = 2$ variables it yields to a decomposition of the FEV into two parts. The first part gives the portion of the FEV which is explained by shocks to the variable itself in time t. The other part gives the portion from the shock to the other variable in t. In the analysis of Diebold and Yilmaz $N = 19$ variables are used. Therefore the variance is decomposed into 19 parts. The results of this decomposition of the FEV can be seen in Table 3 of Diebold and Yilmaz. This Matrix is now used to derive index proposed by the authors. They use the sum of the squared off-diagonal elements of this matrix and divide this by the sum of off and main diagonal elements of the matrix. Multiplying the result by 100 yields a relative value which suggests the spillover. So the Index indicates how much of the forecast error variance comes from innovations to other variables (here countries).

b)

$$y_t = \mu + \mathbf{\Phi}_1 y_{t-1} + \mathbf{\Phi}_2 y_{t-2} + v_t$$

The to this task corresponding code yields the estimation results to the above shown $VAR(2)$ model with $N = 19$ variables. The application of the roots command calculates eigenvalues of the $VAR(2)$. A VAR model specified like this yields 38 eigenvalues, two for each N. We see that all eigenvalues are complex but their modulus is smaller than one. Therefore this $VAR(2)$ is stable.

c)

Running the corresponding code yields Table **??**. We see exactly the same results as provided by Diebold and Yilmaz.

Table 2: Replication of contribution from and to others corresponding to Table 3 of Diebold&Yilmaz.

	US	UK	FRA	GER	HKG	JPN	AUS	IDN	KOR	MYS	PHL	SGP	TAI	THA	ARG	BRA	CHL	MEX	TUR
CFO	6	44	63	72	30	22	43	23	27	31	37	57	26	42	25	34	34	43	14
CTO	292	84	31	11	81	19	11	31	14	16	10	8	6	12	21	9	3	8	7

d)

As can be seen in Table **??**, based on our code the same results are obtained as provided in the paper. The method of variance decomposition allows to sum up spillover effects across markets. The total row sum net the portion of innovations which comes from the variable itself gives the value of "contribution from others" (CFO). The total row sum for each row in of Table **??** is per construction unity (100 or 1, depending on scale). Therefore the CFO values can be obtained by a trivial calculation as shown for the case of Germany. The for this calculation necessary values can be seen in Table **??**.

$$CFO_{Germany} = (40.8 + 15.9 + \cdots + 0.1 + 0.1) - 27.6 = 100 - 27.6 \cong 72$$

It states, that innovations to returns in 18 other countries are responsible for 72% of the error in forecasting variance in forecasting 10-week-ahead Germany returns. The CFO value for Germany is the highest that can be observed in Table 3. Since Germany is huge export nation this result is not a surprise.
Similar the value of "contribution to others" (CTO) can be obtained. If we sum up over the column totally it gives the value of contribution including the corresponding country. This value subtracted by the portion of innovations which comes from the variable itself yields the value of CTO. Again this can be seen best by conducting a little example calculation. For the US the CTO value can be obtained as follows.

$$CTO_{US} = (\text{Contributions including own})_{US} - 93.6 = (93.6 + \cdots + 3.0) - 93.6 \cong 292$$

Therefore the innovation to US returns are responsible for 292% of the error variance in forecasting 10-weeks-ahead the returns of the other 18 countries. This is the highest CTO value which can be observed in Table 3. The result for CTO_{US} is based on a logic background, too. Since the USA is the largest economy and one of the countries which imports most in the world, shocks to the US may effect other countries heavily.

e)

The variables here we choose to conduct our own spillover effect time series test are 2 Year US Treasury Bond Yield ($2\_Y\_B\_Y$), S&P500 Index ($S\&P500$), WTI(WTI), Gold Price ($Gold$) and Foreign Exchange Rate Euro to Dollar ($Euro\_Dollar\_FX$). The time span is from May, 2018 to May, 2019.[1] The reasons we choose these variables and the time span are the following:

1) The US investors usually transfer their money among the bond market, stock market, oil market, commodity market and FX market to seek their profit and money parking opportunities. For example, if the stock market suffers from depression, investors will withdraw their money from stock market and invest in bond market. Therefore, we could expect that the turbulence in stock market will lead to fluctuation in bonds market, which indicates spillover effect.

