Structural Microeconometric Methods:

Problem Set 3: Estimating an Optimal Stopping Model of Schooling

Data

- The Data come from *Génération 98*, a large scale survey conducted in France by Céreq.

- It was already described in the Problem Set devoted to the analysis of the grade progression model (hazard function analysis)

Model Structure:

- **Two states**
  - **school (every individual starts there)**
  - **Full Time work**
  - $\mathbf{t = 1, 2, ...T}$ **periods to allocate (T=60 years)**

- Schooling is constructed as follows: $s = 1, 2, S$ grade levels
  - * $s = 1$:Lycee (entre), duree= 1 an
    * $s = 2$: lycee (CAP/BEP), duree=1an

- $s = 3$: $lycee$ (bacc), duree=1an
- $s = 4$: Enseignement sup (sans diplome), duree=1
- $s = 5$:Bacc+2, duree= 1 an
- $s = 6$: Bacc+3: duree= 1 an
- $s = 7$: Bacc+4, duree= 1an
- $s = 8$: Bacc+5 (ou plus)=1 an

- For each grade level, s,
  - Assume a standard # of years for duration of grade s : In general, it will be 1 year.

- Transform each grade into a quantitative measure (say Bacc+2= ?? years of schooling)

- Assume the “normal" age level of entry in the market (A_s) : In other words, and individual who completes grade s, must enter the labor market at age A_s (in case he does not go further)

- In general, at any date/age t, accumulated experience is $t - A_s$.

- At final date T, Grade-level-specific maximum) experience, is $T - A_s = 60 - A_s$

* Before estimating the model: estimate the log(earnings) regression using earnings data (schooling, experience, repeat indicator)

The Utility Equations

$$U_i^s(.) = \; X_i' \delta_x + \delta_L \cdot I(repeat) + \varepsilon_i^s$$

with

$$\varepsilon_i^s \sim i.i.d N(0, \sigma = 1)$$

$$U_i^w(s,t) = \ln(w_i). = \varphi_1 + \varphi_2 \cdot educ(years) + \varphi_3.(t - A_s) + \varphi_4. \cdot I(repeat)$$

where the variable t denotes age.

In last period, the utility of work is:

$U_i^w(s, 60) = \ln(w_i). = \varphi_1 + \varphi_2 \cdot educ(years) + \varphi_3.(60 - A_s) + \varphi_4. \cdot I(repeat)$

Note that because we are using numbers that have been estimated prior to estimation of the model, $U_i^w(s, 60), U_i^w(s, 59), \ldots$.contains only known constants.

Value of Attending Grade s:

$$\begin{aligned} V^s(s) &= X_i'\delta_x + \delta_L \cdot I(repeat) + \varepsilon_i^s \\ &\quad + \beta E(Max\{V^s(s+1), V^w(s, A_s)\}) \end{aligned}$$

Value of entering the labor market with grade s, at age A_s

$$
\begin{aligned}
V^{w}(s, A_s) &= \varphi_1 + \varphi_s \cdot I(s) + \quad \phi_4 \cdot I(repeat) \\
&\quad + \beta\{\varphi_1 + \varphi_2 \cdot s + \varphi_3.(t - A_s) + \varphi_4. \cdot I(repeat) \\
&\quad + \ldots.. \\
&\quad + \beta^{(60 - A_s - 1)}\{\varphi_1 + \varphi_2 \cdot s + \varphi_3.(T - A_s) + \varphi_4. \cdot I(repeat) \\
&= \sum_{j=A_s}^{60} \beta^{j - A_s - 1}\{\varphi_1 + \varphi_2 \cdot s + \varphi_3.(j - A_s) + \varphi_4. \cdot I(repeat)
\end{aligned}
$$

.

- start from $s = 8$

  - it is clear that the value function of entering grade 8 (and automatically starting full-time work at age A_8 since grade 8 is terminal), $V^s(8)$, has a trivial structure, since no Emax(.) needs to be taken:

$$V^s(8) = X_i'\delta_x + \delta_L \cdot I(repeat) + \varepsilon_i^8 + V^w(8, A_8)$$

- The future utility associated to entering grade 8, $V^w(8, A_8)$, is trivial (sum of life cycle earnings in logs from age A_8 to age 60.

- When contemplating entering grade 8, the choice requires a comparison between $V^s(8)$ and $V^w(7, A_7)$

- So, the choice probability is:

$$
\begin{aligned}
\Pr(d_8 &= 1) = \Pr V^s(8) > V^w(7, A_7) \\
&= \Pr(X_i'\delta_x + \delta_L \cdot I(repeat) + \varepsilon_i^8 + V^w(8, A_8) > V^w(7, A_7) \\
&= \Pr\{\varepsilon_i^8 > V^w(7, A_7) - V^w(8, A_8) - X_i'\delta_x - \delta_L \cdot I(repeat)\} \\
&= \Phi\{V^w(7, A_7) - V^w(8, A_8) - X_i'\delta_x - \delta_L \cdot I(repeat)\}
\end{aligned}
$$

Obviously, the choice of not attending grade 8 (**stopping at grade 7**) is

$$\begin{aligned}
\mathrm{Pr}(d_8 &= 0) = \mathrm{Pr}\, V^s(8) \leq V^w(7, A_7) \\
&= 1 - \Phi\{V^w(7, A_7) - V^w(8, A_8) - X_i'\delta_x - \delta_8 - \delta_L \cdot I(repeat)\}
\end{aligned}$$

