Introduction to Software-Defined Radio: Analog Demodulation of signals using GNURadio

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Summary:

Introduction

Within the course Protocols for connected Objects, we applied our knowledges on the case of reception of real communication signals. We have studied different MAC protocols, network architectures or radio technologies used in Wireless Sensor Network. Then, through this paper we will learn a different aspect in IoT which is the **reception of the radio signal**.

In order to decode a received signal we can use a transceiver, an **Universal Software Radio Peripheral (USRP)**, for example the USRP-2900 transceiver which allows us to transmit and receive on a wide band going from 70 MHz to 6 GHz with a high sampling frequency. Then the USRP is connected via a **Gigabit Ethernet connection to the computer**, and is composed of two stages: **the transposition of the signal** around the zero frequency, and a **conversion from the analog signal to the digital samples (ADC)[1].** Finally, after the conversion, we can use a **Software-Defined Radio (SDR)**, for instance GNU Radio a friendly free radio software, in order to **process digitally the received signal**, for example to filter it or demodulated it[2]. GNU Radio can be also used for the transmission of signal through the USRP.

Figure 1: National Instruments USRP-2900 transceiver

Before we deal with all the process to decode the signal, we will see **mathematical aspects** which are essential for the following parts in order to demodulate the received signal. Then, we will focus on two different bands: the **High Frequency Band (HF)**, between 3 MHz and 30 MHz, and the **Very High Frequency Band (VHF)**, between 30 MHz and 300 MHz, respectively used by citizens, or intercontinental communication, and used by television or FM radio[3].

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[1] USRP-2900 transceiver: http://www.ni.com/fr-fr/support/model.usrp-2900.html
[2] GNU Radio: https://en.wikipedia.org/wiki/GNU_Radio
[3] Electromagnetic Spectrum: https://en.wikipedia.org/wiki/Electromagnetic_spectrum

First part: **Presentation of the acquisition device: In-Phase/Quadrature SDR transceiver**

To really understand the two stages of the USRP transceiver and the mathematical aspects done in the SDR, we will focus on the operations done by the acquisition chain.

1. Modulation of the signal

We can note the **communication transmitted signal** $s_{RF}(t)$, which is transmitted around a carrier frequency f_0 :

$$s_{RF}(t) = A(t).cos(2\pi f_0 t + \varphi(t)),\ t \in \Re$$

With $A(t) \in \Re$ the envelope and $\varphi(t) \in \Re$ the phase of the signal.

We can assume that the received signal $r_{RF}(t)$ is approximately equal to the transmitted signal $s_{RF}(t)$, so we have $s_{RF}(t) = r_{RF}(t)$.

As we know, we have different manners to code a signal $m(t)$ according the modulation chosen of the signal $s_{RF}(t)$, we can modulate the signal with the amplitude $A(t)$ (**Amplitude Modulation - AM**), the phase $\varphi(t)$ (**Phase Modulation - PM**), the frequency (**Frequency Modulation - FM**), or a **combination of these three**.

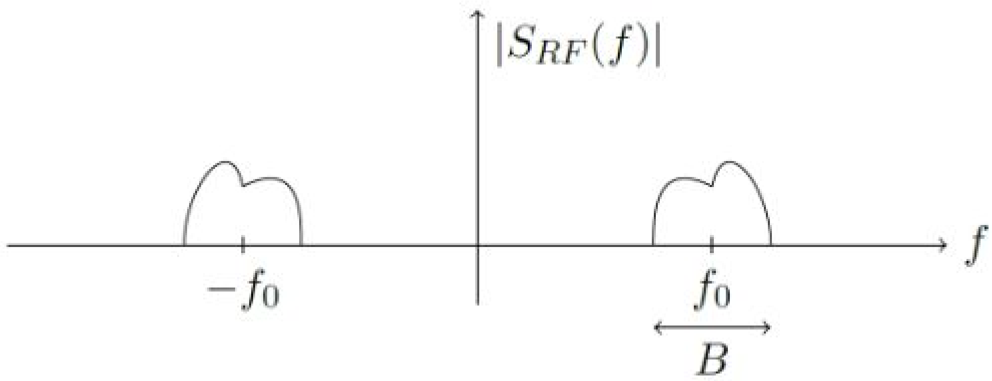


Figure 2: Spectrum of the transmitted signal $s_{RF}(t)$

We know the representation of the transmitted signal in the domain of frequencies.

2. In-phase/Quadrature representation of the received signal

We note the $s_R(t)$ and $s_I(t)$ respectively the **channels in-phase and quadrature**, with a supposed bandwidth $\frac{B}{2} < f_o$, which are equal to :

$$s_R(t) = A(t).cos(\varphi(t)),\ t \in \Re \text{ and } s_R(t) = A(t).sin(\varphi(t)),\ t \in \Re$$

So we can rewrite the transmitted signal as follow:

$$s_{RF}(t) = s_R(t).cos(2\pi f_0 t) - s_I(t).sin(2\pi f_0 t),\ t \in \Re$$

This means that the receiver is capable to reconstruct the both of the in-phase and quadrature channel of the signal.

