Homework 1 (Due: March 20th)

P. 58–61

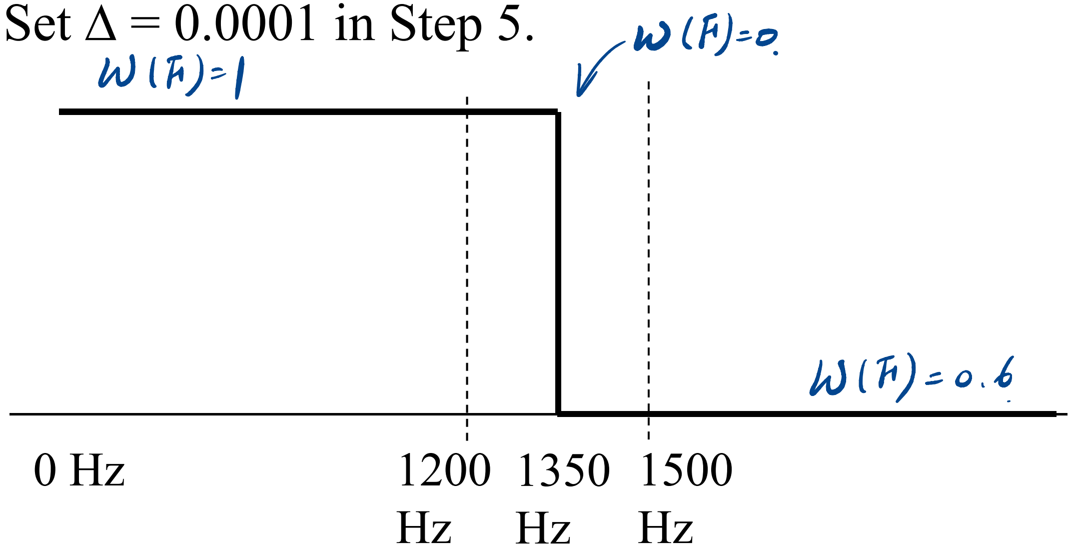
(1) Design a Mini-max **lowpass** FIR filter such that (40 scores)

① Filter length = 17, ② Sampling frequency f_s = 6000Hz,

③ Pass Band 0~1200Hz ④ Transition band: 1200~1500 Hz,

⑤ Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.6 for stop band .

⑥ Set Δ = 0.0001 in Step 5.



※ The code should be handed out by NTUCool, too.

Show (a) the frequency response, (b) the impulse response $h[n]$, and (c) the maximal error for each iteration.

$$\begin{cases} u[n]=1 & \text{for } n \geq 0 \\ u[n]=0 & \text{for } n < 0 \end{cases}$$

IIR filter

講義

(2) How do we implement $y[n] = x[n] *(0.8^n u[n] + 0.5^n u[n])$ efficiently where * means convolution and $u[n]$ is the unit step function? (10 scores)

(3) (a) What are the two main advantages of the Fourier transform (FT)? (b) What are the two main problems to implement the FT? (10 scores)

(4) Suppose that $x[n] = y(0.002n)$ and the length of $x[n]$ is 2000. If $X[m]$ is the FFT of $x[n]$, which frequencies do (a) $X[200]$ and (b) $X[1600]$ correspond to? (10 scores)

fast Fourier transform.

(5) Why (a) the step invariance method and (b) the bilinear transform can reduce or avoid the aliasing effect in IIR filter design? (10 scores)

$h[n] = -h[-n]$ $h[n] = h[-n]$

(6) (a) Which of the following filters are usually even? (b) Which of the following filters are usually odd? (i) Notch filter; (ii) highpass filter; (iii) edge detector; (iv) integral; (v) differentiation 4 times; (vi) particle filter; (vii) matched filter. (10 scores)

(7) Use the MSE method to design the 7-point FIR filter that approximates the highpass filter of $H_d(F) = 1$ for $|F| < 0.25$ and $H_d(F) = 0$ for $0.25 < |F| < 0.5$.

lowpass.

