

# Introduction to Computational Advertising

MS&E 239

Stanford University

Autumn 2011

Instructors: Dr. Andrei Broder and Dr. Vanja Josifovski  
Yahoo! Research

# General course info

- Course Website: <http://www.stanford.edu/class/msande239/>
- Instructors
  - **Dr. Andrei Broder**, Yahoo! Research, [broder@yahoo-inc.com](mailto:broder@yahoo-inc.com)
  - **Dr. Vanja Josifovski**, Yahoo! Research, [vanjaj@yahoo-inc.com](mailto:vanjaj@yahoo-inc.com)
- TA: **Krishnamurthy Iyer**
  - Office hours: Tuesdays 6:00pm-7:30pm, Huang
- Course email lists
  - Staff: [msande239-aut1112-staff](mailto:msande239-aut1112-staff)
  - All: [msande239-aut1112-students](mailto:msande239-aut1112-students)
  - Please use the staff list to communicate with the staff
- Lectures: 10am ~ 12:30pm Fridays in HP
- Office Hours:
  - After class and by appointment
  - Andrei and Vanja will be on campus for 2 times each to meet and discuss with students. Feel free to come and chat about even issues that go beyond the class.

# Course Overview (subject to change)

1. 09/30 Overview and Introduction
2. 10/07 Marketplace and Economics
3. 10/14 Textual Advertising 1: Sponsored Search
4. 10/21 Textual Advertising 2: Contextual Advertising
5. 10/28 Display Advertising 1
6. 11/04 Display Advertising 2
7. 11/11 Targeting
8. 11/18 Recommender Systems
9. 12/02 Mobile, Video and other Emerging Formats
10. 12/09 Project Presentations

# Lecture 05: Display Advertising

## Part 1

# Disclaimers

- This talk presents the opinions of the authors. It does not necessarily reflect the views of Yahoo! inc or any other entity.
- Algorithms, techniques, features, etc mentioned here might or might not be in use by Yahoo! or any other company.
- These lectures benefitted from the contributions of many colleagues and co-authors at Yahoo! and elsewhere. Their help is gratefully acknowledged.

# Lecture plan for today

- Quick feedback discussion [10 min]
- Display advertising [65 min]
- Break [20 min!]
- Presentation on Kontera [15 min incl discussion]
- Display Advertising [45 min]
- Quiz [15 min]

# Feedback on feedback

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# Some suggestions we got

- Trying to cover too much
- 3 hours is long and hard → We will have a 20 min break (30 min?)
- Ways to prepare better before class → We will try to post some preparatory material when appropriate
- Outside text → None exists. We are working on a book but are far from done. Will try to put references in advance.
- Balance of technical to non-technical (20% want less tech, 80 % want more in depth but less topics) → No way to make everyone happy, but we will try to cover fewer tech topics in more depth.
- TA hours → move to Thursday?
- Record the class → Will start today!!!



# We are trying to cover a lot, and the connections are not entirely clear

- Imagine we are studying car engine production ...
- Main topic– understand how the engine works  
→ **Computational Advertising = intersection of many disciplines**
- Should also understand how to drive a car  
→ **Advertising project**
- Should also understand the automotive marketplace  
→ **Class presentations**
  - Please relate the company you discuss to concepts learned in class – what do they try to achieve? Whose utility are they trying to increase (publisher, advertiser, consumer)? Etc.
  - Please take a critical point of view – would you use their services? When does their stuff work and when does not? What would be the best way to use them?
  - Try to send them a few days before class so you can get some feedback.

# Display Advertising

Based in large part on

- A. Ghosh, P. McAfee, K. Papineni, and S. Vassilvitskii, Bidding for Representative Allocations for Display Advertising. WINE 2009, 208-219 (full paper in [http://arxiv.org/PS\\_cache/arxiv/pdf/0910/0910.0880v1.pdf](http://arxiv.org/PS_cache/arxiv/pdf/0910/0910.0880v1.pdf) )
- B. Slideware by Kishore Papineni
- C. Help from Uri Feige, Vahab Mirrokni, Jai Shanmugasundaram and John Tomlin

# Computational Advertising and Market Design



# Overview

- Overview of the market
- How display advertisements are sold
  - Guaranteed delivery (GD)
  - Non-guaranteed delivery (NGD)
- The intermediary's (e.g. Yahoo's) role
- Advertiser view
- Components of a GD system

# Example

“Banner”

**YAHOO! SPORTS**

Search Web Search

Home NFL MLB NBA NHL NCAAF NCAAB NASCAR Golf UFC Boxing Soccer Tennis Action Sports More Shop Fantasy

TRENDING NOW: LeBron Jackson Martin Brodeur David Ortiz Paul Pierce Aubrey Huff

**THE NEXT THREE DAYS**  
IN THEATERS NOVEMBER 19  
CLICK TO WATCH TRAILER

**Headlines**

- Sparky Anderson
- Report: Ex-player
- NFL hands Harrison third fine of 2010
- Red Sox exercise \$12.5M option on Ortiz
- Eagles' Sims says \$50K fine 'outrageous'
- Seahawks QB Hasselbeck out vs. Giants
- Giants drop World Series MVP Renteria
- Blazers C Oberto retires due to heart

More News | Calendar | My Sports News | Video

**AdChoices**

**THE NEXT THREE DAYS**  
IN THEATERS NOVEMBER 19  
CLICK TO WATCH TRAILER

**Lasting impact**

Sparky Anderson's style and iconic smile reassured Tigers fans that their team, and the city of Detroit, was a winner. [Remembering Sparky](#)

Dead at 76 | True baseball man | Peers' praise | Legacy had charm

**Same old corrupt game**

Allegations Cameron Newton was shopped by middleman show nothing has changed. [Wetzel: Heads in the sand](#)

**Steelers safety bashes boss**

Pittsburgh's Troy Polamalu sounded off on the commish and the crackdown on hits. [What he said](#) | [Predictions](#)

**Reluctant UFC star**

Cain Velasquez isn't enjoying everything about being the new heavyweight champion. ['Not my thing'](#) | [TUF 12 picks](#)

**Fantasy football**

Expect another Ochostinko outing for the loudmouth Bengal. [Sit vs. start: WRs](#) | [Time to grab WR Rice](#)

**YAHOO! SPORTS EXPERTS**

**Newton scandal shows game stays same**  
Dan Wetzel November 4, 2010

[Read More](#)

**VIDEO SPOTLIGHT**

**Kobe's big night a good omen**  
Posted Nov 4 2010  
[More: video](#)

[Play Video](#)

Display Ads

# Impressions

- Every occurrence of an ad within the page = impression

The screenshot shows the front page of The New York Times website. Three red arrows highlight specific ad placements:

- Top Left:** A large blue ad for United Airlines.
- Top Right:** A blue ad for holiday travel with the text "Plan your holiday travel today."
- Bottom Right:** A green ad for Samsung's 20nm class DDR3 and SSD, featuring the text "Calculate your savings" and the Samsung logo.