3. Block diagrams of the USRP stages

We can represent the acquisition chain of the USRP receiver as follow, with the two stages, the frequency transposition and the analog to digital conversion:

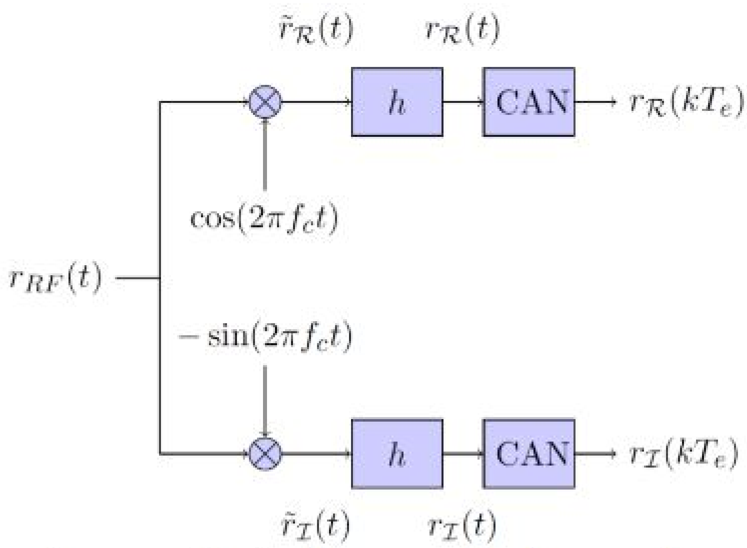


Figure 3: Block diagrams of the receiver

With $r_{RF}(t)$ the received signal, h the filter and CAN the analog to digital converter.

Question 1: We can express the signals $\widehat{r}_R(t)$ and $\widehat{r}_I(t)$ in function of $s_{RF}(t)$, f_c and f_0.

According to the in-phase/Quadrature representation, we have:

$$s_{RF}(t) = s_R(t).cos(2\pi f_0 t) - s_I(t).sin(2\pi f_0 t)$$

And we can deduce from the block diagram:

$$\widehat{r}_R(t) = r_{RF}(t) \times cos(2\pi f_c t)$$
$$\widehat{r}_I(t) = r_{RF}(t) \times sin(2\pi f_c t)$$

And because we can say that the received signal is approximately equal to the transmitted signal: $r_{RF}(t) = s_{RF}(t)$

$$\widehat{r}_R(t) = s_R(t) \times cos(2\pi f_0 t) \times cos(2\pi f_c t) - s_I(t) \times sin(2\pi f_0 t) \times cos(2\pi f_c t)$$
$$\widehat{r}_I(t) =- s_R(t) \times cos(2\pi f_0 t) \times sin(2\pi f_c t) + s_I(t) \times sin(2\pi f_0 t) \times sin(2\pi f_c t)$$

Finally we obtain:

$$\widehat{r}_R(t) = \frac{s_R(t)}{2} \times [cos(2\pi t(f_0 + f_c)) + cos(2\pi t(f_0 - f_c))]$$
$$- \frac{s_I(t)}{2} \times [sin(2\pi t(f_0 + f_c)) + sin(2\pi t(f_0 - f_c)]$$
$$\widehat{r}_I(t) = \frac{s_I(t)}{2} \times [cos(2\pi t(f_0 - f_c)) - cos(2\pi t(f_0 + f_c))]$$
$$- \frac{s_R(t)}{2} \times [sin(2\pi t(f_0 + f_c)) - sin(2\pi t(f_0 - f_c)]$$

4. Low Pass Filter Characteristics

Question 2: What would be the characteristics of the filters h if we want to have $r_R(t) = s_R(t)$ and $r_I(t) = s_I(t)$?

Assuming that $f_c = f_0$ and according to question 1 we can deduce:

$$\widehat{r}_R(t) = \frac{s_R(t)}{2} \times [cos(2\pi t \times 2f_0) + 1] - \frac{s_I(t)}{2} \times [sin(2\pi t \times 2f_0)]$$
$$\widehat{r}_I(t) = \frac{s_I(t)}{2} \times [1 - cos(2\pi t \times 2f_0)] - \frac{s_R(t)}{2} \times [sin(2\pi t \times 2f_0)]$$

We want to keep $s_I(t)$ and $s_R(t)$, so we need to remove the cosine and the sine. To do that, we use a *h-filter* to cut the frequency $2f_0$ $(|f| > f_c)$ and amplify by 2 the frequency band ($|f| \leq f_c$). Thus, the filter has to have the following characteristics:

$$H(w) = 2 \qquad \text{for } |f| \leq f_c$$

$$H(w) = 0 \qquad \text{for } |f| > f_c$$

<u>with:</u> fc $\approx \frac{B}{2}$

Here the spectrum of the both signals $\widehat{r}_R(t)$ and $\widehat{r}_I(t)$, the original transmitted signal $s_{RF}(t)$, and we can also represent the characteristics that the filter h should have:

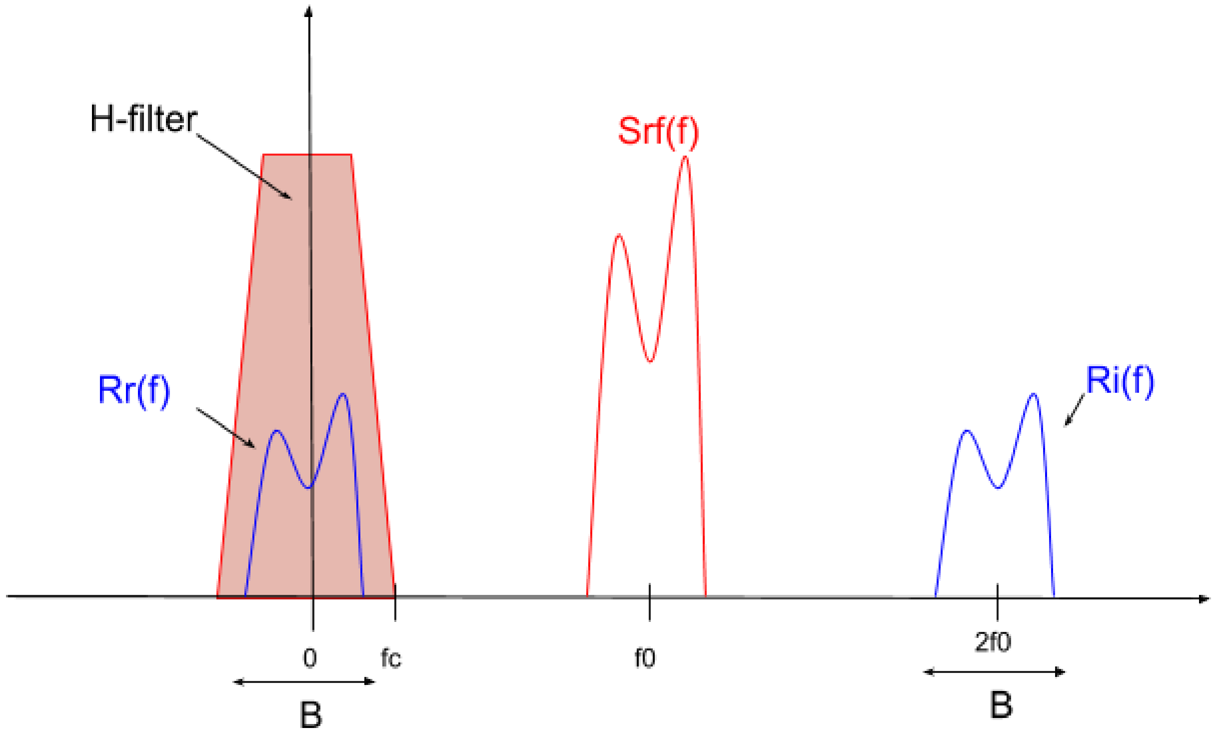


Figure 4: Spectrum Signals and H-filter representation

The amplitude of $s_{RF}(t)$ is the double of the signals $\widehat{r}_R(t)$ and $\widehat{r}_I(t)$, because $r_{RF}(t)$ is separated in two signals. Then the cutoff frequency of the filter must be inferior of $\frac{B}{2}$.

Question 3: Can the receiver work with wide-band signals ?

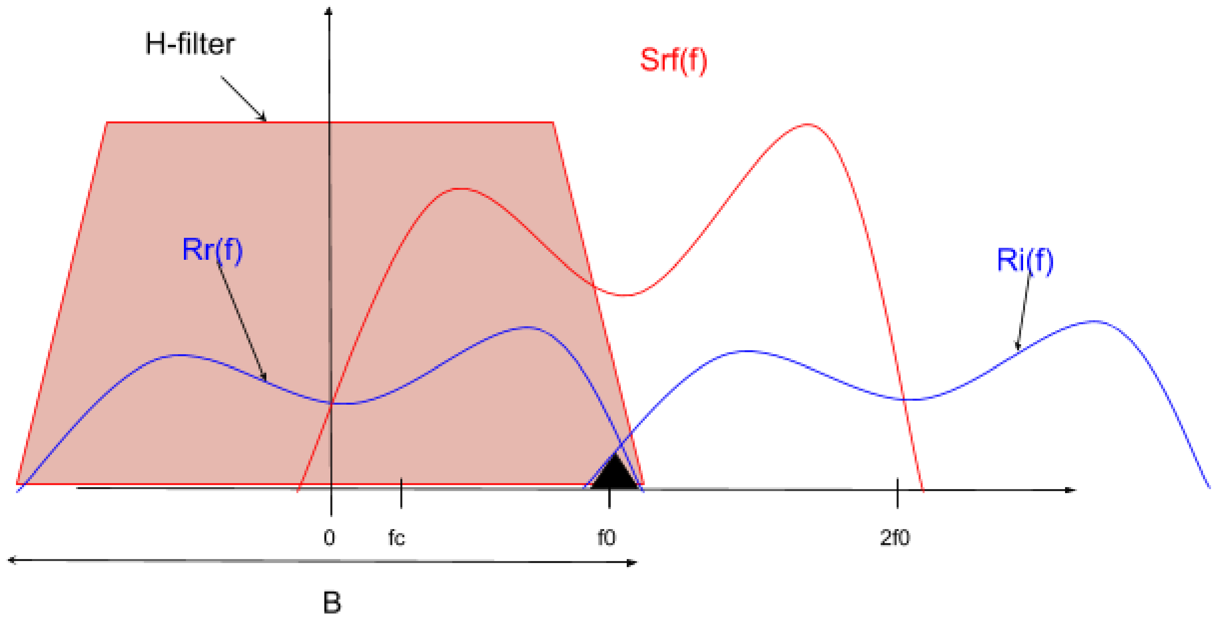


Figure 5: Spectrum Signals and H-filter representation

Here, a representation of a wide-band signal, this means that $f > \frac{B}{2}$.

The two stages of the receiver cannot work for these signals because if we use it we are going to have an overlap (the black part) of the signals, and so lose information.

Question 4: How must the sampling period T_e be chosen in order to recover the analog signal from the digital one?

In order to find the signal $r_R(t),\ t \in \Re$ from the signal $r_R(kT_e),\ k \in Z$, we have to take a sampling period so that we respect the Shannon theorem.

