Quizzes - Topology 0*

December 3, 2025

1 Session 03/12/2025

1. Can two metrics on $\mathbb{R}$ give rise to two different topologies? Three metrics?
2. Find a set with infinitely many non homeomorphic topologies?
3. Let (X, τ) be a topological space. If $A \subseteq X$ contains its boundary, is A closed?
4. Give examples of a topological space (X, τ) such that
   - X is homeomorphic to a proper subspace $A \subsetneq X$ with the subspace topology.
   - X is *not* homeomorphic to *any* proper subspace $A \subsetneq X$ with the subspace topology.
5. In a Hausdorff space, can we separate points by closed sets?
6. If A, B are connected subspaaces of (X, τ), and $A \subseteq C \subseteq B$, is C connected?
7. Let $\tau_1 \subseteq \tau_2$ be two different topologies on X. If (X, τ_1) is connected, resp. path-connected, resp. compact, is (X, τ_2) connected, resp. path-connected, resp. compact ? What about the other implication?
8. If $f : X \to Y$ is a continuous map between topological spaces, and X is path connected, is Y path connected?
9. If $f : \mathbb{S}^1 \to \mathbb{R}$ is continuous, does there exist $x \in \mathbb{S}^1$ with $f(x) = f(-x)$?
10. If (X, τ) is *not* simply connected, and $f : X \to Y$ is continuous, can $f(X)$ be simply connected? If instead X is simply connected, is $f(X)$ always simply connected?
11. If X is not simply connected, can $X \times Y$ be simply connected?
12. Let (X, τ) be a topological space and take a subset $A \subseteq X$ with the subspace topology. If A is compact, is $\overline{A}$ (the closure of A) compact? If $\overline{A}$ is compact, is A compact?

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