The page content includes the newspaper's masthead, navigation links, a search bar, and several news articles such as "Bundlers' for Obama Have Active Ties to Lobbying" and "Experiencing Las Vegas, in Queens".



# Standard sizes (agreed by IAB =Internet Advertising Bureau )

		Recommended Maximum Initial Download Fileweight	Recommended Animation Length (Seconds)
<b>Banners and Buttons</b>			
468 x 60 IMU - (Full Banner)	<a href="#">View IMU</a>	40k	:15
234 x 60 IMU - (Half Banner)	<a href="#">View IMU</a>	30k	:15
88 x 31 IMU - (Micro Bar)	<a href="#">View IMU</a>	10k	:15
120 x 90 IMU - (Button 1)	<a href="#">View IMU</a>	20k	:15
120 x 60 IMU - (Button 2)	<a href="#">View IMU</a>	20k	:15
120 x 240 IMU - (Vertical Banner)	<a href="#">View IMU</a>	30k	:15
125 x 125 IMU - (Square Button)	<a href="#">View IMU</a>	30k	:15
<b>728 x 90 IMU - (Leaderboard)</b>	<a href="#">View IMU</a>	40k	:15



# Historical note: banners

- Banners seem to be the oldest standard format in use
- According to Wikipedia the first banner ad ever was sold in 1993 by **Global Network Navigator** (GNN) to Heller, Ehrman, White, & McAuliffe, a legal firm popular in Silicon Valley.
  - Tidbits:
    - **GNN** was a popular pre-Yahoo! directory eventually sold to AOL in 1995 (Yahoo = Yet Another Hierarchical Official Oracle started as a directory of web sites)
    - **Heller Ehrman White & McAuliffe** was started in 1890 and went bankrupt in 2008. In 1929 they negotiated the financing of the Bay Bridge!



# Types of advertising by objective

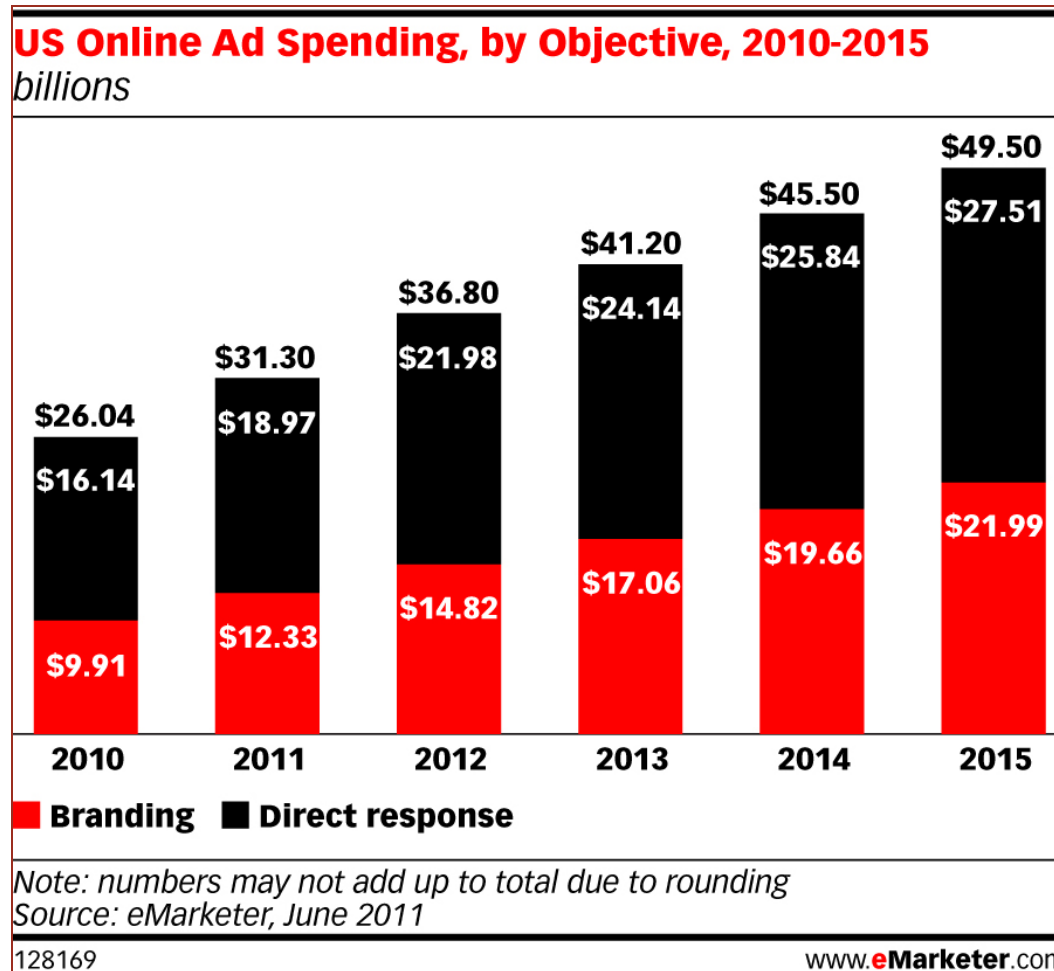
## 1. **Brand advertising**

- Goal: create a distinct favorable image
- Mostly display

## 2. **Direct marketing**

- Goal: a "direct response": buy, subscribe, vote, donate, etc, now or soon
- Majority via paid search but display catching up

# US Online Spending share by objective



# US Market Size

## US Online Ad Spending, by Format, 2010-2015

*billions*

	<b>2010</b>	<b>2011</b>	<b>2012</b>	<b>2013</b>	<b>2014</b>	<b>2015</b>
Search	\$12.00	\$14.38	\$17.03	\$18.85	\$20.19	\$21.53
Banner ads	\$6.23	\$7.61	\$8.94	\$9.93	\$10.97	\$11.73
Classifieds and directories	\$2.60	\$3.00	\$3.35	\$3.65	\$3.98	\$4.29
Video	\$1.42	\$2.16	\$3.09	\$4.20	\$5.64	\$7.11
Rich media	\$1.54	\$1.66	\$1.73	\$1.74	\$1.73	\$1.68
Lead generation	\$1.34	\$1.42	\$1.45	\$1.47	\$1.50	\$1.52
Sponsorships	\$0.72	\$0.91	\$1.05	\$1.18	\$1.32	\$1.47
Email	\$0.20	\$0.16	\$0.16	\$0.17	\$0.17	\$0.18
<b>Total</b>	<b>\$26.04</b>	<b>\$31.30</b>	<b>\$36.80</b>	<b>\$41.20</b>	<b>\$45.50</b>	<b>\$49.50</b>