We set the frequency of $\widehat{r}_R(t)$ or $\widehat{r}_I(t)$ to $f_{max} = \frac{B}{2}$, and the one of the ADC to f_e. So we need to have:

$$f_e \geq 2f_{max} \ \Leftrightarrow f_e \geq 2 \times \frac{B}{2} \ \Leftrightarrow \ f_e \geq B \ \Leftrightarrow \ T_e \leq \frac{1}{B}$$

Finally, T_e has to respect the following relation:

$$T_e \leq \frac{1}{B}$$

Question 5: Why the frequency transposition stage has to be before the analog to digital conversion stage ?

To answer this question, we can take the example of the 3G, for which the frequency of the signals is about 2100MHz.

So to sample a signal at this frequency, you need to have an expensive hardware, that's why we want to reduce this frequency by applying a frequency transposition before applying the analog to digital conversion.

For example, we can have a look to the following LTC2153-14[4] ADC, it can sample signal with an input bandwidth of 1.25GHz, but it costs 144,26$ on DigiKey which is too expensive.

[4] Datasheet: https://www.analog.com/media/en/technical-documentation/data-sheets/215314fa.pdf

Question 6: Supposing a real narrow-band signal, we can express its analytic signal and its complex envelope

We assume a real narrow-band signal with the following expression:

$$s_{RF}(t) = A(t).cos(2\pi t f_0 \; + \; \varphi(t))$$

From the in-phase and quadrature representation, we have:

$$s_{RF}(t) = s_R(t).cos(2\pi f_0 t) - s_I(t).sin(2\pi f_0 t)$$

Then, we know that the analytic signal $S_a(f)$, the complex-valued function that has no negative frequency components, can be modelled by:

$$S_a(f) = S_{RF}(f) + j(-j \times sign(f) \times S_{RF}(f)$$

So: $S_a(f) = 2S_{RF}(f)$ if $f \geq 0$ and $S_a(f) = 0$ if $f < 0$

$$\begin{aligned}
&\Leftrightarrow S_{RF}(f) = \mathfrak{I}\{s_{RF}\}(f) = \mathfrak{I}\{s_R(t).cos(2\pi f_0 t) - s_I(t).sin(2\pi f_0 t)\}(f) \\
&\Leftrightarrow S_{RF}(f) = \mathfrak{I}\{s_R(t).cos(2\pi f_0 t)\}(f) - \mathfrak{I}\{s_I(t).sin(2\pi f_0 t)\}(f) \\
&\Leftrightarrow S_{RF}(f) = S_R(f) * \tfrac{1}{2}|\delta(f - f_0) + \delta(f + f_0)| - S_I(f) * \tfrac{j}{2}|\delta(f - f_0) - \delta(f + f_0)| \\
&\Leftrightarrow S_{RF}(f) = \tfrac{1}{2}[\; S_R(f - f_0) + S_R(f + f_0)] + \tfrac{j}{2}[S_I(f - f_0) - S_I(f + f_0)] \\
&\Leftrightarrow S_{RF}(f) = S_R(f - f_0) + S_R(f + f_0) + jS_I(f - f_0) - jS_I(f + f_0)
\end{aligned}$$

With: $S_R(f + f_0) = 0$ and $S_I(f + f_0) = 0$

So we have the complex envelope of the transmitted signal:

$$S_{RF}(f) = S_R(f - f_0) + jS_I(f - f_0)$$

Now we have seen all the theoretical elements of the receiver acquisition chain, we can move on the next part, the **Software-Defined Radio**, GNURadio, which can process the received signal and decode the information.

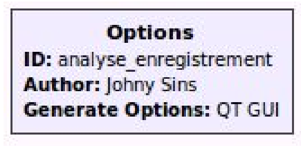
<u>Second part:</u> Reception of frequency modulation (FM) broadcasting

In this part, we will use a signal acquired by the USRP in the Very High Frequency (VHF) where the well known FM radio subband is. Theoretically 203 stations radio are simultaneously broadcasted, because they are spaced by at least 100 kHz.

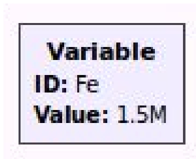
1. <u>Frequency analysis of the recording</u>

Question 7: Present the role of each block used in the processing chain.

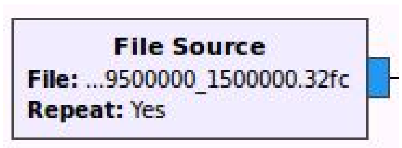
We use GNU Radio to process the recorded file. We can explain the role of all the blocks we used.



- ***Options :*** The options block sets special parameters for the flow graph.

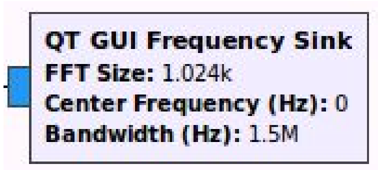


- ***Variable :*** This block maps a value to a unique variable.



- ***File Source :*** Opens and read a file as a source of items into a flowgraph. The data is expected to be in binary format, item after item.

- ***Throttle :*** Interprete the file as a sound file and allows to have a continuous flow in enter of the *QT GUI Frequency Sink* block.



- ***QT GUI Frequency Sink :*** A graphical sink to display multiple signals in frequency.

