Homework 1
CS2233

1. Propositions

- p := “You passed CS2233”
- q := “You passed CS3333”
- r := “You can register for CS3343”
- s := “You understand Boolean Algebra”

a: You have not passed CS2233 but you understand boolean algebra.
Rewrite : You understand boolean algebra and you have not passed CS2233.

s ¬p

b: You can not register for CS3343 only if you have not passed both CS2233 and CS3333
Rewrite : If you have not passed CS2233 and CS3333 then you cannot register for CS3343

¬p ¬q ¬r

c: If you can register for CS3343 then you have passed CS2233 and you understand boolean algebra if you passed CS2233.
Rewrite: If you passed CS2233 then you can register for CS3343 and you understand boolean algebra.

p → (r ^ s)

2: Equivalences

1: Show that (¬q∧(p∨ p)) → ¬q is a tautology (i.e. (¬q∧(p∨ p)) → ¬q ≡ T)

1a: Show using a truth table

p	q	¬q	(¬q p)	(¬q p)→¬q	T	(¬q p)→¬q T
T	T	F	F	T	T	T
T	F	T	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T

1b: Show using a sequence of equivalences

(¬q p) ¬q T	Idempotent Law : p p p
¬(¬q p) ¬q T	Table 7 Line 1
LHS = ¬(¬q p) ¬q , RHS = T	Distribute ¬
LHS = ¬(¬q) ¬p ¬q	Double Negation Law : ¬(¬q) q
LHS = q ¬p ¬q	Negation Law : p ¬p T
LHS = T ¬p	Domination law : p T T
LHS = T	
LHS RHS Tautology	

1b alternate sequence:

Idempotent Law	p p=p	¬q p ¬q
Table 7, Line 1	u ¬w = ¬u ¬w, u=(¬ q p), w=(¬q)	¬(¬q p) ¬q
De Morgan's Law	¬(¬q p) = q ¬p	q ¬p ¬q
Negation Law	q ¬q = T	¬p T
Domination Laws	u T = T, u=(¬p)	T

2: Show that ¬q → (p r) (¬q → r) & (q p)

2a: Show using Truth tables

p	q	r	¬q	p r	¬q (p r)	¬q r	q p	(¬q r) (q p)	¬q (p r) (¬q r) (q p)
T	T	T	F	T	T	T	T	T	T
T	T	F	F	F	T	T	T	T	T
T	F	T	T	T	T	T	T	T	T
T	F	F	T	F	F	F	T	F	T
F	T	T	F	F	T	T	T	T	T
F	T	F	F	F	T	T	T	T	T
F	F	T	T	F	F	T	F	F	T
F	F	F	T	F	F	F	F	F	T

2b: Show using a sequence of equivalences

¬q → (p r) (¬q → r) (q p)

q (p r)	Table 7, line 1
(q p) and (q r)	Distributive Law
LHS = (q p) (q r)	
RHS = (¬q → r) (p q)	Commutative law
(q r) (p q)	Table 7 Line 1
RHS = (q p) (q r)	

LHS RHS

Tautology

2b alternate sequence :

		¬q → (p r) (¬q→ r) (q p)
Table 7, Line 1	u→w = ¬u w	¬(¬q) (p r) (¬(¬q) r) (q p) u= ¬q w= (p r), r
Double Negation Law	¬(¬q) = q	q (p r) (q r) (q p)
Distributive Law	(q r) (q p) = q (r p)	q (p r) q (p r)

3. Additional Operators

1: Using truth tables, show that   is associative. i.e., $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

p	q	r	q r	(p (q r))	p q	((p q) r)	((p q) r) (p (q r))
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	F	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	T	F	F	F	T
F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	F	F	F	F	T

2: Using truth tables show that $\uparrow$ is not associative,
i.e., $p \uparrow (q \uparrow r) \neq (p \uparrow q) \uparrow r$

p	q	r	$q \uparrow r$	$p \uparrow (q \uparrow r)$	$p \uparrow q$	$(p \uparrow q) \uparrow r$	$((p \uparrow q) \uparrow r) \quad (p \uparrow (q \uparrow r))$
T	T	T	F	T	F	T	T
T	T	F	T	F	F	T	F
T	F	T	T	F	T	F	T
T	F	F	T	F	T	T	F
F	T	T	F	T	T	F	F
F	T	F	T	T	T	T	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T