*Note: eMarketer benchmarks its US online ad spending projections against the IAB/PwC data, for which the last full year measured was 2010*

*Source: eMarketer, June 2011*

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www.eMarketer.com

# Growth by format

## US Online Ad Spending Growth, by Format, 2010-2015 % change

	2010	2011	2012	2013	2014	2015
Video	39.6%	52.1%	43.1%	35.9%	34.3%	26.0%
Sponsorships	87.5%	26.4%	16.0%	12.3%	11.6%	11.0%
Banner ads	23.1%	22.1%	17.6%	11.0%	10.4%	7.0%
Search	12.2%	19.8%	18.4%	10.7%	7.1%	6.6%
Classifieds and directories	15.2%	15.7%	11.4%	9.0%	8.9%	7.8%
Rich media	2.2%	7.9%	4.3%	0.8%	-0.8%	-2.7%
Lead generation	-7.7%	6.1%	1.8%	1.7%	2.1%	1.2%
Email	-33.2%	-16.5%	-0.5%	3.3%	3.4%	3.1%
<b>Total</b>	<b>14.9%</b>	<b>20.2%</b>	<b>17.6%</b>	<b>12.0%</b>	<b>10.4%</b>	<b>8.8%</b>

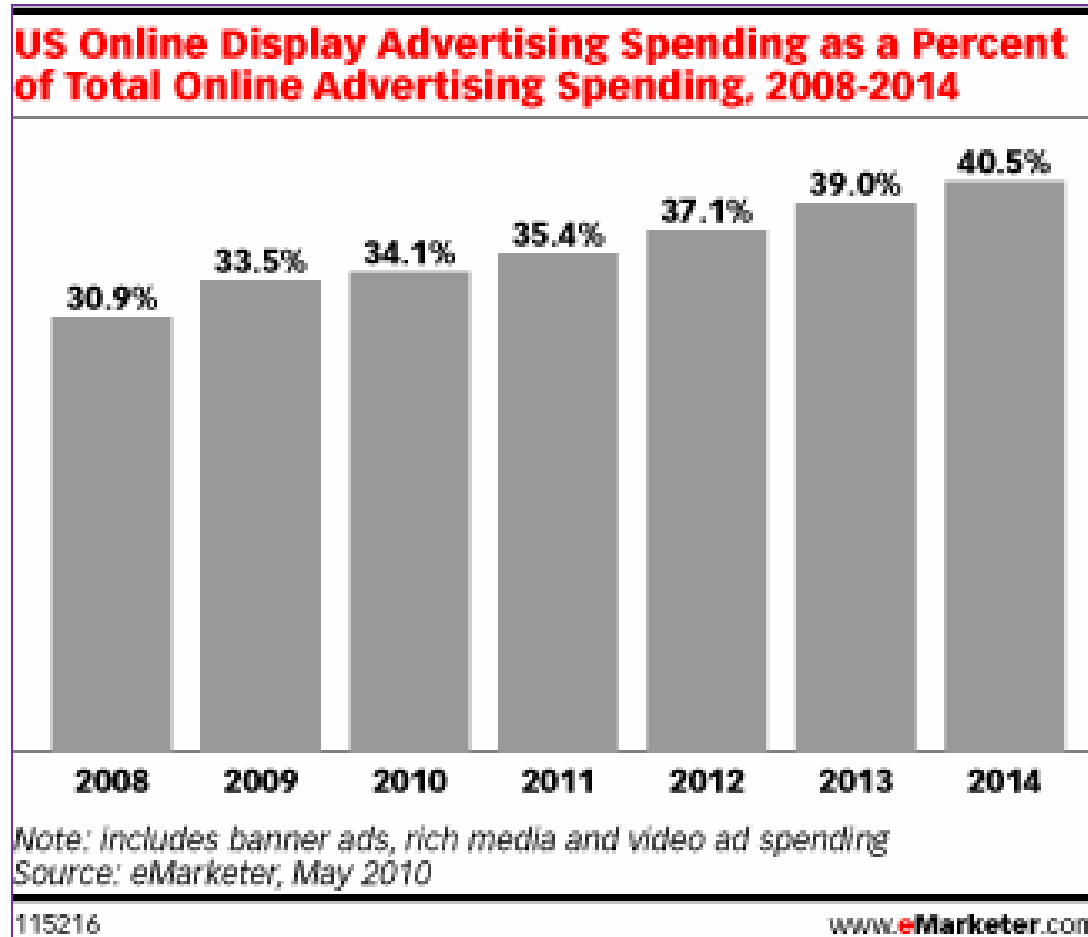
*Note: eMarketer benchmarks its US online ad spending projections against the IAB/PwC data, for which the last full year measured was 2010*