Question 8: Specify the value of the missing variables

The recorded file was obtained in Toulouse, with the following characteristics:

- ***Sampling frequency :*** $F_e = 1.5\ MHz$
- ***Transposition Frequency :*** $F_c = 99.5\ MHz$
- ***Sample rate :*** 1,5 MHz
- ***Center Frequency :*** $f_o = 99.5\ MHz$
- ***Bandwidth :*** $1.5\ MHz$ (the bandwidth of the whole FM radio is $[87.5\ MHz;\ 108\ MHz] \Rightarrow 20.5\ MHz$)

We have to use these parameters to process the recorded file.

Here you can find the diagram we use to plot the frequencies in the recorded file:

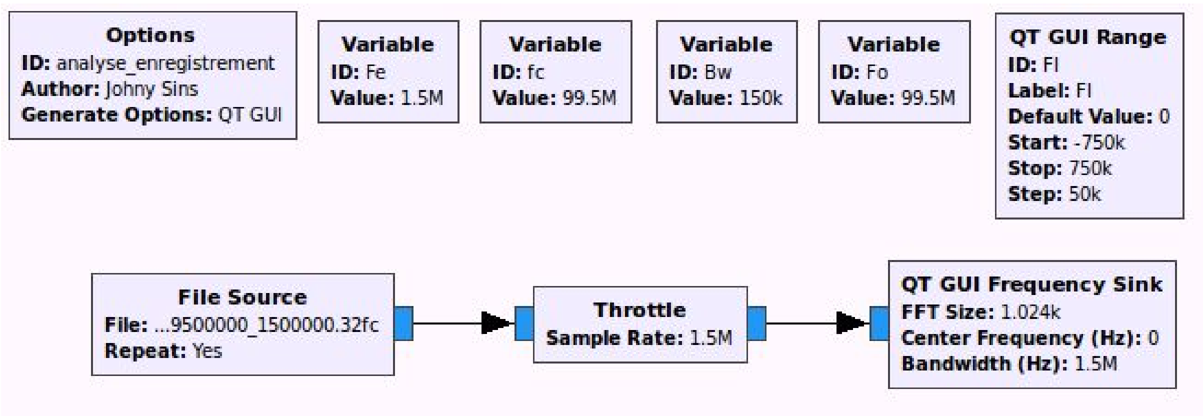


Figure 6: GNU Radio diagram to plot the frequencies from the recorded file

Question 9: How many frequency channels (L) do you observe? Which stations are observed?

We observe 2 different frequencies in the recorded file: $f_1 = 99\,MHz$ and $f_2 = 100\,MHz$. They are respectively the frequency of RFM and Skyrock.

We can also notice an other tiny signal at the frequency $f = 99,5\,MHz$, corresponding to Nostalgie, but this one is very attenuated.

You can see on the following figure all the three frequencies we can observe:

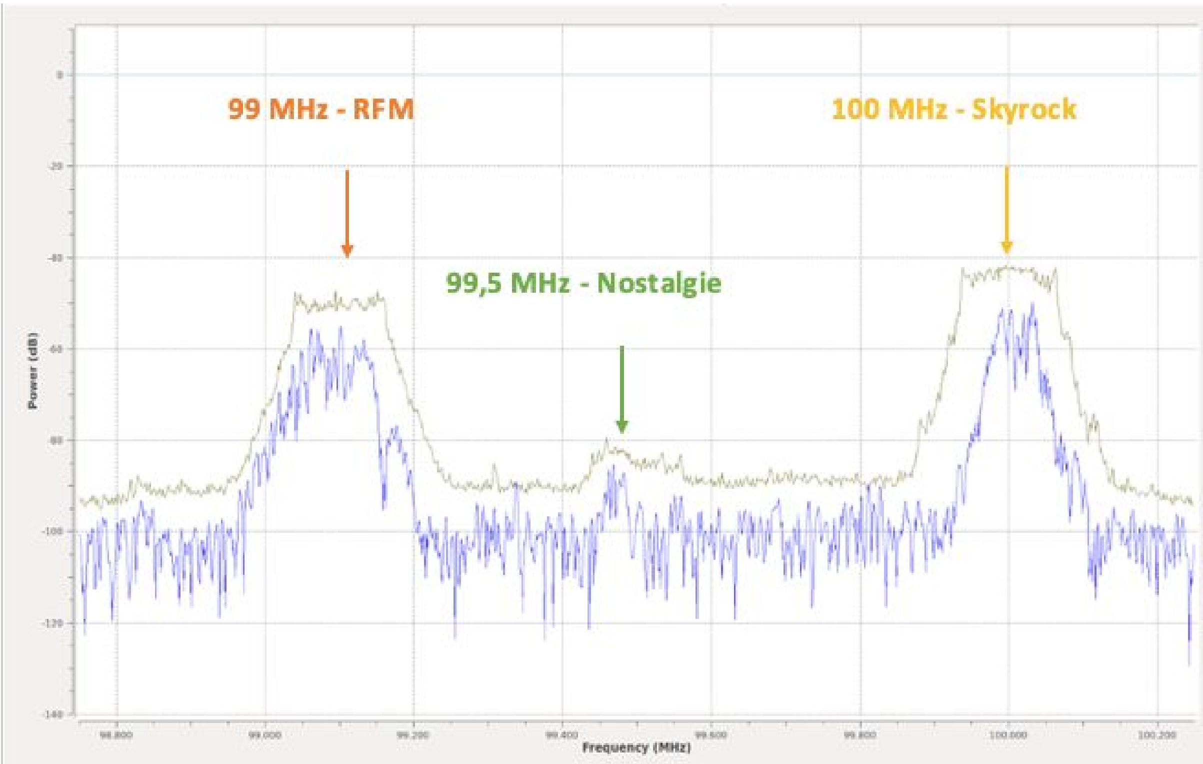


Figure 7: Viewing of the frequency channels of the recording

Question 10: What is the measured signal-to-noise ratio in decibel ?