2) The time span is between May, 2018 and May, 2019, as in this period so many important events happened in these markets. In November 2018 due to the sanction from US to Iran,

[1] Note, that we conducted our analysis in the same procedure as in the previous task: $VAR(2)$ model, variance decomposition horizon $h = 10$.

Table 3: Spillover Effect among markets in US

	From					
To	2_Y_B_Y	S&P500	WTI	Gold	Euro_Dollar_FX	CFO
2_Y_B_Y	92.2	0.2	1.6	0.5	5.5	8
S&P500	8.4	91.2	0.2	0.1	0.1	9
WTI	1.7	16.3	75.4	0.2	6.5	25
Gold	13.6	1.0	0.2	84.2	1.0	16
Euro_Dollar_FX	1.4	1.5	1.6	3.2	92.4	8
CTO	25	19	3	4	13	
CIO	117	110	79	88	105	

the oil price world wide starts to fluctuate. Besides, in the second part of 2018, US Treasury Bond Yield went up due to the market's poor anticipation of US government debt issue. Of course, the trading war should be also mentioned here. Therefore, we may be able to observe strong spillover effect among those market in this time span. As we can see from Table **??**, stock market ($S\&P500$) and bond market ($2\_Y\_B\_Y$) have the strongest influencing power on other markets and are not easily influenced by other markets. In comparison the oil market (WTI) and Gold ($Gold$) are more vulnerable to other markets' fluctuations.

2. German macro data

a)

See *R*-file. Since we always used logarithmic values, we decided to specify GDP and M1 as log values, before we calculated the fourth differences. To clarify the notation, we state $\triangle_4 bip$ as the 4-th difference of the logarithm of GDP and $\triangle_4 m1r$ as the 4-th difference of the logarithm of M1.

b)

We estimated the following $VAR(2)$ model for the $N = 3$ variables given in this task $y_t = (\triangle_4 bip, \triangle_4 m1r, glsr)'$.

$$y_t = \mu + \mathbf{\Phi}_1 y_{t-1} + \mathbf{\Phi}_2 y_{t-2} + v_t$$

The maximum likelihood estimates are

$$\widehat{\mathbf{\Phi}}_1 = \begin{bmatrix} -0.200 & -0.9531 & 0.0203 \\ 0.4206 & -1.6673 & 0.0002 \\ -0.8216 & 0.8883 & 1.2816 \end{bmatrix}, \widehat{\mathbf{\Phi}}_2 = \begin{bmatrix} -0.4378 & -0.1384 & -0.0252 \\ 0.1370 & -0.4939 & -0.0019 \\ -2.0453 & 0.7930 & -0.4468 \end{bmatrix}, \widehat{\mu} = \begin{bmatrix} 0.0071 \\ 0.0029 \\ 0.2005 \end{bmatrix}.$$

The log-likelihood function evaluated at $\widehat{\Theta}$ is $L_T(\widehat{\Theta}) = 156.741$.

c)

After running a VAR(2), one can continue with the causality command for Granger causality tests. First we choose to test for simple Granger causality. Table **??** shows the result of a simple Granger causality test. Here the H_0: $\triangle_4 bip \nrightarrow \triangle_4 m1r$, $glsr$ tests, whether $\triangle_4 bip$ Granger causes $\triangle_4 m1r$ and $glsr$. The p-values of this test is small enough to reject this H_0. This means that future values of $\triangle_4 m1r$ and $glsr$ can be predicted better if current and past values of $\triangle_4 m1r$ are used. This statement is also true for the Granger causality test in row two of table **??**. For the third Granger causality test we conduced the H_0 can not be rejected. Therefore gls not Granger-causes $\triangle_4 bip$ and $\triangle_4 m1r$. Secondly we also want to test for instantaneous Granger causality.