*Source: eMarketer, June 2011*

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www.eMarketer.com

# Display advertising as a percent of total online spending in US



# Some metrics

## US Online Display Ad Revenues and Metrics, 2009-2014

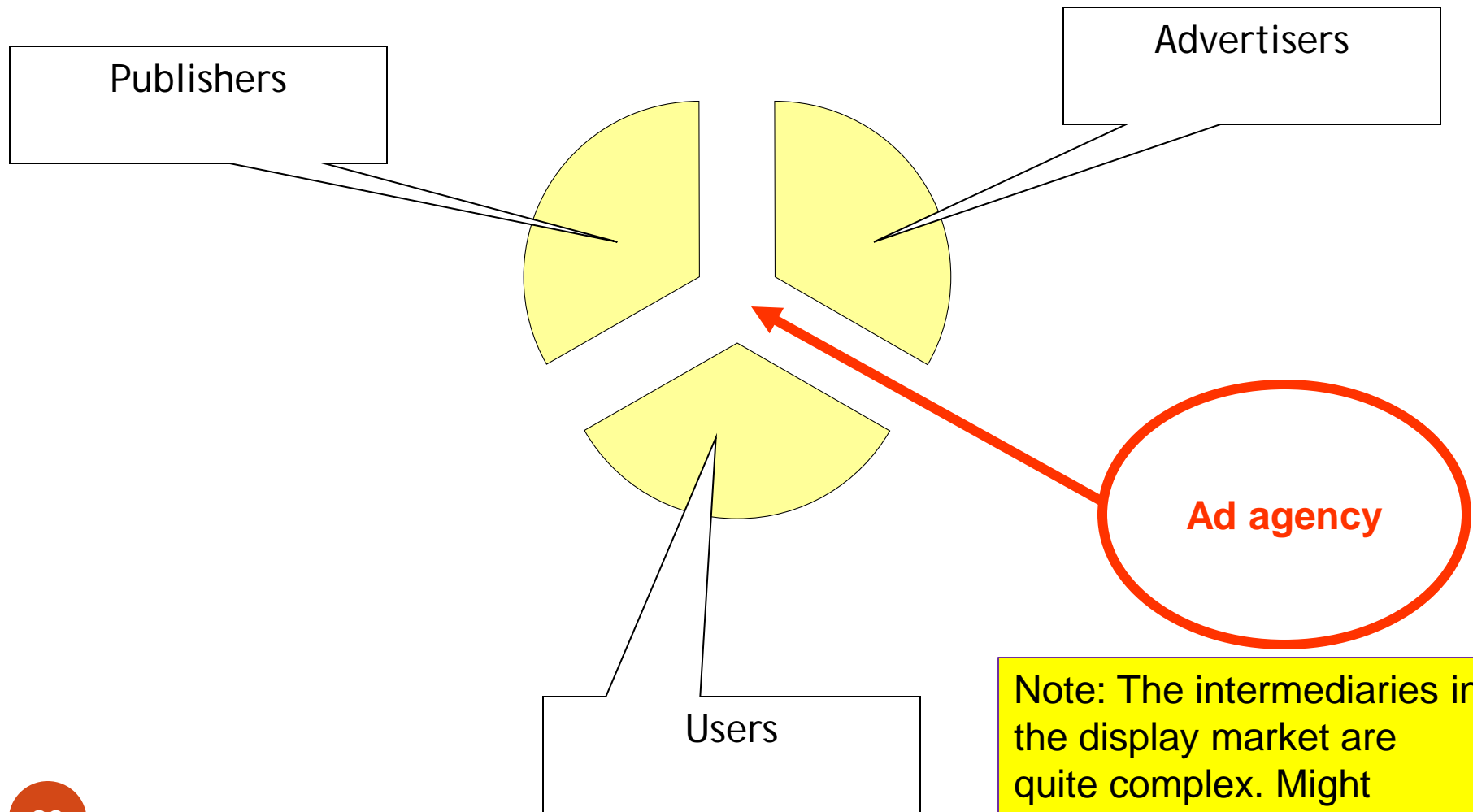
	2009	2010	2011	2012	2013	2014	CAGR
Internet population (millions)	222	227	231	235	239	243	1.8%
Pages viewed per user per day	53	57	61	65	68	72	6.3%
Total pages viewed (billions)	4,307	4,737	5,164	5,577	5,967	6,385	8.2%
Impressions per page	0.60	0.61	0.61	0.62	0.63	0.64	1.3%
<b>Total impressions (billions)</b>	<b>2,584</b>	<b>2,890</b>	<b>3,150</b>	<b>3,458</b>	<b>3,759</b>	<b>4,086</b>	<b>9.6%</b>
CPM (per 1,000 impressions)	\$3.05	\$3.13	\$3.25	\$3.25	\$3.25	\$3.20	1.0%
RPM (per 1,000 pages)	\$1.83	\$1.91	\$1.98	\$2.02	\$2.05	\$2.05	2.3%
<b>Total revenues (billions)</b>	<b>\$7.88</b>	<b>\$9.05</b>	<b>\$10.24</b>	<b>\$11.24</b>	<b>\$12.22</b>	<b>\$13.08</b>	<b>10.7%</b>
% change	3%	15%	13%	10%	9%	7%	-

Source: J.P. Morgan, "Nothing But Net," provided to eMarketer, Jan 3, 2011

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www.eMarketer.com

# The usual actors: Publishers, Advertisers, Users, & “Ad agency”



Note: The intermediaries in the display market are quite complex. Might discuss in next lecture

# Graphical ads marketplace

## Two types of online graphical advertising

### 1. **Guaranteed delivery (GD)**

- Contract booked based on targeting attributes.
  - Hundreds of dimensions available: user dimensions (age, gender, interests, ...), page dimensions (content category, type, ...), context dimensions (location, mobile/PC, ...)
  - Example: Male, 25-40, Interested in Sports, Travel Enthusiast, ..., Sports Page, NFL, NBA, ..., California, iPhone, ...
- Each contract has a duration and a desired number of impressions
- Publishers guarantee the desired number of user visits many months in advance
- Advertisers pay for ads delivered to user visits
- Publishers are responsible for any shortfall in the guaranteed user visits

### 2. **Non Guaranteed Delivery (NGD)**

- Impressions sold at auction in (approximately) real-time





Sports Search

POPULAR SEARCHES:

[Mark McGwire](#)[Cliff Lee](#)[Yankees](#)[Caroline Wozniacki](#)[David Beckham](#)

IT'S **YOUR** SPORTS. SITES. NEWS. MAIL.  
YOUR WAY.



YAHOO!

Go Now

NFL

[COWBOYS GIVE LB WARE \\$78M OVER 6 YEARS](#)
[NBA](#)
[NCAAF](#)
[NCAAB](#)

Headlines

[Blogs](#)
[Rumors](#)
[Buzzed](#)

## Team of destiny?


[PLAY VIDEO](#)

NBAE/Getty Images

The clock is ticking on the Cavs. Can Shaquille O'Neal deliver a title and keep LeBron James smiling in Cleveland? [Analysis](#) | [Previews](#)

[Contenders: East - West](#)
[MVP predictions](#)
[Play Fantasy](#)

### World Series breakdown

The Yanks and Phils are full of talent. Let's see how they stack up. [Brown: Matchups](#)  
[Experience matters?](#)



### Agent: No deal for Rondo

Rajon Rondo will join next summer's big free-agent class. [Marc Spears story](#)  
• [Clips' Griffin sidelined](#)



### College football

- [Rivals ranks: UT climbs?](#)
- [Contenders' issues](#)
- [Big 12 North down, out](#)
- [Dr. Saturday](#) | [More](#)



### Fantasy football

A big Jets injury makes Shonn Greene a must-have at RB. [Waiver wire gems](#)  
[Studs](#) [Duds](#) | [More](#)



