Math problems

2020/09/24

1) 1.1, $\frac{y^{58}}{y^4 \cdot y^{12}} = \frac{y^{58}}{y^{12+4}} = \frac{y^{58}}{y^{16}} = y^{58-16} = \underline{y^{42}}$

1.2, $8^2 \cdot 2^x = 2^9 \therefore (2^3)^2 \cdot 2^x = 2^9 \therefore 2^{6+x} = 2^9 \therefore 6 + x = 9 \quad \underline{x = 3}$

1.3, $\frac{x}{y} = 3$ then $\frac{y^2}{x^2} = \left(\frac{y}{x}\right)^2 = \left(\left(\frac{x}{y}\right)^{-1}\right)^2 = (3^{-1})^2 = \left(\frac{1}{3}\right)^2 = \underline{\frac{1}{9}}$

1.4, $\frac{\sqrt{2^{13}}}{\sqrt{8^3}} = \sqrt{\frac{2^{13}}{8^3}} = \left(\frac{2^{13}}{8^3}\right)^{\frac{1}{2}} = \left(\frac{2^{13}}{(2^3)^3}\right)^{\frac{1}{2}} = \left(\frac{2^{13}}{2^9}\right)^{\frac{1}{2}} = (2^{13-9})^{\frac{1}{2}} = (2^4)^{\frac{1}{2}} = 2^{\frac{4}{2}} = 2^2 = \underline{4}$

1.5, a, $x + y = y + x$ True

b, $x(y+z) = xy + xz$ True

c, $x^{(y+z)} = x^y x^z$ - True

d, $\frac{x^y}{x^z} = x^{y-z}$ True

1.6, $\frac{x^2 - 25}{x - 5} = 3 \therefore \frac{(x+5)\cancel{(x-5)}}{\cancel{(x-5)}} = 3 \therefore x + 5 = 3 \therefore \underline{x = -2}$

2) 2.1, $0°K = -460°F$ $\quad 1000°K = 1340°F$ $\quad m = \frac{1340 - (-460)}{1000 - 0} = 1.8$

$\therefore F = 1.8K + b$

$(-460) = 1.8(0) + b$

$-460 = b$

$F = 1.8K - 460$

$K = 0.556F + b$

$0 = 0.556(-460) + b$

$0 = -255.56 + b$

$255.56 = b$

$K = 0.556F + 255.56$

$1.8\cancel{K} - 460 = 0.556\cancel{K} + 255.56$

$1.244x = 715.56$

$\underline{\underline{x = 575}}$

at 575° they are equal

2.2, $f(x) = 2x + 3 \quad f(y) = 17$

$f(y) = 17 = 2y + 3 \therefore 14 = 2y \quad \underline{\underline{7 = y}}$

2.3, $3^{2x^2 - 4x + 3} = 27 = 3^3 \therefore 2x^2 - 4x + 3 = 3 \quad 2x^2 - 4x = 0$

$x(2x - 4) = 0 \quad x = 0$ or $2x - 4 = 0$ $\quad \underline{x = 0 \text{ or } 2}$

$2x = 4$

$x = 2$

2, 2.~~3~~4, an. GDP growth 1% how long to double GDP?

$$1.5 = (1+0.01)^n \therefore \ln(1.5) = \ln(1.01)^n \therefore \ln(1.5) = n\ln(1.01)$$

$$n = \frac{\ln(1.5)}{\ln(1.01)} = \underline{\underline{40.75}} \text{ years}$$

2.5, $\ln\left(\frac{e^2}{e^3}\right) = \ln(e^{2-3}) = \ln(e^{-1}) = -1\ln(e) = -1(1) = \underline{\underline{-1}}$

3, 3.1, $\sum_{i=0}^{\infty}\left(\frac{1}{6^i} + 0.25^i\right) = \sum_{i=0}^{\infty}\left(\left(\frac{1}{6}\right)^i + \left(\frac{1}{4}\right)^i\right)$

if $i=0$ $\left(\frac{1}{6}\right)^0 + \left(\frac{1}{4}\right)^0 = 1+1 = 2$

if $i=1$ $\left(\frac{1}{6}\right)^1 + \left(\frac{1}{4}\right)^1 = \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$

if $i=2$ $\left(\frac{1}{6}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{1}{36} + \frac{1}{16} = \frac{13}{144}$

sequence $\left(2, \frac{5}{12}, \frac{13}{144}\right)$

sum a1 sum a2 sum a3

2 2.4 3

if $i=3$ $= 0.0202$

if $i=4$ $= 0.0046$

if $i=5$ $= 0.0011$

sum at $i=5 = 2.532$

therefore sum is approaching ~~2.53~~ $\underline{\underline{2.533}}$

3.2, $\lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3}\frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = \lim_{x\to 3}(x+3) \therefore 3+3 = \underline{\underline{6}}$

