CS 2233 Homework 3

1. Contrapositive and contradiction (4 points)

Consider the following claim:

For all integers m and n, if (m − n) is odd then m is odd or n is odd.

SET UP

p := (m-n is odd) ::::: (m-n = 2c+1) for c $\in \mathbb{Z}$

x := (m is odd) :::: (m = 2a+1) for a $\in \mathbb{Z}$

y := (n is odd) :::: (n = 2b+1) for b $\in \mathbb{Z}$

q := (x or y)

(1) (2 points) Prove the claim using a proof by contrapositive

$p \rightarrow q$

$\neg q \rightarrow \neg p$ (Contrapositive)

$\neg(x \lor y) \rightarrow \neg p$ (Replace q with its definition)

$\neg(x \lor y) == \neg x \land \neg y$ (DeMorgan's Law)

$\neg x \land \neg y \rightarrow \neg p$ (Replace line 3 with the new DeMorgan's version)

"if m is not odd and n is not odd, then m-n is not odd"

Extra:

$\neg(m = 2a+1) \rightarrow m = 2a$ for $a \in \mathbb{Z}$ Showing that m is even
$\neg(n = 2b+1) \rightarrow n = 2b$ for $b \in \mathbb{Z}$ Showing that n is even

$m - n = 2a - 2b = 2(a-b)$ therefore m-n is even by the definition of an even number.

$\neg(m\text{-}n = 2c) \rightarrow (m\text{-}n = 2c+1)$

(2) (2 points) Prove the claim using a proof by contradiction

Assuming m-n is odd , if m and n are both even:

$m = 2k, k\in\mathbb{Z}$

$n = 2g, g\in\mathbb{Z}$

$m\text{-}n = 2k - 2g$

$a = k - g$

$m\text{-}n = 2a$

m-n is even

Contradiction

$$\forall x \in \mathbb{R} : x \in \mathbb{Q} \Leftrightarrow x-5 \in \mathbb{Q} \Leftrightarrow x/3 \in \mathbb{Q}$$

$$p := x \in \mathbb{Q}$$
$$q := x-5 \in \mathbb{Q}$$
$$r := x/3 \in \mathbb{Q}$$

$$p \Leftrightarrow q \Leftrightarrow r \equiv (p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)$$

$$(p \Leftrightarrow q) \Leftrightarrow r$$
$$s := (p \Leftrightarrow q) \equiv (x \in \mathbb{Q} \rightarrow x-5 \in \mathbb{Q}) \wedge (x-5 \in \mathbb{Q} \rightarrow x \in \mathbb{Q})$$
$$x \in \mathbb{Q} \rightarrow x = \frac{a}{b}$$
$$x-5 \in \mathbb{Q} \rightarrow x-5 = \left(\frac{a}{b}\right) - 5$$
$$(x \in \mathbb{Q} \rightarrow x-5 \in \mathbb{Q}) = true, (x-5 \in \mathbb{Q} \rightarrow x \in \mathbb{Q}) = true$$
$$s = true$$

$$t := (s \Leftrightarrow r) \equiv (s \rightarrow x/3 \in \mathbb{Q}) \wedge (x/3 \in \mathbb{Q} \rightarrow s)$$

$$x/3 \in \mathbb{Q} \rightarrow \frac{\left(\frac{a}{b}\right)}{3} = \left(\frac{a}{b}\right) * \left(\frac{1}{3}\right)$$
$$r = true$$

$$(s \rightarrow r) = true, (r \rightarrow s) = true$$
$$(s \Leftrightarrow r) = true$$
$$s = p \Leftrightarrow q$$

$$(p \Leftrightarrow q) \Leftrightarrow r = true$$

$$p \Leftrightarrow q \Leftrightarrow r = true$$

$$x \in \mathbb{Q} \Leftrightarrow x-5 \in \mathbb{Q} \Leftrightarrow x/3 \in \mathbb{Q} = true$$

2.

3.