Table 4: Results of a simple Granger causality test

Null hypothesis			F statistic	DOF	p-values
$\triangle_4 bip$	$\nrightarrow$	$\triangle_4 m1r$, $glsr$	15.375	4	0.000
$\triangle_4 m1r$	$\nrightarrow$	$\triangle_4 bip$, $glsr$	23.256	4	0.000
$glsr$	$\nrightarrow$	$\triangle_4 bip$, $\triangle_4 m1r$	1.200	4	0.3109

Table 5: Results of a instantaneous Granger causality test

Null hypothesis			χ^2 statistic	DOF	p-values
$\triangle_4 bip$	$\not-$	$\triangle_4 m1r$, $glsr$	42.942	2	0.000
$\triangle_4 m1r$	$\not-$	$\triangle_4 bip$, $glsr$	42.744	2	0.000
$glsr$	$\not-$	$\triangle_4 bip$, $\triangle_4 m1r$	2.020	2	0.3643

Instantaneous Granger causality states that future values of y_t can be better predicted, if the future of other variables are used in addition to the current and past value of the other variable. First entity of Table **??** expresses the short form of the H_0 of no instantaneous causality between $\triangle_4 bip$ and $\triangle_4 m1r$ as well as $glsr$. Again only gls not instantaneous Granger causes $\triangle_4 bip$ and $\triangle_4 m1r$, since there is no possibility to reject the H_0 at any common significance level.

d)

Our choice of the Cholesky ordering is based on the previous works of the relations among GDP (*bip*), M1 ($m1r$) and the interest rate spread between long and short run rates ($glsr$). The researches from Mishkin (1981) and Mishkin (1982) shed a light into that the $m1r$ can affect the short term and long term interest rate, which is $glsr$ in our context. Besides, according to Gebhard Kirchgässner and Marcel Savioz (2001), the $glsr$ is Granger causal to real *bip* growth. The work of Sauer and Scheide (1995) also indicates this causality. The book of Gebhard Kirchgässner and Jürgen Wolters (2007) explains the Granger causality between $m1r$ and *bip*. This is confirmed by Rami (2010) and Ogunmuyiwa and Ekone (2010),by using data from India and Nigeria respectively. Based on all the researches mentioned above, our Cholesky ordering is then $m1r$, $glsr$ and *bip*. The estimation results of the orthogonalized one and two standard deviation impulse responses are:

$$IRF_1 = \widehat{\boldsymbol{S}} = \begin{bmatrix} 0.07 & 0.00 & 0.00 \\ 0.03 & 0.71 & 0.00 \\ 0.10 & 0.01 & 0.08 \end{bmatrix}, IRF_2 = \widehat{\boldsymbol{\Phi}}_2 \widehat{\boldsymbol{S}} = \begin{bmatrix} -0.07 & 0.00 & 0.04 \\ 0.01 & 0.90 & -0.07 \\ -0.08 & 0.01 & -0.02 \end{bmatrix}.$$

e)

Here we choose $h = 10$ for the variance decomposition horizon. The ordering of the variables is done in the same way as suggested in task 2d. The results are given in Table **??**. As already stated in the first task, variance decomposition allows to sum up over rows and/or columns. By definition the total row sum is unity. So one can state, that the contribution of $\triangle_4 m1r$ to the total variance of the $h = 10$ steps ahead forecast error in the $\triangle_4 bip$ equation is 62.92%. Further 0.97% is of $glsr$ and 36.11% is of $\triangle_4 bip$ itself. If we have a specific look on to the main diagonal, we see that the total variance of the $h = 10$ steps ahead forecast error of the variables $\triangle_4 m1r$ and $glsr$ are mainly driven by shocks to the variable itself. That $\triangle_4 m1r$ is the main driver for the total variance of the 10-steps ahead forecast of $\triangle_4 bip$ interesting from an economic point

Table 6: Variance decomposition at $h = 10$

	$\triangle_4 m1r$	$glsr$	$\triangle_4 bip$
$\triangle_4 m1r$	0.8015	0.0007	0.1978
$glsr$	0.0108	0.9555	0.0337
$\triangle_4 bip$	0.6292	0.0097	0.3611

of view. In particular this states that $\triangle_4 bip$ envolves in dependence to $\triangle_4 m1r$. This satisfies same insights of the basic IS-LM model.[2]

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[2] The option fevd of the vars package deliver the output we see in Table **??**. However one can also obtain the R output for h-step ahead variance decomposition by calculation by hand. To show that we more or less know what we are doing here we connected a little example calculation in the corresponding part of the R-file for the 1-step ahead variance decomposition.