3.3, $f(x) = x^3 - 4$ at $(-1, -5)$

$f'(x) = 3x^2$ $f'(-1) = 3(-1)^2 = +3$

3.4, $\left(\frac{x^2+3}{x+2}\right)'$ $f' = \frac{(x^2+3)'(x+2) - (x^2+3)(x+2)'}{(x+2)^2} = \frac{(2x)(x+2) - (x^2+3)(1)}{(x+2)^2}$

$$= \frac{2x^2+4x-x^2+3}{(x+2)^2} = \frac{x^2+4x+3}{(x+2)^2} = \underline{\frac{(x+1)(x+3)}{(x+2)^2}}$$

3.5, $f(x) = x^7 + 4x^2$ $f'(x) = 7x^6 + 4(2)x = 7x^6 + 8x$

$f''(x) = 7(6)x^5 + 8 = \underline{42x^5 + 8}$

3.6, $f(x) = \frac{(x^4+4^x)}{\ln(x)}$ $f'(x) = \frac{(x^4+4^x)'(\ln x) - (x^4+4^x)(\ln x)'}{(\ln x)^2}$

$$= \frac{(4x^3 + 4^x \cdot \ln 4)(\ln x) - (x^4+4^x)\left(\frac{1}{x}\right)}{(\ln x)^2} = \underline{\frac{4x^3(\ln x) + 4^x(\ln 4)(\ln x) - x^3 + 4^x}{(\ln(x))^2}}$$

3, ~~3.7, $f(x) = 3x^{2} - 9x$~~

~~$f'(x) = 3(2)x - 9 = 6x - 9$~~ $\quad 6x - 9 = 0 \quad \therefore 6x = 9 \quad x = \frac{9}{6} = \frac{3}{2}$

~~$f''(x) = 6 \rightarrow$ increasing $\quad \therefore \frac{3}{2}$ is a local minimum~~

3.8, $f(x,y) = x^2 + 2y^3 \qquad f(2,3) = (2)^2 + 2(3)^3 = 4 + 2(27) = 4 + 54 = \underline{\underline{58}}$

3.9, $f(x,y) = \ln(2x - y) =$ $\qquad$ domain of $\ln(x) = \mathbb{R}$ if $x > 0$

$\therefore = \mathbb{R}^+$

$f'_x(x,y) = \left(\frac{1}{2x-y}\right) \cdot 2 = \frac{2}{2x-y}$

$f'_y(x,y) = \left(\frac{1}{2x-y}\right) \cdot 1 = \frac{1}{2x-y}$

therefore domain of $\ln(2x-y)$

$2x - y > 0 \quad x, y = \mathbb{R}$

$2x > y \quad \therefore 2 > \frac{y}{x}$

3.10, $f(x,y) = x^5 e^y + x^2 y^3$

$\frac{df(x,y)}{dx} = 5x^4 e^y + 2xy^3$

$\frac{df(x,y)}{dy} = x^5 e^y + x^2 3y^2$

3.11, $f(x,y) = \sqrt{xy} - 0.7x - 0.7y = (xy)^{\frac{1}{2}} - 0.7x - 0.7y = x^{\frac{1}{2}} \cdot y^{\frac{1}{2}} - 0.7x - 0.7y$

$\frac{df(x,y)}{dx} = y^{\frac{1}{2}} \cdot \frac{1}{2} x^{-\frac{1}{2}} - 0.7$

$= \frac{1}{2} \frac{\sqrt{y}}{\sqrt{x}} - 0.7 = \frac{y}{2\sqrt{xy}} - 0.7$

$\frac{df(x,y)}{dy} = x^{\frac{1}{2}} \frac{1}{2} y^{-\frac{1}{2}} - 0.7$

$= \frac{\sqrt{x}}{2\sqrt{y}} - 0.7 = \frac{x}{2\sqrt{yx}} - 0.7$

~~$x = y \quad \frac{x}{2\sqrt{xx}} - 0.7 = 0 \quad \therefore \frac{x}{2x} = 0.7$~~

$\sqrt{y} = \frac{x}{\sqrt{y}}$

$\underline{y = x}$

$\frac{y}{2\sqrt{xy}} - 0.7 = \frac{x}{2\sqrt{xy}} - 0.7 \qquad \frac{\sqrt{y}}{2\sqrt{x}} - 0.7 = \frac{\sqrt{x}}{2\sqrt{y}} - 0.7$