- [Lesnar postpones UFC title fight](#) | [Blog](#)
- [Clippers F Griffin could miss 6 weeks](#)
- [Jackson's TDs lift Eagles over Redskins](#)
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- [Cowboys give LB Ware \\$78M over 6 years](#)
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ADVERTISEMENT

**I feel the need...**  
visit your dealer today

BASKETBALL 09

YAHOO! FANTASY SPORTS

PLAY NOW

## YAHOO! SPORTS EXPERTS

[Johnson wrecking his foes' hopes](#)
[Dan Wetzel](#) October 25, 2009


After a strong run at Martinsville, perhaps

## VIDEO SPOTLIGHT



[Yankees in World Series, Good for Baseball?](#)

Posted Oct 26 2009

# The life of that ad

- **Early January 2009:** Yahoo! sales person receives an RFP (Request for Proposal) from Top Gun Motors
- **Mid January 2009:** Yahoo! confirms and guarantees order with Top Gun Motors
  - **Target:** Males in California visiting Yahoo! Sports
  - **Duration:** October 1-29, 2009
  - **Quantity:** 25 million user visits (impressions)
  - **Price:** \$6.25 CPM (Cost Per Mille = 1000 user visits)
- **27 Oct 2009, 4:02 PM:** Male from CA visits Sports page
- **27 Oct 2009, 4:02.5 PM:** Sports page shown with Top Gun Motors ad
- **30 Nov 2009:** Top Gun Motors pays for delivered ads, Yahoo! incurs penalties/make-goods for shortfall



# Why GD?

- Currently, GD commands premium prices
- Advertisers may have timed reach goals
  - E.g. GM knows will launch a new model targeted to single, young, irresponsible males in Nov 2011...
- In general: quality of inventory (page views) given to GD contracts > non-guaranteed
  - NB Quality of a page is really about the users!

# Advertiser's view – recap

- Advertiser can buy the ad space
  1. in advance (1-24 months) as GD
    - Pay a premium
    - Get premium inventory
    - Many targeting attributes
  2. on the spot market (at the time of page view) as NGD
- Same advertiser might buy both GD and NGD
- Goal: maximize ROI (e.g. cost per conversion, cost per lead, etc)

# Publisher's view -- basics

- Publishers sells impressions
- Some impressions clearly more valuable than others – price should somehow be determined by supply and demand
  - More competition for “females, 30-50, high income” than for “teenager drop-outs”
  - More competition for “Yahoo finance” than for “Yahoo horoscopes”
- Must decide whether to sell GD contracts in advance (at what price?) or spot market online (NGD)
- Wants to maximize revenue but also not make their users (readers) unhappy by excessive/intrusive advertisers

# Components of a GD system

1. Forecast supply and demand
2. Forecast NGD pricing
3. Admission control & pricing
  - Advertiser wants: 30m impressions in May 2012 on yahoo sports
  - Should we accept the contract? Can we meet the guarantee? What price should we charge? How are other contracts impacted?

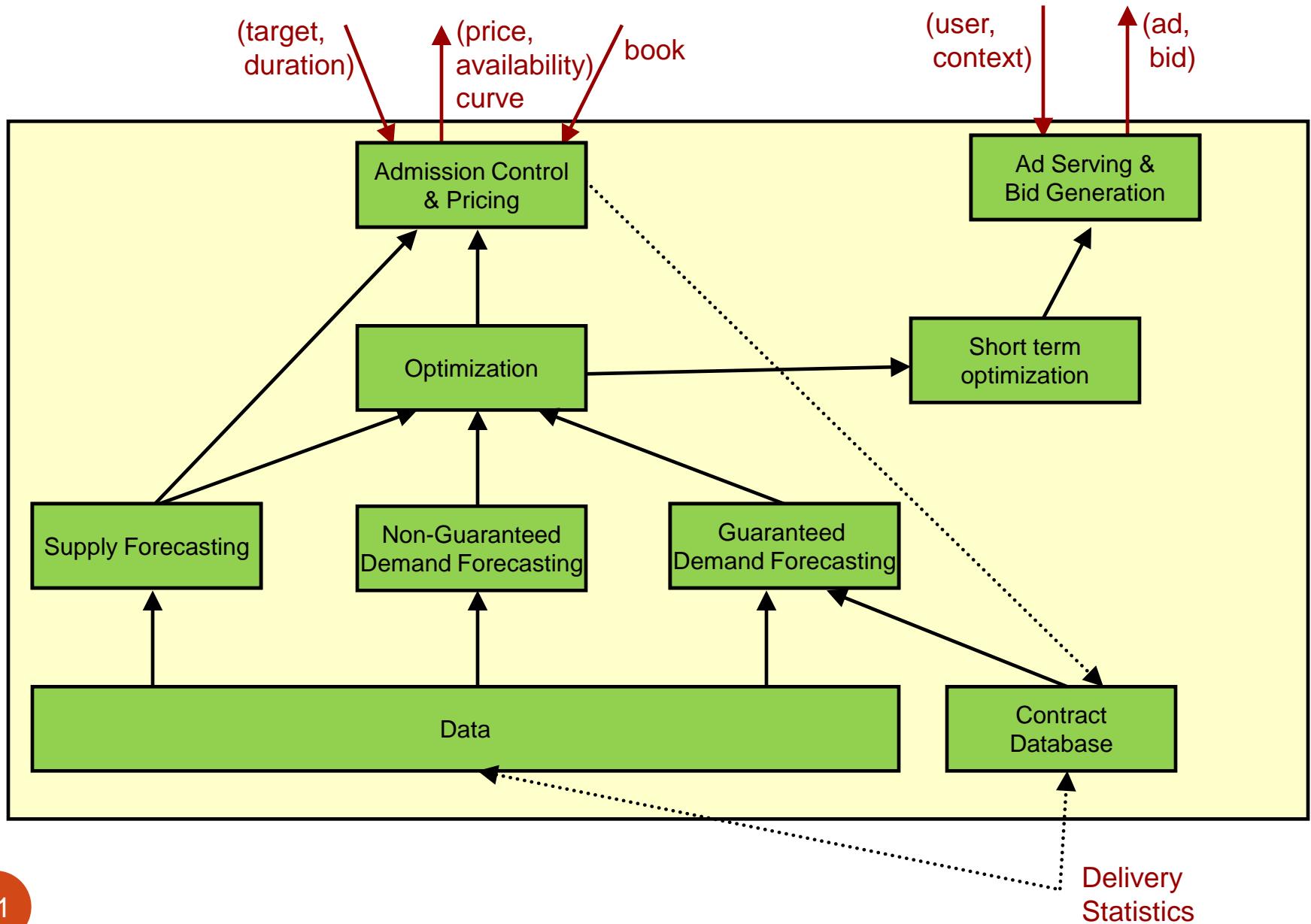
Sell time

4. Optimal allocation of impressions to active contracts
  - Objective function?
  - Can not re-run after every impression:
    - Need short term delivery plans
5. Ad serving

Delivery time

- Components 1-4 are obviously interrelated!
- Demand (long term) depends on quality of allocation!

