

# Character corruption

On 3<sup>rd</sup> page, it misreads the characters for every item in a set, and for one item in a set

Homework 2  
CS2233

## 1. Predicates and Quantifiers (7 points)

(1) (3 points) Let  $L(x)$  denote the statement “ $x$  has used calculus.” and  $D$  denote the domain of all students in our class. Express each of these quantified propositions in English:

(a)  $\forall x \in D : L(x)$

“For every student in the domain of all students in our class, it is the case that the student has used calculus”

(b)  $\neg \exists x \in D : L(x)$

$\neg \exists x \in D : L(x) \quad \forall x \in D : \neg L(x) \quad \leftarrow$  DeMorgan’s Law for Quantifiers

“There is no student in the domain of all students in our class such that the student has used calculus”  
or

“For every student in the domain of all students in our class, it is the case that the student has not used calculus”  $\leftarrow$  DeMorgan’s Law version

(c)  $\exists x \in D : \neg L(x)$

“There exists a student in the domain of all students in our class such that the student has not used calculus”

1:(2) (2 points) Let  $Q(x)$  denote the statement “ $2^x > 3x$ ” and  $Z$  denote all integers (i.e.,  $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$ ). Determine the truth value of each of these statements:

(a)  $Q(2)$

$$Q(2) = 2^{(2)} > 3(2) = \text{False}$$

(b)  $Q(4)$

$$Q(4) = 2^{(4)} > 3(4) = \text{True}$$

$$(c) \forall x \in \mathbb{Z} : (Q(x) \vee x < 4)$$

True

$$(d) \exists x \in \mathbb{Z} : (Q(x) \wedge x < 4)$$

True

(3) (2 points) Translate the following statements to English where  $B(x)$  is “x understands boolean algebra” and  $M(x)$  is “x has taken discrete math” and the domain  $D$  is all students at UTSA.

$$(a) \forall x \in D : (M(x) \rightarrow B(x))$$

“For every student in the domain of all students at UTSA it is the case that if the student has taken discrete math, then the student understands boolean algebra”

$$(b) \exists x \in D : (B(x) \wedge \neg M(x))$$

“There exists a student in the domain of all students at UTSA such that the student understands boolean algebra and has not taken discrete math”

## 2. Nested Quantifiers (3 points)

Let  $K(x, y)$  denote the statement “x knows y” and  $D$  denote the domain of all people. Express the following English sentences as a quantified proposition using the definitions above:

(1) Everybody knows somebody.

$$\forall x \in D \exists y \in D : K(x, y)$$

(2) There is somebody that no one knows.

$$\exists y \in D \forall x \in D : \neg K(x, y)$$

(3) There is no one who knows everybody.

$$\neg \exists x \in D \forall y \in D : K(x, y)$$

### 3. Negating Quantifiers (3 points)

Rewrite each of these statements such that all of the negation symbols (i.e.,  $\neg$ ) are in front of the propositional functions P or Q.

(1)  $\neg \exists x : (P(x) \wedge Q(x))$

$$\forall x : \neg P(x) \vee \neg Q(x)$$

(2)  $\neg \forall x \exists y \forall z : (P(x, y) \rightarrow Q(z, y))$

$$\exists x \forall y \exists z : P(x, y) \wedge \neg Q(z, y)$$

### 4. Negation (6 points)

Let  $S(x, y)$  denote the statement “x has seen y” and D denote the set of all students in our class and M be the set of all movies.

(1) (2 points) Express the following English sentence as a quantified proposition using the definitions above:

“For every movie there is a pair of students in our class who have both seen it.”

$$\forall y \in M \exists x \exists z \in D : S(x, y) \wedge S(z, y)$$

(2) (2 points) Negate the quantified proposition you wrote for part (1) (i.e., place a “ $\neg$ ” in front of it). Use de Morgan’s law for quantifiers to move the negation inside the quantifiers.

$$\neg (\forall y \in M \exists x \exists z \in D : S(x, y) \wedge S(z, y))$$

$$\exists y \in M \neg (\exists x \exists z \in D : S(x, y) \wedge S(z, y))$$

$$\exists y \in M \forall x \forall z \in D : \neg (S(x, y) \wedge S(z, y))$$

$$\exists y \in M \forall x \forall z \in D : \neg S(x, y) \vee \neg S(z, y)$$

(3) (2 points) Translate your answer for part (2) back to plain English.

“There exists some movie such that out of all of the students in our class, no two have seen it”