(15 scores)

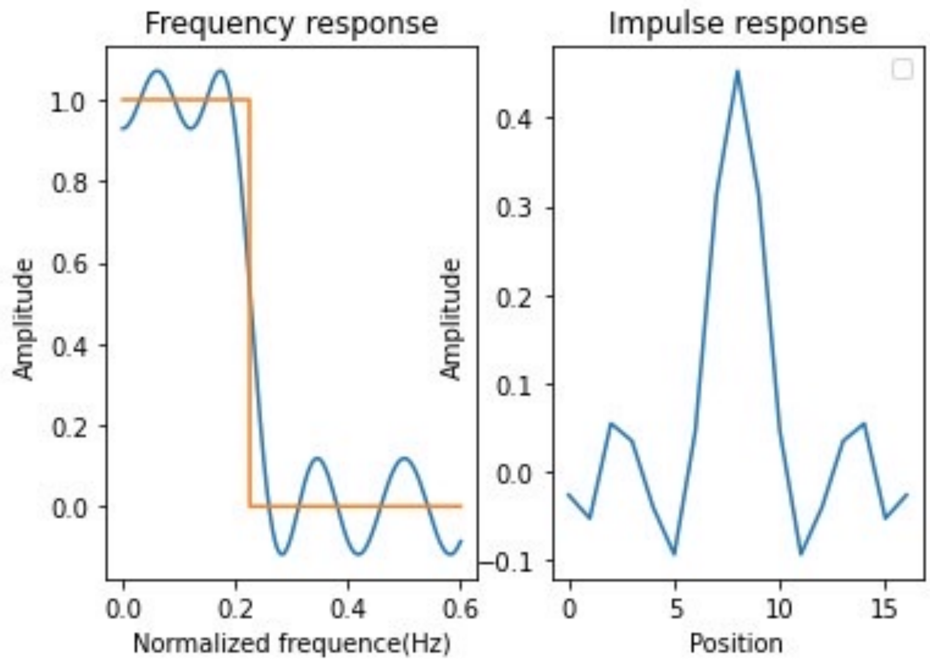
(Extra): Answer the questions according to your student ID number.

(ended with 0, 1, 2, 3, 5, 6, 7, 8)

(8) for $f = -150\,Hz$ $m = ?$

$$f = m\Delta f = \frac{m}{N\Delta t} = \frac{m f_s}{N}$$

(1)



[0.11154526203134667, 0.07725717699330047, 0.0712000982779512, 0.07117214412329143]

(2) $y[n] = x[n] * (0.8^n u[n] + 0.5^n u[n])$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} 0.8^n z^{-n} + \sum_{n=0}^{\infty} 0.5^n z^{-n}$$

$$= \frac{1}{1-0.8z^{-1}} + \frac{1}{1-0.5z^{-1}}$$

z transform $\downarrow$

$$Y(z) = X(z) H(z)$$

$$Y(z) = X(z)\left(\frac{1}{1-0.8z^{-1}} + \frac{1}{1-0.5z^{-1}}\right)$$

$$Y(z) = X(z)\left(\frac{2-1.3z^{-1}}{(1-0.8z^{-1})(1-0.5z^{-1})}\right)$$

$$Y(z) = X(z)\left(\frac{2-1.3z^{-1}}{1-1.3z^{-1}+0.4z^{-2}}\right)$$

$$(1-1.3z^{-1}+0.4z^{-2})\, Y(z) = X(z)\,(2-1.3z^{-1})$$

$$Y(z) - 1.3z^{-1}Y(z) + 0.4z^{-2}Y(z) = 2X(z) - 1.3z^{-1}X(z)$$

$z^{-1} \downarrow$

$$y[n] - 1.3y[n-1] + 0.4y[n-2] = 2x[n] - 1.3x[n-1]$$

$$y[n] = 2x[n] - 1.3x[n-1] + 1.3y[n-1] - 0.4y[n-2]$$ #

(3) ⓐ advantages:

1. Frequency analysis:

By transforming signals from the time domain to the frequency domain, it becomes easier to analyze and manipulate signals, particalarly for applications such as filtering, modulation, and spectral analysis.

2. Solving Differential Equations:

It can simplify the solution of differential equations, particalarly linear ordinary and partial differential equations, by transforming them into algebraic equations in the frequency domain. (convolution → multiplication)

(b) disadvantages:

1. Computational Complexity:

The computation of the Fourier Transform involves a significant amount of arithmetic operations, particalarly for large datasets.

2. Windowing and Boundary Effects:

When performing the Fourier Transform on finite-duration signals, windowing and boundary effects can distort the frequency domain representation of signal.