# Logical diagram



# Admission control

- Assume we know the future supply in perfect detail
- Assume that we have a well defined “impression quality” metric
- To decide whether to accept a contract or not need to
  1. Figure out of all contracts agreed to + new contract can be satisfied
    - Some old contracts might now be under-delivered
    - The quality assigned to some contracts might decline
  2. Figure out the expected revenue with/without the contract
  3. Figure out the price
- Need to solve an optimal allocation problem!



# Detour: Pricing GD contracts

- Publishers need to effectively price guaranteed contracts that are sold months in advance
  - **Over-pricing → reduced sales → reduced revenue**
  - **Under-pricing → reduced revenue**
- The price needs to be available for any targeting combination since advertisers can target any combination
  - **Many trillions of possible combinations**
  - **Up to tens of billions of user visits *per day***
- The price needs to be produced in 100s of milliseconds to satisfy interactive queries/bookings
- Note: Price used as starting point for negotiation by the sales team
- More in the next lecture

# Optimal allocation

- Maximize a stated objective function subject to
  - Supply constraints
  - Demand constraints
- What objective?
  - Value of the remaining inventory?
    - Good for publisher
  - Maximize quality?
    - Good for advertiser
  - As usual need to balance utilities: publisher, advertiser, user, & network!
    - Maintain representativeness!

# Quality of a page view

- Really about the user
  - Demographics
  - Context
- Ultimately about the user's receptiveness to the ad and/or propensity to (eventually) buy what the advertiser wants to sell (more later)
  - Remember: interesting ads are better for both users and advertisers
- Subjective and fuzzy
  - Examples
    - Female, 25-34
    - Male, CA Bay Area, finance interest
    - Male, US, auto interests
  - Based on both (semi) reliable and soft data: registration data, previous activity, inferred demographics
  - Usually only a “best commercial effort” promise

# Reverse cherry picking & representativeness

[A. Ghosh & al, Bidding for Representative Allocations for Display Advertising, [http://arxiv.org/PS\\_cache/arxiv/pdf/0910/0910.0880v1.pdf](http://arxiv.org/PS_cache/arxiv/pdf/0910/0910.0880v1.pdf) ]

- Unless the targeting is very fine-grained, in the inventory matching a typical contract, there is a wide spectrum of quality
- Contract says: Male, US, auto interests
  - What should be supply to this contract?
    - Is it OK to supply 100% 15 year-old males, daydreams about cars, weekly allowances \$25 ?
    - Advertiser probably wants/expects a representative sample of car-buying US male population
- Lack of representativeness leads to
  - loss of advertisers' trusts in targeting → lower price (“cat in the bag”)
  - mis-formulation of targets → inefficient market

# One more factor: spot market

At serving time:

- Should we sell the current opportunity on the spot market or use it to satisfy one of the contracts?

Remember: Spot market prices vary!

- Spot market prices for the targeted slot (Male, US, auto interests) depends on supply/demand for the given slot
- Advertisers may prefer
  - 9 a.m. viewers over 1 a.m. viewers
  - New York viewers over Bangalore viewers
  - Certain users, depending on the advertisers private info
  - Etc
- Spot market for the target slot depends on the entire market → price is highly variable, but reflects “true value”

# Publisher potential strategies

- Assume publisher has just one GD contract
- Suboptimal strategy:
  - Deliver first all impressions to the contract
  - Only after the contract met, sell in spot market
- Bad for the publisher because the GD pageviews may fetch lot more money on the spot than the contract value

# Publisher potential strategy

- Better strategy
  - Put up every pageview on auction (as a seller)
  - Also place a bid on it for the contract (as a buyer)
    - Value determined by probability & penalty of not fulfilling the contract

# Publisher-optimal bid strategy

- If target is 30 million, place the smallest constant bid in each round so that exactly 30 million pageviews are won
- All excess inventory will be sold to someone else (not the GD contract) at a higher price.
- Unfair to the GD contract
  - All impressions delivered are of low value
    - 2 a.m. viewers
    - viewers from poor neighborhoods
    - basically, viewers nobody wanted! (Winners' curse!)