- When we will solve the problem recursively, the first non-trivial exercise will be when contemplating grade 7:
  - you will need to compute $EMax\{V^s(s+1), V^w(s)\}$
  - or, in other words, $EMax\{V^s(8), V^w(7, A_7)\}$
  - to reduce notation burden, denote $EMax\{V^s(8), V^w(7, A_7)\}$ by $EV(8)$ since this choice is made at the beginning of period 8,

$$
\begin{aligned}
EV(8) &= pr(d_8 = 1) \cdot E(V^s(8) \mid V^s(8) > V^w(7, A_7)\ ) \\
+pr(d_8 &= 0) \cdot E(V^w(7, A_7) \mid V^w(7, A_7) > V^s(8)\ ) \\
&= pr(d_8 = 1) \cdot E\{V^s(8) \mid V^s(8) > V^w(7, A_7)\ \} \\
+pr(d_8 &= 0) \cdot E(V^w(7, A_7)
\end{aligned}
$$

- Note that
  - $V^w(7, A_7)$ is non stochastic, so $E(V^w(7, A_7) = V^w(7, A_7)$ and is already known.
  - only need to evaluate $E(V^s(8) \mid V^s(8) > V^w(7, A_7)$
  - Formally, this is

$$
\begin{aligned}
E(V^s(8) \quad &| \quad V^s(8) > V^w(7, A_7) \;= \\
E\{X_i'\delta_x + \delta_L \cdot I(repeat) + \varepsilon_i^8 + V^w(8, A_8) \quad &| \\
X_i'\delta_x + \delta_L \cdot I(repeat) + \varepsilon_i^8 + V^w(8, A_8) \quad &> \quad V^w(7, A_7)
\end{aligned}
$$

This is equal to

$$
\begin{aligned}
&= X_i'\delta_x + \delta_8 + \delta_L \cdot I(repeat) + V^w(8, A_8) \\
+E(\varepsilon_i^8 \quad &| \quad \varepsilon_i^8 > V^w(7, A_7) - V^w(8, A_8) - X_i'\delta_x - \delta_L \cdot I(repeat)\}
\end{aligned}
$$

- Recall that $\varepsilon_i^8 \sim N(\mu, \sigma^2)$
- $\mu = 0, \sigma^2 = 1$
- $E(\varepsilon_i^8 \mid \varepsilon_i^8 > c) = \mu + \frac{\phi(c-\mu)/\sigma}{1-\Phi(c-\mu)/\sigma}$

- where $c = V^w(7, A_7) - V^w(8, A_8) - X_i'\delta_x - \delta_8 - \delta_L \cdot I(repeat)\}$

- Keep $EV(8)$ in memory

- Proceed recursively until you reach the initial period (say the minimum schooling).

- In the preceding period/choice, we get
  - Compare $V^s(7)$ with $V^w(6, A_6)$
  - where $V^s(7)$ is simply

$$\begin{aligned} V^s(7) &= X_i'\delta_x + \delta_L \cdot I(repeat) + \varepsilon_i^7 + E\max\{V^s(8), V^w(7, A_7)\} \\ &= X_i'\delta_x + \delta_L \cdot I(repeat) + \varepsilon_i^7 + EV(8) \end{aligned}$$

where $EV(8)$ is now given

- Compute
  * $\Pr(d_7 = 1)$
  * $\Pr(d_7 = 0)$
  * $E(V^s(7) \mid V^s(7) > V^w(6, A_6))$
  * $EV(7)$

- Compare $V^s(6)$ with $V^w(5, A_5)$
  - etc.....

- Estimation is achieved by maximum likelihood.

- the probability of having reached grade level $\tilde{s}$, is

$$\begin{aligned} L^s &= Pr[(d_1 = 1), (d_2 = 1) .... (d_{\tilde{s}} = 1), (d_{\tilde{s}+1} = 0)] \\ &= Pr(d_1 = 1) \cdot \mathrm{Pr}(d_2 = 1) .... \mathrm{Pr}(d_{\tilde{s}} = 1) \cdot \mathrm{Pr}(d_{\tilde{s}+1} = 0) \end{aligned}$$

except for the probability of having reached grade level 8, which is

$$\begin{aligned} L^s &= Pr[(d_1 = 1), (d_2 = 1) .... (d_{\tilde{s}} = 1), (d_8 = 1)] \\ &= Pr(d_1 = 1) \cdot \mathrm{Pr}(d_2 = 1) .... \mathrm{Pr}(d_{\tilde{s}} = 1) \cdot \mathrm{Pr}(d_8 = 1) \end{aligned}$$

To estimate wages, we assume that observed wages (earnings) are expressed as:

$$\ln(w_i^{obs}). = \varphi_1 + \varphi_2 \cdot educ + \varphi_3.(60 - A_s) + \varphi_4. \cdot I(repeat) + \varepsilon_{it}^m$$

where ε_{it}^m is measurement error (i.i.d. across time and individuals).

Problem Set : Estimating an Optimal Stopping Model of Schooling

Due date: May 2017

1. Estimate the structural parameters $\{\delta_x, \delta_L, \beta\}$ by maximum likelihood. Compute the mean level of schooling in the population and the predicted distribution by years (grade)?

2. Answer the following question: Suppose that following technological change, the effect of education on log wages is multiplied by 2, what will happen to changes in mean schooling?