STATE-SPACE METHODS OF DECONVOLUTION FOR GEOPHYSICAL DATA PROCESSING

S-F. Yiu, P. C. Young and B. Robinson

Control and Systems Group, Centre for Research on Environmental Systems, Institute of Environmental and Biological Sciences, University of Lancaster, Lancaster LA1 4YQ, UK

Abstract. Seismic data processing is of great economic importance and it is not surprising that many different procedures have been proposed, particularly since the development of modern digital computers. An important innovation in seismic analysis was proposed by Mendel (1977), who was the first to utilise state-space methodology in performing "minimum-variance deconvolution". This novel approach seems to work well in the seismic context but appears to have some limitations when applied in related areas of geophysical data processing.

In this paper, we describe a new approach to the deconvolution problem, based on optimal state-space smoothing procedures, which has been developed for a new geophysical prospecting system. It proves useful if the multiple signals from the system are combined by principal component analysis (PCA) to yield two orthogonal signals which carry most of the information that is useful for the deconvolution analysis. Recursive smoothing algorithms are then applied to the analysis of these orthogonal components with a non-minimum phase wavelet model, which is required to define the algorithm, obtained by optimal instrumental variable methods of estimation. The Expectation and Maximization (EM) algorithm is utilised to estimate the noise process variances. In effect, the smoothing algorithm acts as an off-line, lag-free wave shaping filter, which attenuates the high frequency noise components in the deconvolved trace. The results of the new method are compared with those obtained by Mendel's minimum-variance deconvolution procedure.

Keywords. State-space methods; Kalman filters; optimal filtering; deconvolution; Refined Instrumental Variable method; minimum-variance deconvolution; EM algorithm.

INTRODUCTION

The geophysical exploration system under consideration is designed to investigate the top few metres of the Earth in order to detect buried objects. At an arbitrary position on the ground surface, the total signal received is composed of various components: (1) those due to any target objects below the receiver; and (2) those due to ground clutter, equipment limitations and other "noise" effects. A typical, single, received signal, as shown in Fig. 1, contains both of these effects but is, unfortunately, dominated by the noise components.

Fig. 1. A typical single received signal

In practice, multiple signals, each similar to that shown in Fig. 1, are obtained from the receiver. However, following the procedure used by Kittler and Young (1973) for analysing vector electrocardiogram signals, it has been found that most of the information in these signals is contained in two of the orthogonal components, obtained by the application of Principal Components Analysis (PCA). The details of this part of the data pre-processing are described elsewhere (Yiu, Young and Robinson, 1988), but Fig. 2 shows a pair of orthogonal components, denoted by I and Q, as obtained from a set of measurements made at one location point over made ground composed mainly of crushed dolomite. The lower frequency oscillations in the latter part of each trace are associated with the response to a buried object, while the clutter noise, which has been reduced substantially by the PCA analysis, still dominates the front portion of the trace. The object of subsequent deconvolution of these two orthogonal time series is to improve the detection of the target; in other words, to enhance the target reflections in the latter part of the response.

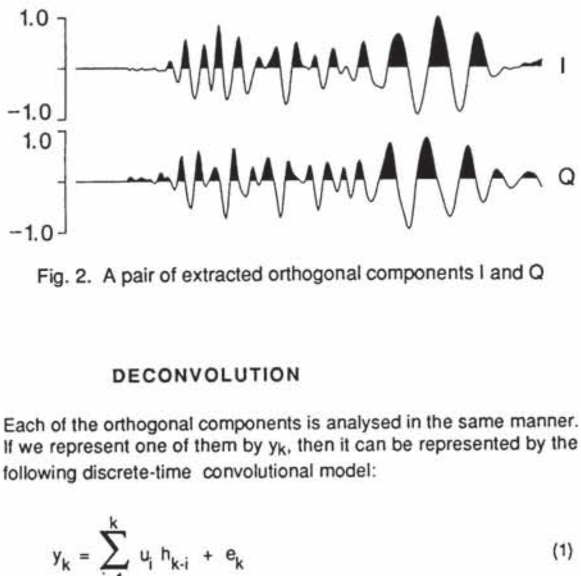

Fig. 2. A pair of extracted orthogonal components I and Q

DECONVOLUTION

Each of the orthogonal components is analysed in the same manner. If we represent one of them by y_k, then it can be represented by the following discrete-time convolutional model:

$$y_k = \sum_{i=1}^{k} u_i h_{k-i} + e_k \tag{1}$$

where h_i, i=0, 1, 2, ..., denotes the impulse response of the system; u_k is the reflectivity sequence, representing the response of the interfaces between the different materials, such as the soil and the target; and e_k is measurement noise. The series h_i can be considered as a response wavelet, similar to a seismic wavelet used in geophysical data processing (Robinson and Treitel, 1980).

Ricker (1940) succinctly summarises the purpose of deconvolution as follows: "The object of deconvolution is to remove the effects of the source wavelet and the noise term from the observed signal, so that one is left with the desired reflection signal, or at least an estimate thereof".

The most common approach to deconvolution has been the design of Wiener inverse filters (Wiener, 1949), which produce a desired output corresponding to each reflected wavelet. In the Wiener filtering approach, the desired signal u_k and the observed signal y_k are assumed to be stationary stochastic processes. The application of this approach to deconvolution has been adequately described by Robinson and Treitel (1980) and many others. However, there are many other methods for solving the deconvolution problem, such as homomorphic filtering (Ulrych, 1971).

The state-space approach to deconvolution is valuable because it allows us to eliminate the restrictive modelling assumptions of previous methods, such as: stationarity of u_k and/or y_k; the need for a minimum-phase wavelet h_i; and, finally, the time-invariance of h_i. Crump (1974) developed a state-space method for the deconvolution problem by augmenting u_k into the state vector and then estimating the state vector by means of a Kalman filter. In the mid-1970s, Mendel (1977) invented an important new approach to deconvolution based on Kalman filtering, which is referred to as minimum-variance deconvolution (MVD). It relies not only on Kalman filtering, but also on optimal smoothing. In MVD, all the model parameters of both the source wavelet and the noise statistics are assumed to be known in advance and the data are processed linearly. This novel approach can achieve a high resolution output when: the source wavelet is exactly represented by an ARMA model; the input sequence u_k and measurement noise e_k are both white noise processes; and the variances of these two processes are known exactly.

This MVD approach seems to work well in the seismic context but appears to have some limitations when applied to the kind of geophysical data considered in this paper. Firstly, the assumption that the input sequence u_k is white may not be true. Secondly, the high resolving power of the MVD filter is designed to detect high frequency components in y_k but it can, in some circumstances, enhance high frequency noise as well. We have found that this causes problems in the accuracy of detection when applied to the type of geophysical data considered here. One method of improving this situation is to use a wavelet shaping filter (Mendel, 1983). Thirdly, the clutter noise remaining in the two orthogonal components in the present application is not white, due to the receiver characteristics. Because of practical limitations, the available reference source wavelet is considerably different from the wavelet reflected in the ground measurements due to frequency dependent absorption effects. Also, there is no prior knowledge of the two noise variances.

Due to the practical constraints, we propose an alternative approach to the deconvolution problem which also is based on optimal state-space smoothing procedures but which is rather different from the previous seismic procedures. In this smoothing approach, the characteristic response of the system h_i is first modelled by the recursive optimal Instrumental Variable (IV) method proposed by Young (1985). The variances are then estimated by the Expectation and Maximization (EM) algorithm, which basically is an iterative maximum likelihood method (Dempster, Laird and Rubin, 1977).

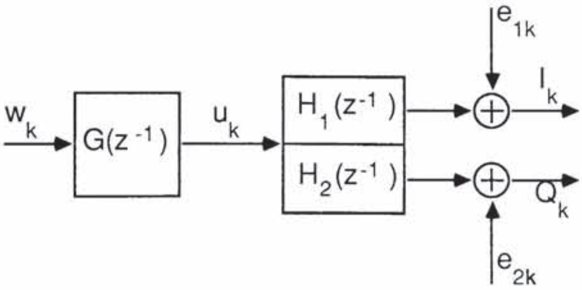

Fig. 3 Single input system with two orthogonal outputs

OPTIMAL SMOOTHING DECONVOLUTION

We assume that the two orthogonal components I and Q may be represented by the discrete-time convolution model described by Eq. (1) and shown in Fig. 3. For simplicity, the impulse response h_i for each orthogonal signal of the system is assumed to be of the form associated with an nth-order autoregressive moving average (ARMA) model, whose Z-transform is

$$H_j(z^{-1}) = \frac{\sum_{i=1}^{n} b_{i-1} z^{-i}}{1 - \sum_{i=1}^{n} a_i z^{-i}} \quad ; \quad j=1,2 \tag{2}$$