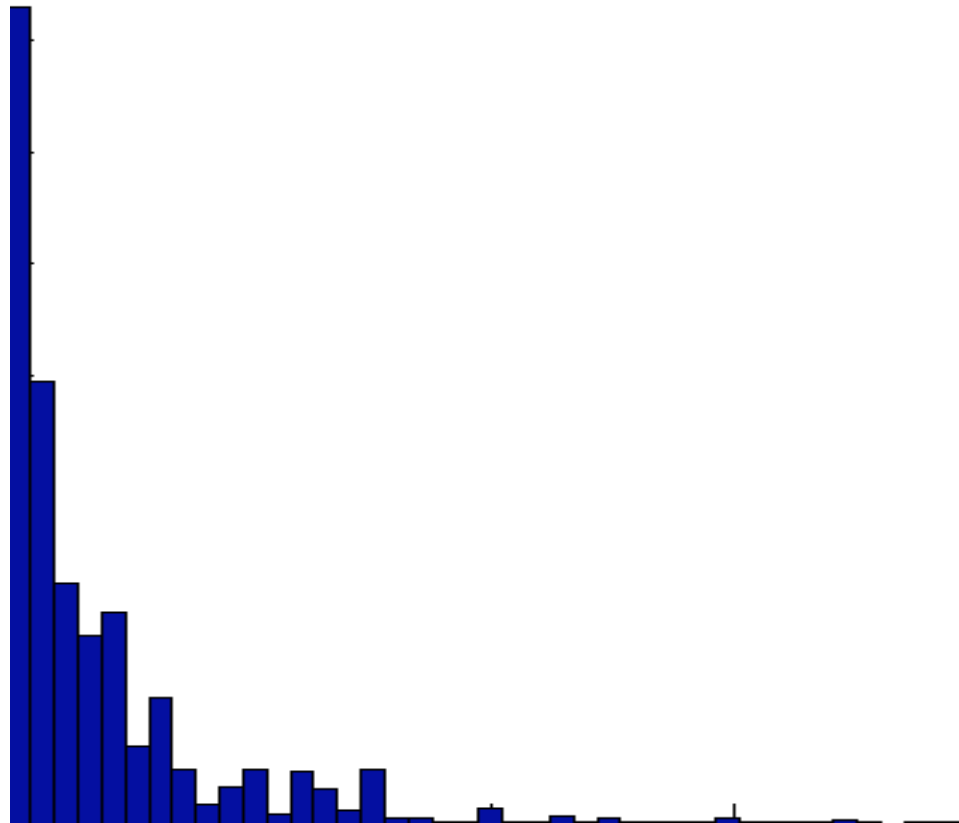


# Problem setting

- Assume the publisher knows the distribution of the external winning bid on the spot market
- Notation
  - $p$  = price (winning bid)
  - $f(p)$  = price density = the highest bid is drawn i.i.d from  $f$
  - $s$  = total supply (inventory) of impressions
  - $d$  = demand (GD volume) for the contract
  - $t$  = target spend per impression (budget)
- $d/s$  is the fraction of the total supply that needs to be delivered to the (**unique!**) contract

# Real-world example: Volume vs.. price of winning bids on spot market

**Volume =  
number of  
impressions  
sold at p**

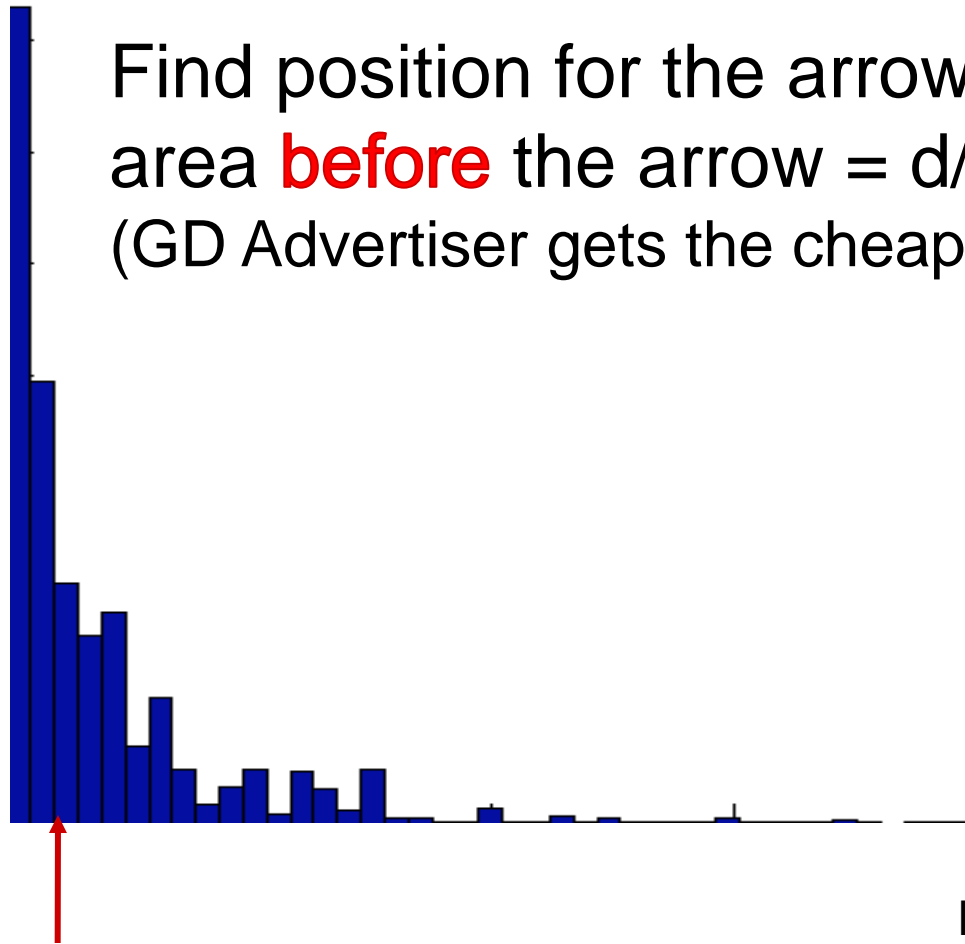


**Price**

# Publisher-Optimal

Volume

Find position for the arrow such that  
area **before** the arrow =  $d/s$   
(GD Advertiser gets the cheapest stuff)

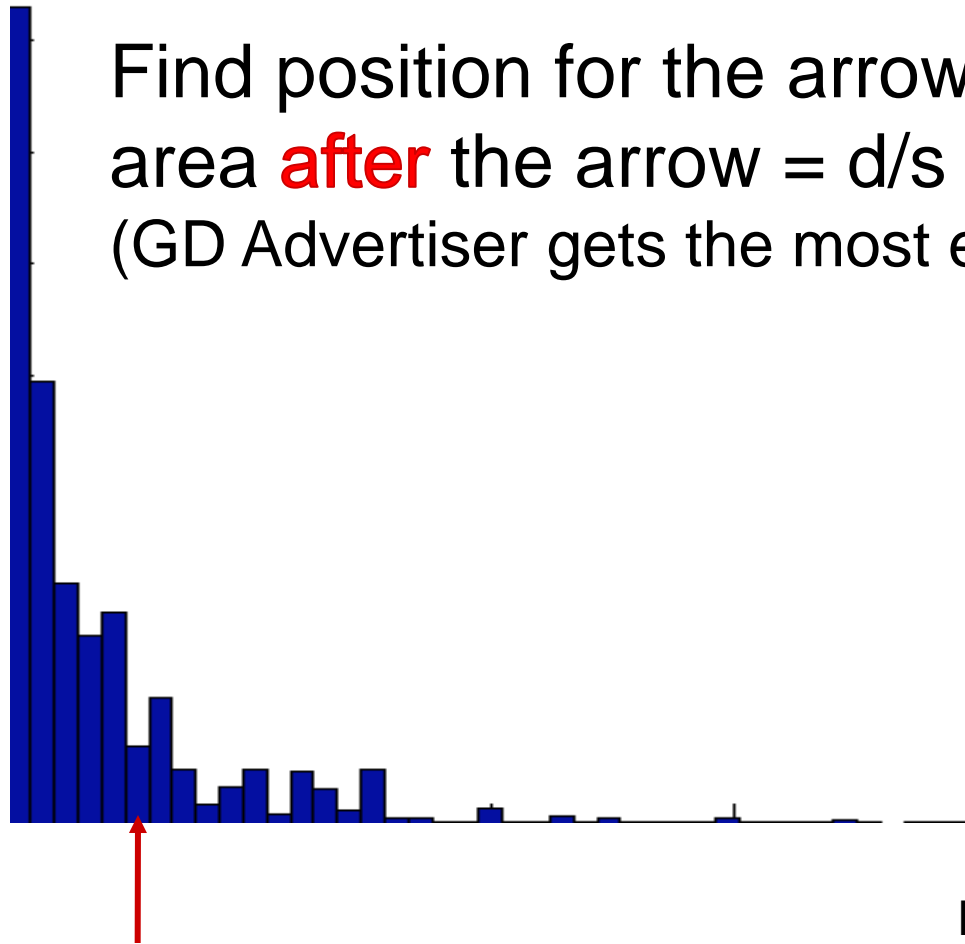


Price

# Advertiser-Optimal

Volume

Find position for the arrow such that  
area **after** the arrow =  $d/s$   
(GD Advertiser gets the most expensive stuff)