(4)

$x[n] = y(0.002n)$ $\quad N = 2000 \quad \Delta t = 0.002$

(a) $X[m] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi mn}{N}} \Rightarrow f = \frac{m}{N} f_s \ , \ f_s = \frac{1}{\Delta t}$

$X[200] \Rightarrow \frac{200}{2000} \times 500 = 50$ #

(b)

$X[1600] \Rightarrow \frac{1600}{2000} \times 500 = 400$

$\because 1600 > \frac{N}{2}$

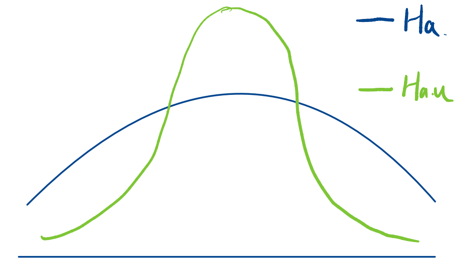
$\therefore 400 - 500 = -100$ #

(5) (a) step invariance:

Calculate the convolution of $h_a(t)$ and $u(t)$

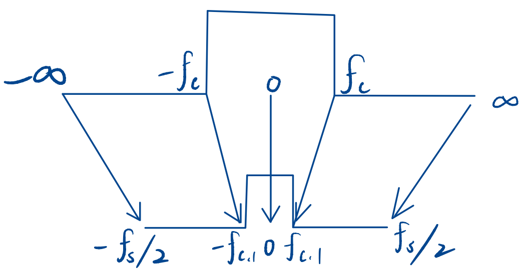
$$h_{a.u}(t) = h_a(t) * u(t) = \int_{-\infty}^{\infty} h_a(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} h_a(\tau)\, d\tau$$

$$H_{a.u}(f) = \frac{H_a(f)}{j2\pi f}$$ f值越大 $H_{a.u}$值越小



aliasing effect 常發生在高頻 → 壓低高頻成份可使 aliasing effect 降低

(b) Bilinear Transform



The bilinear transform maps the analog frequency axis into the digital frequency axis using a nonlinear transformation. This transformation effectively maps the entire analog frequency range ($-\infty$ to ∞) into the digital frequency range ($-\pi$ to π)

$$f_{c,1} = \frac{f_s}{\pi}\operatorname{atan}\left(\frac{2\pi}{c} f_c\right) \qquad -\infty < f_c < \infty$$

(b)

(a) Usually even filters :

(i) notch filter (ii) highpass filter (v) differentiation 4 times filter

(b) Usually odd filters :

(iii) edge detector (iv) integral filter

(c) 不可判斷

(vi) particle filter (vii) matched filter

(7) $H_d(F) = 1$ for $|F| < 0.25$,

$H_d(F) = 0$ for $0.25 < |F| < 0.5$ $\quad n = 7$

$$S[0] = \int_{-1/2}^{1/2} H_d(F)\,dF = \int_{-\frac{1}{4}}^{\frac{1}{4}} 1\,dF = \frac{1}{2}$$

$$S[n] = 2\int_{-1/2}^{1/2} \cos(2\pi n F) H_d(F)\,dF = 2\int_{-\frac{1}{4}}^{\frac{1}{4}} \cos(2\pi n F)\,dF$$

$$= 2\cdot\frac{1}{2\pi n}\cdot\left[\sin(2\pi n F)\Big|_{-\frac{1}{4}}^{\frac{1}{4}}\right] = \frac{1}{\pi n}\left(\sin\left(\frac{1}{2}n\pi\right) + \sin\left(-\frac{1}{2}n\pi\right)\right)$$

$$= \frac{2\sin\left(\frac{1}{2}n\pi\right)}{\pi n}.$$

Finally, set $h[k] = S[0]$ $\quad k \Rightarrow \left\lfloor \frac{7}{2} \right\rfloor = 3$.

$h[k+n] = S[n]/2 \quad h[k-n] = S[n]/2$.

$h[0] = h[6] = S[3]/2 = \dfrac{2\sin\left(\frac{3}{2}\pi\right)}{3\pi} \times \dfrac{1}{2} \doteq -0.106$

$h[1] = h[5] = S[2]/2 = \dfrac{2\sin(\pi)}{2\pi} \times \dfrac{1}{2} = 0$

$h[2] = h[4] = S[1]/2 = \dfrac{2\sin\left(\frac{1}{2}\pi\right)}{\pi} \times \dfrac{1}{2} \doteq 0.3183$

$h[3] = S[0] = \frac{1}{2}$

(8) for $f=-150\,Hz$ $m=?$ 取樣頻率 6000 $N=30000$

$$f=m\Delta f=\frac{m}{N\Delta t}=\frac{mf_s}{N} \qquad f_s=\frac{1}{\Delta t}$$

$$X[m]\Delta t \simeq Y\left((m-N)\frac{f_s}{N}\right)=Y\left(m\frac{f_s}{N}-f_s\right)=Y(5850)$$

$$5850=m\times\frac{6000}{30000} \Rightarrow m=29250$$