The input sequence u_k is also assumed to be described by an ARMA model, of the following kind, as shown in Fig. 3, i.e.

$$G(z^{-1}) = \frac{\sum_{i=1}^{m+1} d_{i-1} z^{-i+1}}{1 - \sum_{i=1}^{m} c_i z^{-i}} \tag{3}$$

The ARMA model $G(z^{-1})$ has an input w_k, which is a zero mean, mutually uncorrelated white noise series with variance q. Also, each of the measurement noise terms e_{1k} and e_{2k}, with variances r_1 and r_2 respectively, are assumed to be uncorrelated.

The advantages of assuming that the input sequence u_k is a coloured noise sequence are threefold: (1) with this assumption and the use of smoothing deconvolution, we can better suppress the clutter noise; (2) it allows us to overcome the difficulty caused by using a reference wavelet whose bandwidth is much wider than the bandwidth of the reflected ground wavelet; and (3) the transformation $G(z^{-1})$ can be considered as a waveshaping filter.

In order to exploit the advantages of optimal recursive estimation theory, Eqs. (2) and (3) must be represented in state-space terms, with the input u_k considered as one of the state-variables in the state vector $\underline{x}_k$

$$\underline{x}_{k+1} = F \underline{x}_k + G w_k \tag{4}$$
$$y_k = H^T \underline{x}_k + e_k \tag{5}$$
$$u_k = h^T \underline{x}_k \tag{6}$$

where

$$F = \begin{bmatrix} -a_1 & 1 \; 0 \ldots 0 & b_0 & 0 \; 0 \ldots 0 \\ -a_2 & 0 \; 1 \ldots 0 & b_1 & 0 \; 0 \ldots 0 \\ \vdots & \vdots & \vdots & \vdots \\ -a_n & 0 \; 0 \ldots 0 & b_{n-1} & 0 \; 0 \ldots 0 \\ 0 & 0 \; 0 \ldots 0 & -c_1 & 1 \; 0 \ldots 0 \\ 0 & 0 \; 0 \ldots 0 & -c_2 & 0 \; 1 \ldots 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 \; 0 \ldots 0 & -c_m & 0 \; 0 \ldots 0 \\ 0 & 0 \; 0 \ldots 0 & 0 & 0 \; 0 \ldots 0 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ d_0 \\ d_1 \\ \vdots \\ d_{m-1} \\ d_m \end{bmatrix} \tag{7}$$

with

$$H^T = (1\;0 \ldots 0 : 0\;0 \ldots 0)$$
$$h^T = (0\;0 \ldots 0 : 1\;0 \ldots 0) \tag{8}$$
$$q = E[w_k^2]$$
$$r = E[e_k^2] \tag{9}$$

If all the parameters in this state-space model are assumed known, then an optimal estimate of u_k can be obtained by using a state-space smoothing procedure. In practice, however, we do not know the model parameters, so it is necessary to identify the statistically most appropriate model structure and to estimate the associated parameters. Moreover, the two variances q and r need to be estimated. We will discuss the estimation procedure used to estimate these parameters in later sections.

The most widely used method of state estimation for linear dynamic systems is that derived by Kalman (1960) based on the method of orthogonal projection. However, a theoretically superior "smoothed" estimate of the state can be obtained if it is possible to utilise future values of measurements. This estimate can be obtained most straightforwardly by an optimal off-line or on-line recursive smoothing procedure. Since all the available data have been collected before the processing commences, off-line, fixed-interval smoothing is the most useful processing procedure in the present context.