To measure the signal-to-noise ratio, we choose to take the value of the signal at the middle of the maximum power. To do that, we just need to subtract $3\ dB$ to the value of the maximum power S_{max} of our signal. As we can see on the picture below $S_{max} = 64.7\ dB$, so $S_{max} -\ 3dB = 61.7\ dB$.

We choose to set the power of the noise to $N\ = 100\ dB$, which corresponds to the mean of the signal that is between the two peaks of the figure below.

Thus, the signal-to-noise ratio is : $|\,S_{max} - N\,|\ =\ 38,3dB$

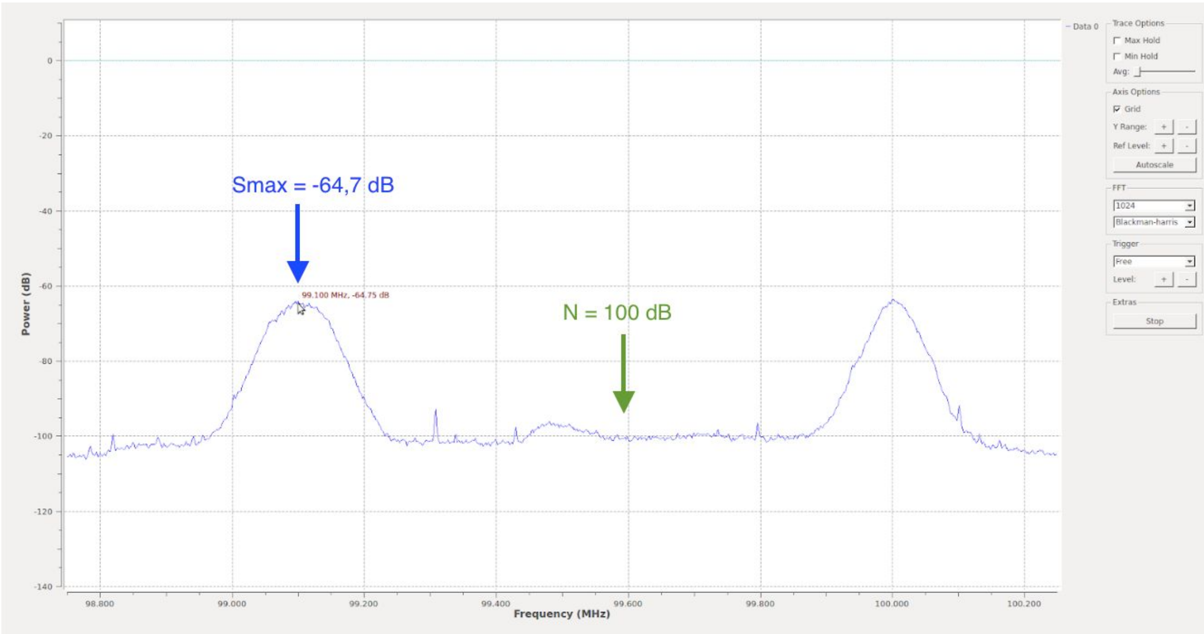


Figure 8: Value of the signal and the value of the noise for the recording

Question 11: What is the approximate bandwidth of the channel ?

Different methods exist to approximate the bandwidth, you can take the difference of frequencies at the half of the maximum signal value. Or you can take the difference at the basis of the signal, in other words, the limit between the signal and the noise.

- <u>First Method :</u> 99.180 - 99.027 = 153 kHz
- <u>Second Method:</u> 99.230 - 98.960 = 270 kHz

2. Channel extraction by frequency Transposition And Low - pass filtering

After identifying the 3 frequencies included in the recorded file, we want to select them separately. In order to do this, we have to multiply the original recorded file by cosine with the offset frequency that we want to apply.

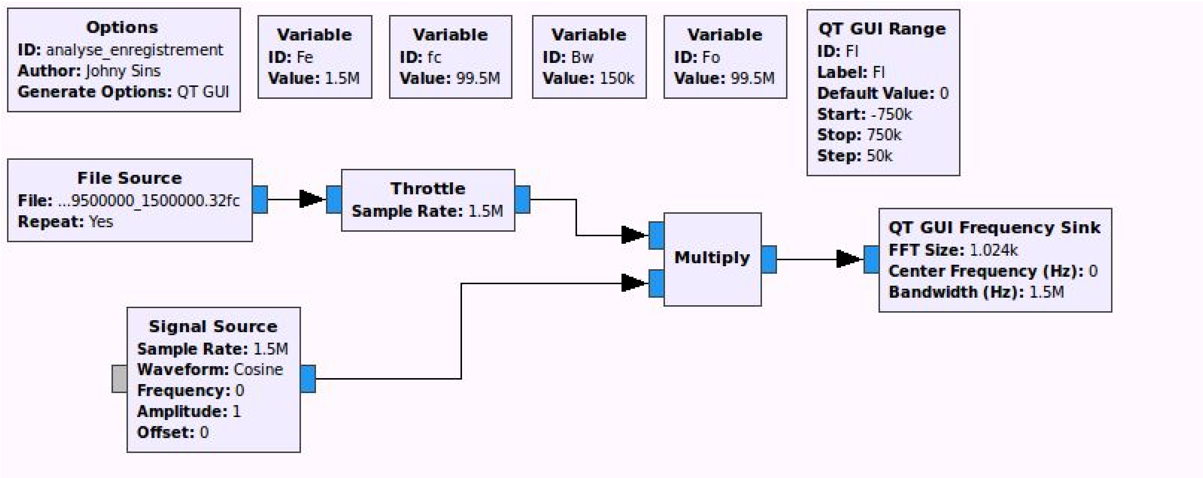


Figure 9: GNU Radio diagram to shift the frequencies

Question 12: What are the frequency offsets needed to center each channel?

