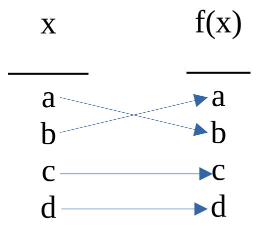
Homework 4
CS2233

1: Functions

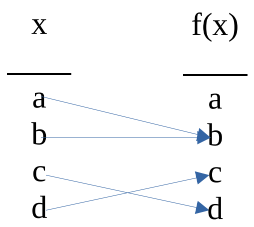
1)

a:



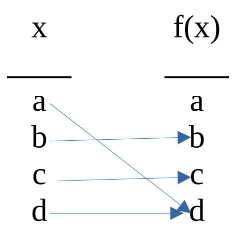
One to One, as each input has a distinct output. Onto because there are no outputs without an input.

b:



Not onto or one-to-one. f(a) and f(b) give the same output. There is no input that gives a as an output.

c:



Not onto or one-to-one. f(a) and f(d) give the same output. There is no input that gives a as an output.

2)

a: Yes, one to one correspondence. Each x has exactly 1 y and all y values have an x.
b: No. Not onto , as all positive y values above 7 have no input. Not one to one, as all y values less than 7 have 2 inputs.
c: Not one to one as multiple x values point to the same y, but onto as all y values have some x value. $f(-2) = 0$, $f(0) = 0$, $f(1) = 0$

3)

$f(x) = 3x+4$, $g(x) = x^2$

$f(g(x)) = f(x^2) = 3(x^2)+4 = 3x^2+4$

$g(f(x)) = (3x+4)^2 = 9x^2+24x+16$

4)

a: f(x) = x^2
b: f(x) = $\lceil x/2 \rceil$
c: f(x) = 1

2: Sequences and Summations

1)

a: $1, -3, 9, -27, 81$ if $\mathbb{N}$ includes 0
b: $2,0,2,0,2$ if $\mathbb{N}$ includes 0

2)

$$\sum_{i=0}^{3} (-2)^i = -5$$

3)

a: $\sum_{n=0}^{\infty}$ 6n+4

b: $\sum_{n=0}^{\infty} 5(3^n)$

c: $\sum_{n=1}^{\infty} 5(-1^n) + 15$

3. Growth of Functions

1)

a: No, O(x)
b: Yes
c: No, O(x^3)
d: No, O(x log x)

2)

$$f(n) = 5n^5 + 4n^4 + 3n^3 + 2n^2 + n$$
$$f(n) \in \Theta(n^5) \quad \Leftrightarrow \quad f(n) \in O(n^5) \wedge f(n) \in \Omega(n^5)$$
$$f(n) \in O(n^5) \rightarrow f(n) \leq C * n^5$$
$$f(n) \in \Omega(n^5) \rightarrow f(n) \geq C * n^5$$

Highest order term is n^5 therefore... $f(n) \in O(n^5) \wedge f(n) \in \Omega(n^5)$
So by definition, $f(n) \in \Theta(n^5)$

$$f(n) = 2n^3 - n + 10$$

$$f(n) \in \Theta(n^3) \quad \Leftrightarrow \quad f(n) \in O(n^3) \wedge f(n) \in \Omega(n^3)$$

3) $$f(n) \in O(n^3) \rightarrow f(n) \leq C * n^3$$

$$f(n) \in \Omega(n^3) \rightarrow f(n) \geq C * n^3$$

Highest order term is n^3 therefore... $f(n) \in O(n^3) \wedge f(n) \in \Omega(n^3)$

So by definition, $f(n) \in \Theta(n^3)$

rule: $f(n) \in O(g(n)) \Leftrightarrow \exists_{(c>0)}, \exists_{((n_0)>0)}, \forall_{(n \geq n_0)} : f(n) \geq c * g(n)$

$$f(n) \in O(h(n)) \rightarrow f(n) \geq a * h(n)$$

$$f(n) = a * n^i$$

$$g(n) \in O(h(n)) \rightarrow g(n) \geq b * h(n)$$

$$g(n) = b * n^i$$

4) $$f(n) \in O(n^i) \wedge g(n) \in O(n^i)$$

$$f(n) + g(n) = (a * n^i) + (b * n^i) = n^i(a+b) : a+b \in \mathbb{R}^+$$

assuming $a+b=c$ ...

$$f(n) + g(n) = c(n^i)$$

$$c(n^i) \in O(n^i)$$