$\frac{y}{2\sqrt{xy}} = \frac{x}{2\sqrt{xy}} \qquad \frac{\sqrt{y}}{2\sqrt{x}} = \frac{\sqrt{x}}{2\sqrt{y}} \quad / \cdot 2 \qquad \frac{x}{2\sqrt{x \cdot x}} - 0.7 = 0$

$\frac{\sqrt{y}}{\sqrt{x}} = \frac{\sqrt{x}}{\sqrt{y}} \qquad \frac{1}{2}\left(\frac{x}{x}\right) = 0.7$

$0.5 \neq 0.7$ inflection point

3.12, $f(x) = x^2 y^2$ st. $x + y = 10$

$\mathcal{L} = x^2 y^2 - \lambda(x + y - 10)$

$\frac{d\mathcal{L}}{dx} = 2xy^2 - \lambda(1) \qquad 2xy^2 - \lambda = 0 \qquad 2yx^2 = 2xy^2$

$2xy^2 = \lambda \qquad \frac{x^2}{2x} = \frac{y^2}{2y} \quad \therefore x = y$

$\frac{d\mathcal{L}}{dy} = 2yx^2 - \lambda(1) \qquad 2yx^2 - \lambda = 0 \qquad x + y - 10 = 0$

$2yx^2 = \lambda \qquad x + x - 10 = 0 \qquad \underline{x = y = 5}$

$2x = 10$

$x = 5$

4, 4.1, $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ $A \cdot B = \begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 6 \\ 5 & 6 & 5 \end{bmatrix}$

$A \cdot B$

$$\begin{array}{c|ccc} & 1 & 4 & 1 \\ & 2 & 1 & 2 \\ \hline 2\,3 & 8 & 11 & 8 \\ 4\,1 & 6 & 17 & 6 \\ 1\,2 & 5 & 6 & 5 \end{array}$$

4.2, $B \cdot A$

$$\begin{array}{c|cc} & 2 & 3 \\ & 4 & 1 \\ & 1 & 2 \\ \hline 1\,4\,1 & 19 & 9 \\ 2\,1\,2 & 10 & 11 \end{array}$$

$B \cdot A = \begin{bmatrix} 19 & 9 \\ 10 & 11 \end{bmatrix}$

4.3, $\underline{C} = \begin{bmatrix} 3.3 & 5.1 \\ 6.1 & 1.23 \\ 45.76 & 0 \end{bmatrix}$ $\underline{C}^T = \begin{bmatrix} 3.3 & 6.1 & 45.76 \\ 5.1 & 1.23 & 0 \end{bmatrix}$

4.4,

$D = \begin{bmatrix} 2 & 3 & 0 \\ 4 & 5 & 2 \\ 2 & 5 & 3 \end{bmatrix}$ det:
$$\begin{matrix} 2 & 3 & 0 & 2 & 3 \\ 4 & 5 & 2 & 4 & 5 \\ 2 & 5 & 3 & 2 & 5 \end{matrix}$$

$2(5)(3) + (3)(2)(2) + (0)(4)(5)$
$- (0)(5)(2) - (2)(2)(5) - (3)(4)(3)$
$= 30 + 12 + 0 - 0 - 20 - 36 = \underline{\underline{-14}}$

5, 5.1, flip a coin 2. sample space $(\Omega) = \{HH, HT, TH, TT\}$

5.2, 30 competitors 1st, 2nd, 3rd $\underline{30}\ \underline{29}\ \underline{28} = \underline{24360}$

5.3, toss a dice 2. at least 1 result being odd odd $\{1,3,5\}$

$\#\Omega = 36$ $\{(1,1)(1,2),(1,3)(1,4)$ ~~~~

$(1,5),(1,6);(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(5,1),(5,2),(5,3),$
$(5,4),(5,5),(5,6),(2,1),(2,3),(2,5),(4,1),(4,3),(4,5),(6,1),(6,3)$
$(6,5)\}$ $\frac{27}{36} = \frac{9}{12} = \underline{\underline{\frac{3}{4}}}$

3, 3.7, $f(x) = 3x^3 - 9x$ $f'(x) = 9x^2 - 9 = 9(x^2-1) = 9(x+1)(x-1)$

$\therefore 9(x+1)(x-1) = 0$ then $\underline{x = 1 \text{ or } -1}$

$f''(x) = 18x$ $f''(1) = 18(1)$ positive ~~not~~ local $\underline{\text{minimum}}$

$f''(-1) = 18(-1)$ negative local $\underline{\text{maximum}}$