		A	B	C	D	E	F	G	H	I
X	Y	Max(x,y)	Min(x,y)	$(A+B)^2$	A*B	C+D	x^2	3xy	y^2	F+G+H
z+1	z	z+1	z	$4z^2+4z+1$	z^2+z	$5z^2+5z+1$	z^2+2z+1	$3z^2+3z$	z^2	$5z^2+5z+1$

z	z+1	z+1	z	$4z^2+4z+1$	z^2+z	$5z^2+5z+1$	z^2	$3z^2+3z$	z^2+2z+1	$5z^2+5z+1$
z	z	z	z	$4z^2$	z^2	$5z^2$	z^2	$3z^2$	z^2	$5z^2$

Link to the work spreadsheet:
https://docs.google.com/spreadsheets/d/1YMrbVXUFwMJAai-7e2hO3nYh69NKyHjTLki3-mS-cPg/edit?usp=sharin

4.

(1)

X = 2, y = ½, both $x \in \mathbb{Q}$ and $y \in \mathbb{Q}$ but 2 to the power ½ (2^(½) = $\sqrt{2}$, which is irrational, therefore it is false, disproven by case, since the result is irrational, but the neither base nor the exponent are irrational

(2)

Proof by contrapositive:

Assume x is rational, $x \in \mathbb{Q}$, $x=p/q$, $x^2 == p^2/q^2$, both $p,q \in \mathbb{Z}$, therefore, since x can be described by an integer over an integer, $x^2 \in \mathbb{Q}$

If x is not irrational, then the square of x is not irrational as well

Work:

//x = a / b -> $x^2 = (a/b)(a/b) = M$, $M \in \mathbb{Q}$

//$x \in \mathbb{Q}$ -> x = (p/q), $p/q \in Z$ :: $p^2, q^2 \in Z$

//$x^2 = (p^2 / q^2) \in \mathbb{Q}$

5. Sets (6 points)

(1) (1.5 points) Use set builder notation to give a description of the set $\{-3, -2, -1, 0, 1, 2, 3, 4, 5\}$.

$\{ x \mid -3 \le x \le 5, x \in \mathbb{Z} \}$

(2) (2 points) Let A = {a, b, c}, B = {x, y}, and C = {5, 10}.

Find $A \times B \times C$ and $C \times A \times B$.

A x B x C = { (a,x,5), (a,x,10), (a,y,5), (a,y,10), (b,x,5), (b,x,10), (c,x,5), (c,y,5), (b,x,10), (b,y,10), (c,x,10), (c,y,10)}

C x A x B = { (5,a,x) , (5,a,y) , (5,b,x) , (5,b,y) , (5,c,x) , (5,c,y) , (10,a,x) , (10,a,y) , (10,b,x) , (10,b,y) , (10,c,x) , (10,c,y)}

(3) Let A = {1,4,8,16}, B = {2,4,16,32,64)

A∪ B = {1,2,4,8,16,32,64}

A∩B = {4,16}

A\B = {1, 8}

B\A = {2, 32, 64}

| P(A) | = 2 to the power (|A|) (2^(|A|)) = 2^4 = 16

6: A ∪ (A ∩ B) = A

Work:
//x ∈ A ∩ B ≡ x ∈ A ∧ x ∈ B
//{x | x ∈ A ∧ x ∈ B}
//x in A ∪ B -> x in A or x in B

P == {x | x∈A}
Q == {x | x∈B}

A ∪ (A ∩ B)	Original problem
A ∩ B = P ^ Q	Definition of Intersection
A ∪ (P ^ Q)	Replacing the Intersection with Definition

$A \cup (P \wedge Q) = P \vee (P \wedge Q)$ Replacing the Union with the Definition of a Union
$P \vee (P \wedge Q) = P$ Absorption Law

P = All x's in A, therefore P is equal to the set of A
Therefore, $A \cup (A \cap B) = P$, $P = A$, $A \cup (A \cap B) = A$

(P := A)
(Q:= B)
($P \cup (P \cap Q) = P$, $A \cup (A \cap B) = A$)