Much has been written on smoothing algorithms (Meditch, 1973). Mendel (1983) suggests that one of the most computationally efficient forms of the fixed-interval smoothing algorithm is the following set of backward filtering equations, applied to the results from an initial recursive Kalman filtering run through the data:

$$\hat{x}_{k|N} = \hat{x}_{k|k-1} + P_{k|k-1} R_{k|N} \tag{10}$$

where k=N-1, N-2, ..., 1. Here $\hat{x}_{k|k-1}$ is the predicted estimate of x_k based on the information up to the (k-1)th data point, $P_{k|k-1}$ is the covariance matrix of the predicted state and

$$R_{k|N} = [\,I - K_k H\,]^T F^T R_{k+1|N} + H\,[\,H P_{k|k-1} H^T + r\,]\,\chi_{k|k-1} \tag{11}$$

where k = N, N-1, ..., 1; $R_{k|N} = 0$; H and F are defined in Eqs. (7) and (8); K_k is the Kalman gain; r is the measurement noise variance; and

$$\chi_{k|k-1} = y_k - H\,\hat{x}_{k|k-1} \tag{12}$$

is the innovations process.

The error covariance matrix $P_{k|N}$ associated with the smoothed state estimate in Eq. (10) can be generated in the following way:

$$P_{k|N} = P_{k|k-1} - P_{k|k-1} S_{k|N} P_{k|k-1} \tag{13}$$

where k=N-1, N-2, ...,1 and the n by n matrix $S_{k|N}$ is obtained from:

$$S_{k|N} = [\,I - K_k H\,]^T F^T S_{k+1|N} F\,[\,I - K_k H\,] + H^T[\,H P_{k|k-1} H^T + r]^{-1} H \tag{14}$$

This smoothing algorithm is computationally very efficient because it requires no matrix inversion since $[\,H P_{k|k-1} H^T + r\,]^{-1}$ is the output of the forward Kalman filter.

MODELLING THE REFERENCE WAVELET

In optimal smoothing deconvolution, the system parameters are assumed to be known. However, in practice there will be no prior knowledge of the model parameters. Consequently, we need a reference waveform to identify the system. One approach uses, as a reference, the return signal received from a test object separated from the receiver only by air. Any correction to this reference signal to allow for ground effects could involve both frequency weighting and time expansion. Since such a correction could be very complicated in practice, we simply use the reference signal directly. Figure 4 shows the pair of reference wavelets I_r and Q_r obtained in this manner.

It has been mentioned before, in the previous section, that the input u_k is assumed to be coloured. This approach allows us flexibility in choosing the shape of the assumed input. A recursive filter is designed to construct the ARMA model for the input, whose bandwidth is suitable for the real situation. In the present case, this input shaping filter is designed by utilising Butterworth polynomials but clearly other approaches are possible. The following filter transfer function was obtained from a knowledge of the frequency spectra associated with the reference wavelet, the clutter noise and the ground reflected wavelet, as shown in Fig. 5:

$$G(z^{-1}) = \frac{0.0407 + 0.1221\,z^{-1} + 0.1221\,z^{-2} + 0.0407\,z^{-3}}{1 - 2.1266\,z^{-1} + 1.5958\,z^{-2} - 0.4118\,z^{-3}} \tag{15}$$

The impulse response of this filter is shown in Fig. 4.

There are many methods of identifying and estimating the transfer function between this input and the reference wavelet but here we base our approach on the refined Instrumental Variable (RIV) method first proposed by Young and Jakeman (1979). The RIV procedure yields estimates of the parameters which are both consistent and asymptotically efficient, provided the noise sequence is statistically uncorrelated with the input series and can be represented as an ARMA process. In this paper, we utilise a simplified refined IV (SRIV) algorithm (Young, 1985), which is particularly appropriate in the present context. It is interesting to note that Kollias and Halkias (1985) have successfully applied ordinary IV estimation to seismic deconvolution. Here, however, we are able to carry out the