Price

# Reasonable compromise

- Allocate to the contract inventory at various prices proportional to the supply
- Example: Suppose  $\frac{2}{3}$  of the supply sells for \$1 and  $\frac{1}{3}$  sells for \$2 and we need 30M impressions
  - GD contract should get 20 million impressions that sell for \$1 and 10 million that sell for \$2

# Alternate view

- Of the supply selling at every price, give d/s fraction to the GD contract.
- Then, price distribution in GD mirrors the intrinsic distribution in the total supply.
- Objective function must penalize deviation from this ideal

# Allocation $a(p)$

- $a(p)/s$  = fractional allocation to GD at price  $p$ , that is:
  - There are  $s \cdot f(p) \cdot dp$  impressions available at price  $p$
  - The GD contract gets
$$a(p)/s * s \cdot f(p) \cdot dp = a(p) \cdot f(p) \cdot dp$$
impressions at price  $p$
- Ideal:  $a(p)/s = d/s$  for all  $p$

# Allocation Constraints

$$0 \leq a(p) \leq s$$

$$\int_0^{\infty} a(p) f(p) dp = d$$

- All integrals are Lebesgue,  $a()$  is not assumed continuous a priori
- If indeed  $a(p)/s = d/s$  for all  $p$  constraint is satisfied!



# Allocation Constraints..

$$\int_0^{\infty} pa(p)f(p)dp$$

= the dollar amount spent on meeting the contract. So we must have

$$\int_0^{\infty} pa(p)f(p)dp \leq td$$

Recall  $t$  = the average budget per impression.  
(Publisher does not pay more than the average he offered per impression. Note that what the publisher is willing to pay for the impressions he buys from the NGD market has nothing to do with what the advertiser paid for the NG contract)

# Objective function

$$\int_0^{\infty} u(a(p), d) f(p) dp$$

where  $u$  is a utility function that measures the divergence between the allocation  $a(p)/s$  and the ideal distribution  $d/s$ .

# Final Optimization Problem

$$\inf_{a(\cdot)} \int_0^{\infty} u(a(p), d) f(p) dp$$

subject to

$$0 \leq a(p) \leq s$$

$$\int_0^{\infty} a(p) f(p) dp = d$$

$$\int_0^{\infty} p a(p) f(p) dp \leq td$$

# Possible distance: Kullback-Leibler divergence

- K-L divergence between two nonnegative functions is

$$\int_0^{\infty} \left[ a(p) \log \frac{a(p)}{b(p)} + b(p) - a(p) \right] dp$$

Maximum entropy when b is uniform

# K-L Optimization Problem

$$\inf_{a(\cdot)} \int_0^{\infty} a(p) \log \frac{a(p)}{d} f(p) dp$$

subject to

$$0 \leq a(p) \leq s$$

$$\int_0^{\infty} a(p) f(p) dp = d$$

$$\int_0^{\infty} p a(p) f(p) dp \leq td$$

# Bidding strategy

- How can we implement the optimal allocation  $a(p)$  in the auction environment?
- We have to bid randomly!
  - Bidding the same amount each round is suboptimal (except when the budget is tightest)

# Stochastic Bidding (simplified)

- Recall  $a(p)/s$  is the fraction of supply available at price  $p$  that should be won for GD
- Let  $H(x)$  be the GD bid distribution (cdf)
- Fix a price  $p$  in the NGD side, what fraction of the supply will be won for GD?
  - Fraction won =  $\text{prob}\{\text{GD bid} > p\} = 1 - H(p)$
- Thus we must define  $H(p)$  via:  $a(p)/s = 1 - H(p)$ 
  - Need  $a(p)$  to be monotone non-increasing!!

# Stochastic Bidding

- With probability  $a(0)$ , we enter the auction.
- If we bid, the bid amount is drawn randomly from  $H$
- Reading assignment: Arpita Ghosh & al: Bidding for Representative Allocations for Display Advertising. WINE 2009, 208-219 (we'll have the file on the web site)



# Reality Objective Function (multiple contracts, L2 norm, etc)

Representative-ness of **guaranteed** - defined as distance of allocation relative to a target allocation.

Expected revenue opportunity from **non-guaranteed** demand

$$\min \Phi = \underbrace{\gamma}_{\text{Guaranteed - Non Guaranteed Tradeoff}} \left\{ \frac{1}{2} \sum_j \underbrace{V^j}_{\text{Campaign Priority}} Y^j \sum_{i \in B^j} \frac{\sum_{k \in B^j} x_k}{x_i} \left\{ \underbrace{\frac{x_i}{\sum_{k \in B^j} x_k}}_{\text{Target Allocation}} \frac{y_i^j}{Y^j} \right\}^2 \right\} - \sum_i [r_i z_i]$$

$$x_i \geq \sum_{j|i \in B^j} y_i^j \quad Y^j = \sum_{i \in B^j} y_i^j$$

$Y^j$ : Requested demand for contract  $j$

$B^j$ : Eligible impressions for contract  $j$

$x_i$ : Available supply for impression  $i$

$r_i$ : Opportunity cost for impression  $i$

$z_i$ : Slack for impression  $i$

$y_i^j$ : Amount of impression  $i$  supply allocated to contract  $j$

# Summary

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# Key points

## Display advertising

- Complex optimization problem – a lot more math than you might suspect
- Interplay of forecasting, optimization, economics
- Need to have solutions for:
  1. Forecast supply, demand, NGD pricing
  2. Admission control
  3. Pricing
  4. Optimal allocation of impressions to contracts
  5. Ad serving

# Questions?

We welcome suggestions about all aspects of the course: [msande239-aut0910-staff](mailto:msande239-aut0910-staff)

# Thank you!

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