The frequency transition for a quantity f_l is given by : $r_l[k] = r[k].e^{-j2\Pi\frac{f_l}{F_e}k}$.

In order to define the frequency offset f_l of the cosine needed to center the RFM & Skyrock channel, we implement a QT GUI Range bloc which allow us to find dynamically f_l during the software execution.

To be accurate, we set the step of f_l in the QT GUI Range bloc to $50\ kHz$. We know the bandwidth is $1.5\ MHz$, and the space between two frequencies is $100\ kHz$, so no need to take a lower step.

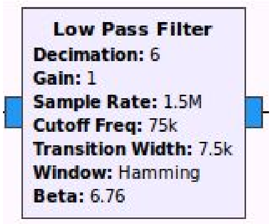
Finally we found that to center the RFM Toulouse radio signal ($99.1\ MHz$), we have to apply an offset of $+400\ kHz$, and for Skyrock radio signal ($100\ MHz$), we have to apply an offset of $-500\ kHz$, it corresponds to the central frequency $99.5\ MHz$.

Question 13: What if the frequency offset is higher than the sampling frequency F_e ?

We know that when we sample a signal, this one is repeated every F_e. So, if the frequency offset is higher than F_e, we will see the repeated signal appeared.

Question 14: What are the low-pass filter parameters, as well as those of the frequency analyser at the output of the filter?

To apply the filter to the signal transposed we use the Low Pass Filter block diagram:



$$Cutoff\ frequency = \frac{BW}{2} = \frac{150\ kHz}{2} = 75\ kHz \text{ (from question 11)}$$
$$Transition\ width = Cutoff\ frequency * 10\% = 0.1 * 75 = 7,5\ kHz$$

After the transposition of the frequencies, we can apply the filter, therefore you can find the GNU Radio diagram we use:

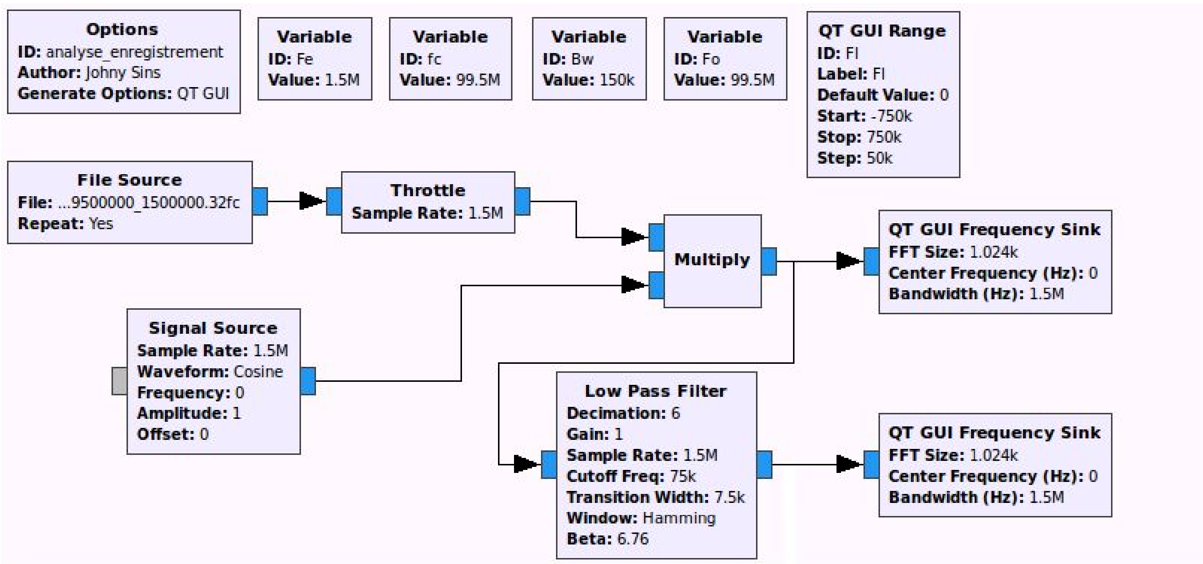


Figure 10: GNU Radio diagram to shift and filter the recorded file

After applying the low-pass filter seen in the question 2, and by setting the frequency transposition f_L to $-500\ kHz$, $0\ Hz$ and $400\ kHz$, we can see that we have filtered the signal of the Skyrock, Nostalgie and RFM radio[5].