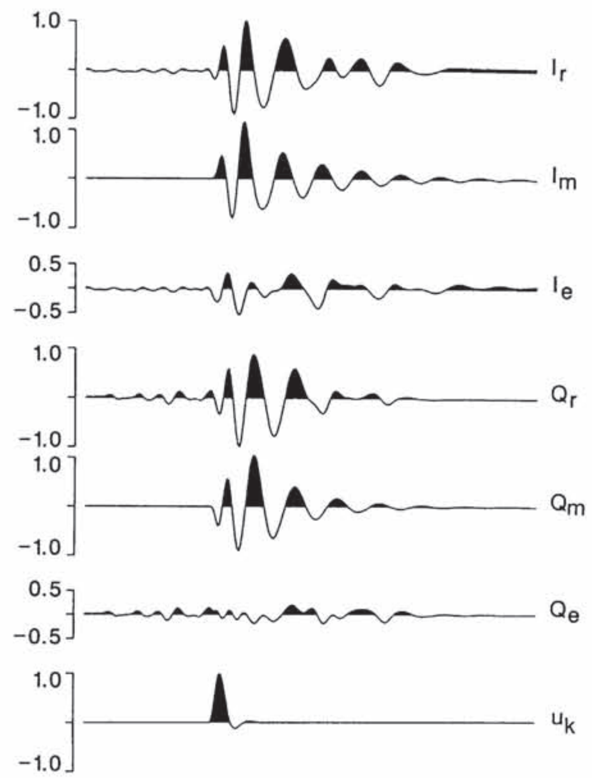

Fig. 4. The model fit to reference wavelets I_r and Q_r using a Butterworth filter impulse response input u_k to produce fitted models I_m and Q_m with their residual model errors I_e and Q_e

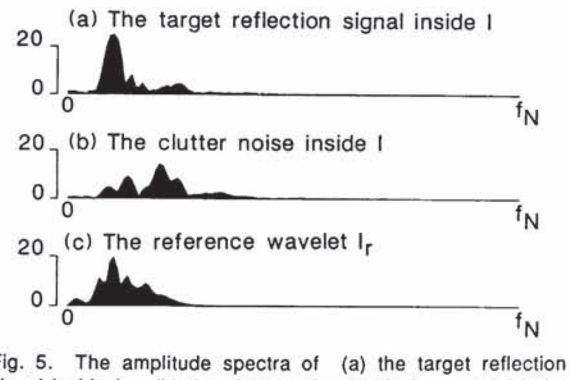

Fig. 5. The amplitude spectra of (a) the target reflection signal inside I, (b) the clutter noise inside I and (c) the reference wavelet I_r (f_N is the Nyquist frequency).

estimation off-line, based on the planned "clear air" experiments, so we have direct access to the input signal required for IV estimation. Kollias and Halkias, on the other hand, had to develop a rather ingenious on-line adaptive algorithm, in which the input "reflectivity" sequence was estimated by fixed lag smoothing methods.

Table 1 The Estimated Parameters of the Transfer Functions

$H_1(z^{-1})$		$H_2(z^{-1})$	
a_1 = -3.3863	b_0 = 0.1843	a_1 = -3.2819	b_0 = -0.5507
a_2 = 4.6008	b_1 = -0.3949	a_2 = 4.1980	b_1 = 1.7621
a_3 = -2.9672	b_2 = 0.2190	a_3 = -2.4835	b_2 = -1.9180
a_4 = 0.7760		a_4 = 0.5811	b_3 = 0.7073

Table 1 shows the parameters of the two identified transfer function models $H_1(z^{-1})$ and $H_2(z^{-1})$ between the input u_k and the reference wavelets I_r and Q_r respectively. Both models explain about 90% of the original reference wavelets. The model responses and the residual errors associated with them are shown in Fig. 4. It is clear that the early part of waveform is modelled very well, while the discrepancies in the tail are thought to be due to equipment and ground interference effects.

THE EM ALGORITHM

The smoothed states from the recursive smoothing algorithm depend on the two variances q and r. If the dynamic system is known, they can be viewed as tuning parameters of a state-space smoother. In order to achieve optimal results, it is necessary to estimate these two variances from the data. In seismic deconvolution, Kormylo and Mendel (1983) have proposed a maximum likelihood method to compute the two variances. However, their method is only applicable to problems in which the input is assumed to have a Bernoulli-Gaussian distribution. Also the method is a rather slow and computationally expensive technique.

