Technical Note: Bootstrapping

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Objective

In this project, we implemented a bootstrapping framework to construct the zero-coupon yield curve using a set of market instruments. Our objective was to extract implied zero-coupon rates from deposits, forward rate agreements (FRAs), and interest rate swaps (IRS), with increasing maturities, while maintaining numerical robustness and flexibility.

1. Instruments Overview

Deposits

We started with deposits as the simplest instruments. Given a market rate R and a maturity T, we compute the corresponding discount factor $P(T)$ as:

$$P(T) = \frac{1}{1 + R \cdot T}$$

From this, we derive the continuously compounded zero rate r using:

$$r = -\frac{\ln P(T)}{T}$$

Forward Rate Agreements (FRAs)

FRAs are used to lock in an interest rate over a future period $[T_1, T_2]$. Assuming we already know the zero rate r_1 at T_1, and using the market-observed fixed forward rate R_f, we compute the implied zero rate r_2 at T_2 with:

$$r_2 = \frac{(T_2 - T_1) \cdot R_f + T_1 \cdot r_1}{T_2}$$

When r_1 is not directly available from previous instruments, we interpolate it using either linear, quadratic, or cubic spline interpolation based on the existing zero-curve.

Interest Rate Swaps (IRS)

For swaps, we take into account a stream of fixed coupon payments made quarterly up to the swap's maturity. The swap rate is the fixed rate that equates the present value of these cash flows to 1. To derive the zero rate r at maturity T, we numerically solve the following equation:

$$\sum_{i=1}^{n} \text{Coupon} \cdot \Delta t \cdot e^{-r_i t_i} + e^{-rT} = 1$$

Where:

- Coupon is the fixed swap rate,

- $\Delta t = 0.25$ represents quarterly intervals,
- r_i is known or interpolated from the existing curve for time t_i,
- T is the final maturity,
- r is the unknown zero rate at T, solved using numerical root-finding.

This approach ensures that all future cash flows are discounted consistently using the evolving zero-curve.

2. Bootstrapping Methodology

We applied the following process:

1. We first sort all instruments by increasing maturity.
2. For each instrument, we compute the zero rate as follows:
   - For Deposits: directly using the formula above.
   - For FRAs: using the forward rate formula, leveraging already bootstrapped or interpolated values.
   - For Swaps: by solving the present value equation numerically.
3. Each newly derived zero rate is stored and used in the computation of subsequent instruments.

3. Interpolation Techniques

We implemented multiple interpolation schemes to handle non-standard maturities and ensure smoothness of the yield curve:

- Linear interpolation
- Quadratic interpolation
- Cubic spline interpolation (used by default for higher accuracy)

4. Visualization of Results

Below, we present the yield curves obtained from our bootstrapping algorithm. The red dots represent the bootstrapped points. The lines show the interpolated curves using different methods.

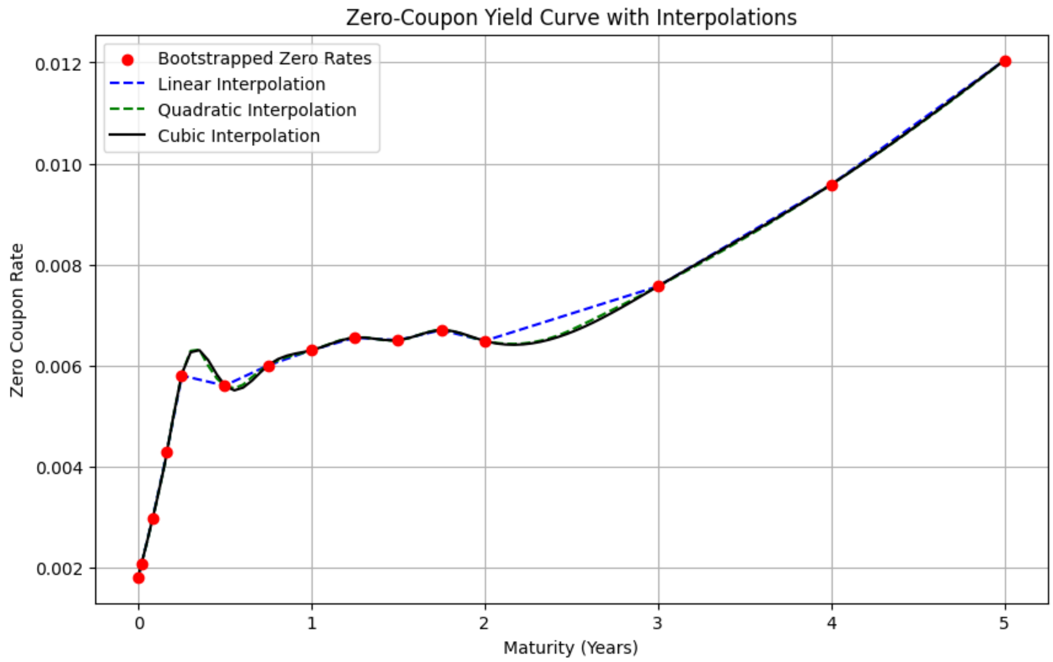

Figure 1: Zero-coupon yield curve using deposits, FRAs, and swaps up to 5 years. Generated using our bootstrapping code.

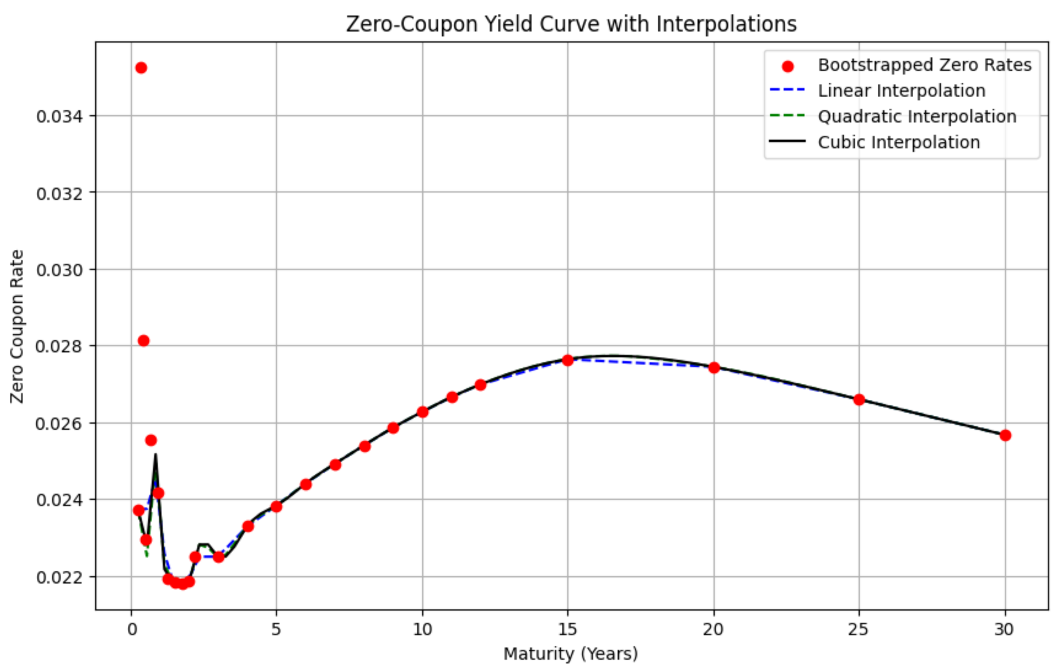

Figure 2: Full yield curve up to 30 years. Generated using our bootstrapping framework.

This result aligns well with the Bloomberg zero-coupon curve shown in the next figure.

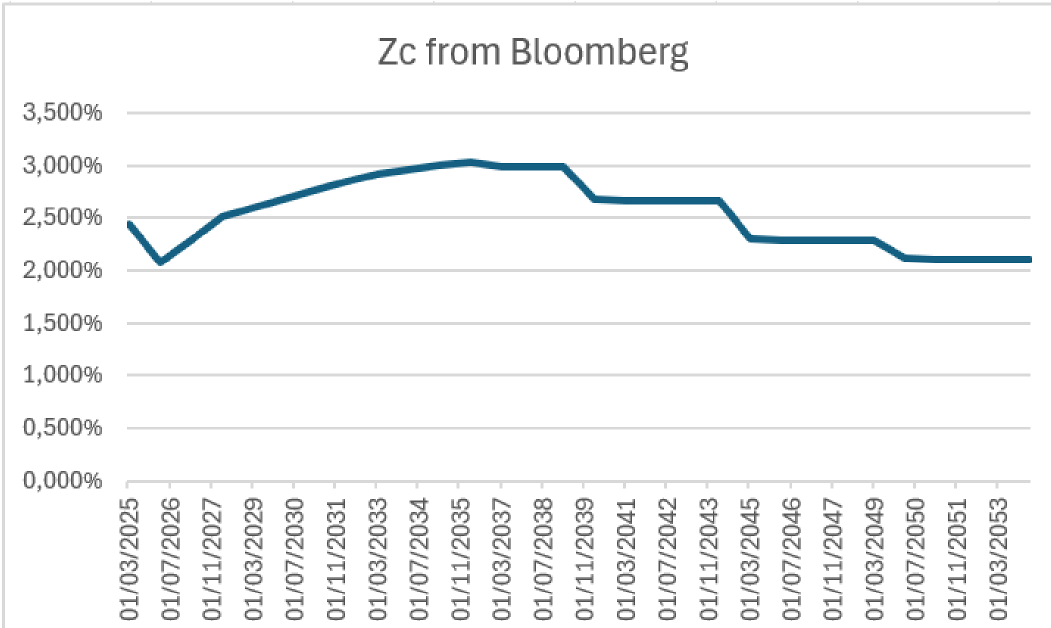

Figure 3: Zero-coupon curve extracted from Bloomberg

5. Conclusion

Through this project, we successfully developed a robust and modular bootstrapping framework capable of deriving a continuous zero-coupon yield curve from real market instruments. We validated our results visually and numerically, and compared them with Bloomberg market data to confirm the realism of the curve. The result shown in Figure 2 is consistent with the Bloomberg benchmark in Figure 3, which validates the accuracy of our model. This tool forms a solid basis for future developments such as curve calibration, sensitivity analysis, and pricing of interest rate derivatives.