Here, we propose the use of the EM algorithm which is a comparatively simpler method. The EM algorithm was introduced by Dempster, Laird and Rubin (1977) and consists of two steps, which are iterated to convergence: an "expectation" step and a "maximization" step. The maximization step calculates the maximum likelihood estimates of all the unknown parameters conditional on a full data set. The expectation step constructs estimates of the sufficient statistics, conditional on the observed data and the parameters. This is a derivative-free method and does not require any evaluation of the likelihood function directly.

Consider the state-space model of Eq. (7) with a time invariant covariance matrix $Q = q\,G\,G^T$ and a noise variance r. The joint log likelihood function of the complete data $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N, y_1, y_2, \ldots, y_N$ when $\underline{x}_0$ and P_0 are fixed, will be

$$\log(L) = -\frac{N}{2}\log r - \frac{1}{2r}\sum_{k=1}^{N}(y_k - H^T\underline{x}_k)^2 - \frac{N}{2}\log|Q|$$
$$-\frac{1}{2}\sum_{k=1}^{N}(\underline{x}_k - F\underline{x}_{k-1})^T Q^{-1}(\underline{x}_k - F\underline{x}_{k-1}) - \frac{1}{2}\log|P_0|$$
$$-\frac{1}{2}(\underline{x}_0 - \mu)^T P_0^{-1}(\underline{x}_0 - \mu) \qquad (16)$$

The unknown parameters are q and r, and are denoted by the vector $\underline{V}$. The EM algorithm proceeds iteratively by evaluating

$$E\left\{ \frac{\partial \log L}{\partial \log \underline{V}} \mid y_1, \ldots, y_N \right\} \qquad (17)$$

conditional on the latest estimate of $\underline{V}$. This mathematical expression is then set to a zero vector and the new estimate of $\underline{V}$ is obtained from the solution of the resulting equation. The procedure is repeated until the likelihood function and estimates are stable.

Harvey and Peters (1986) have developed two equations for the estimates of the two variances based on the above procedures,

$$r = \frac{1}{N}\sum_{k=1}^{N}(s_{k|N}^2 + H^T P_{k|k} H) \qquad (18)$$

where $s_{k|N} = y_k - H^T \underline{x}_{k|N}$ and

$$Q = \frac{1}{N}\sum_{k=1}^{N}[\underline{W}_k \underline{W}_k^T + P_{k|N} + F^T P_{k|N} F - F P^*_{k|N} - P^*_{k|N} F^T] \quad (19)$$

where $\underline{W}_k = \underline{x}_{k|N} - F\underline{x}_{k-1|N}$ and

$$P^*_{k|N} = E[(\underline{x}_{k|N} - \underline{x}_k)(\underline{x}_{k|N} - \underline{x}_k)^T] \qquad (20)$$

$P^*_{k|N}$ is not the direct output of the state-space smoother. A relatively straightforward way to compute $P^*_{k|N}$ is to augment the state vector by lagged values $\underline{x}_{k-1}$. The matrix $P^*_{k|N}$ then appears as the off-diagonal block of the $P_{k|N}$ matrix of this augmented state vector. Since the unknown q is a scalar, there is a modified simple computational equation for estimating q in order to avoid the complicated Eq. (19) (Yiu, Young and Robinson, 1988).

RESULTS

In this section, we present some results which illustrate the performance of our smoothed deconvolution approach to geophysical signal processing. We use the proposed procedures with the two transfer function models $H_1(z^{-1})$ and $H_2(z^{-1})$ of the modelling section and the q and r values estimated from the EM algorithm, in order to deconvolve the two orthogonal data traces I and Q. The signals I_{H1} and Q_{H2}, shown in Fig. 6, denote the traces I and Q deconvolved by the transfer functions $H_1(z^{-1})$ and $H_2(z^{-1})$ respectively. By comparison, I_{D1} and Q_{D2}, also shown in Fig. 6, are the deconvolved traces of I and Q by using the normal MVD method with ARMA models for the impulse response of I_r and Q_r.

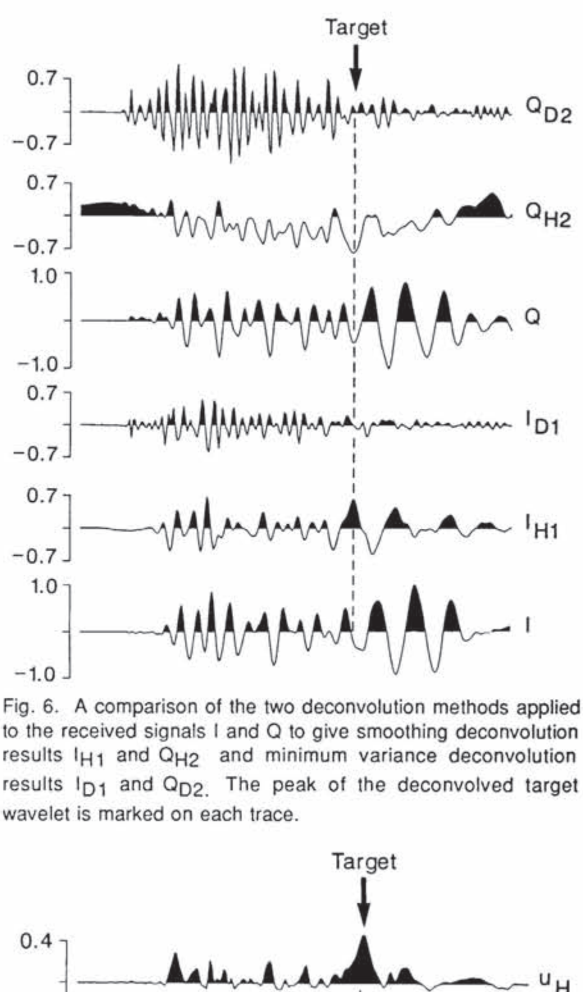

Fig. 6. A comparison of the two deconvolution methods applied to the received signals I and Q to give smoothing deconvolution results I_{H1} and Q_{H2} and minimum variance deconvolution results I_{D1} and Q_{D2}. The peak of the deconvolved target wavelet is marked on each trace.

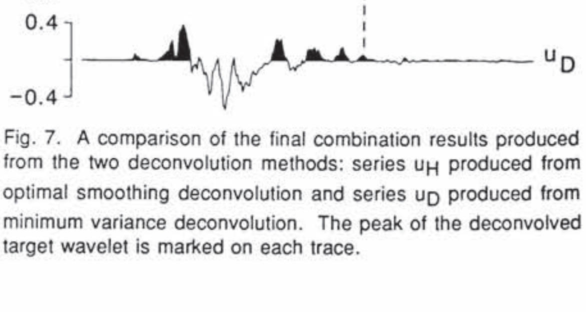

Fig. 7. A comparison of the final combination results produced from the two deconvolution methods: series u_H produced from optimal smoothing deconvolution and series u_D produced from minimum variance deconvolution. The peak of the deconvolved target wavelet is marked on each trace.

The model shown in Fig. 3 suggests that it is desirable to reconstruct the single reflectivity trace u_k from some combination of both the orthogonal traces I and Q, which are 90° out of phase. One advantage of the proposed approach is that the orthogonality of I and Q is still retained in the deconvolved traces. Figure 7 shows an amplitude series of the estimated u_k produced by combining the deconvolved traces from the optimal smoothing method in a special manner which emphasises the signal from the buried object. For comparison, the deconvolved traces from MVD are combined in the same way and produce an alternative amplitude series of the estimated u_k.

CONCLUSION

Although it is clear that the MVD method can have high resolving power in seismic applications, the method can, in certain circumstances, both enhance the clutter noise and suffer the effects of using an imperfect reference wavelet, both of which are important in other geophysical data processing applications. In contrast, the new approach proposed here can transform the reflected wavelet into a fairly narrow envelope and can also suppress clutter noise of the kind encountered in this particular application. This method, coupled with the novel application of principle components orthogonalisation, clearly improves the detection of the buried object, which is the principle objective of the present study. In addition, the EM algorithm used here has the potential for adaptive deconvolution i.e. the ability to re-estimate the system parameters while performing deconvolution of the data (Kollias and Halkias, 1985). The advantages of the systems-based recursive estimation and smoothing techniques are clear from the results presented in this paper.

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