RFM - $f_L = 400\ kHz$ **:**

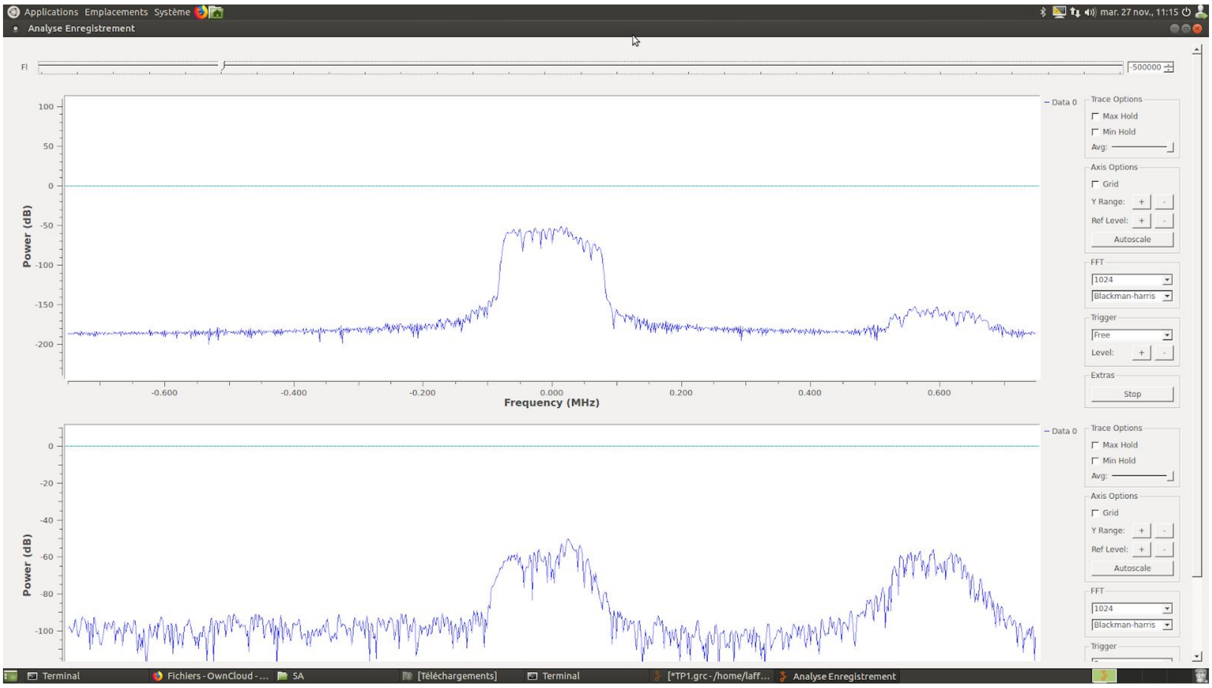
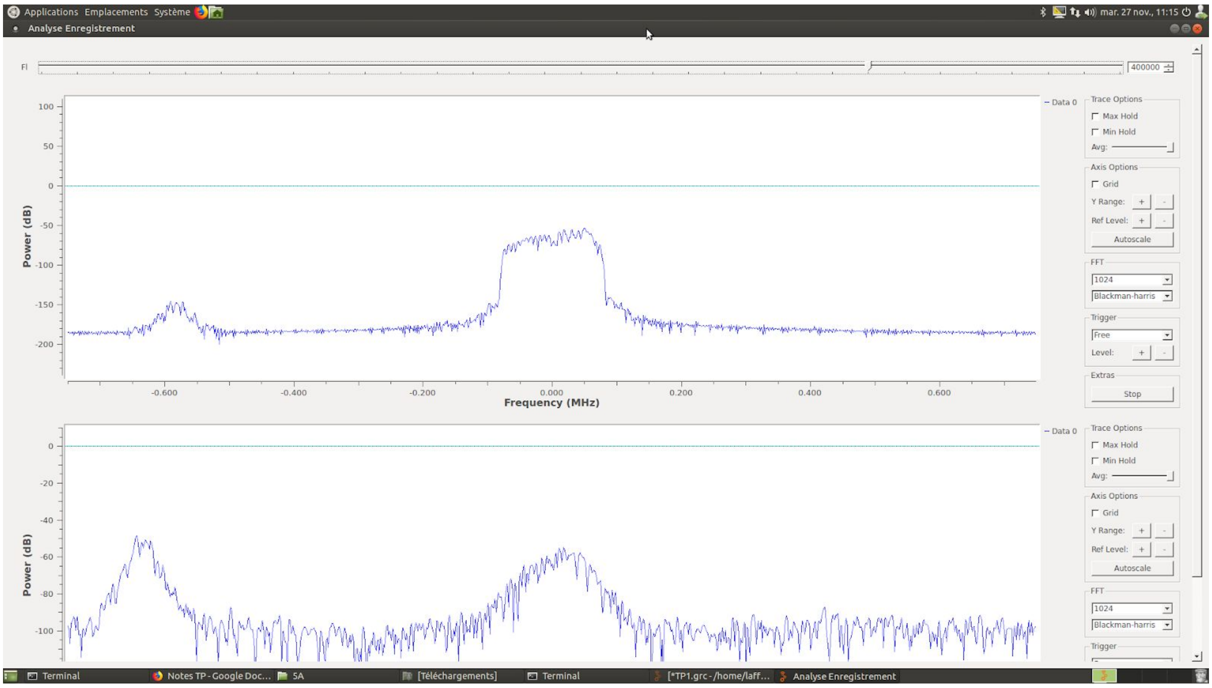


Figure 11: Skyrock channel after low-pass filtering (top) and before filtering (bottom)

Skyrock - $f_L = -500\ kHz$ **:**



[5] Toulouse Radio Stations: https://www.annuradio.fr/index.php?mode=searchville&choixville=TOULOUSE

Figure 12: RFM channel after low-pass filtering (top) and before filtering (bottom)

Nostalgie - $f_L = 0\ Hz$ **:**

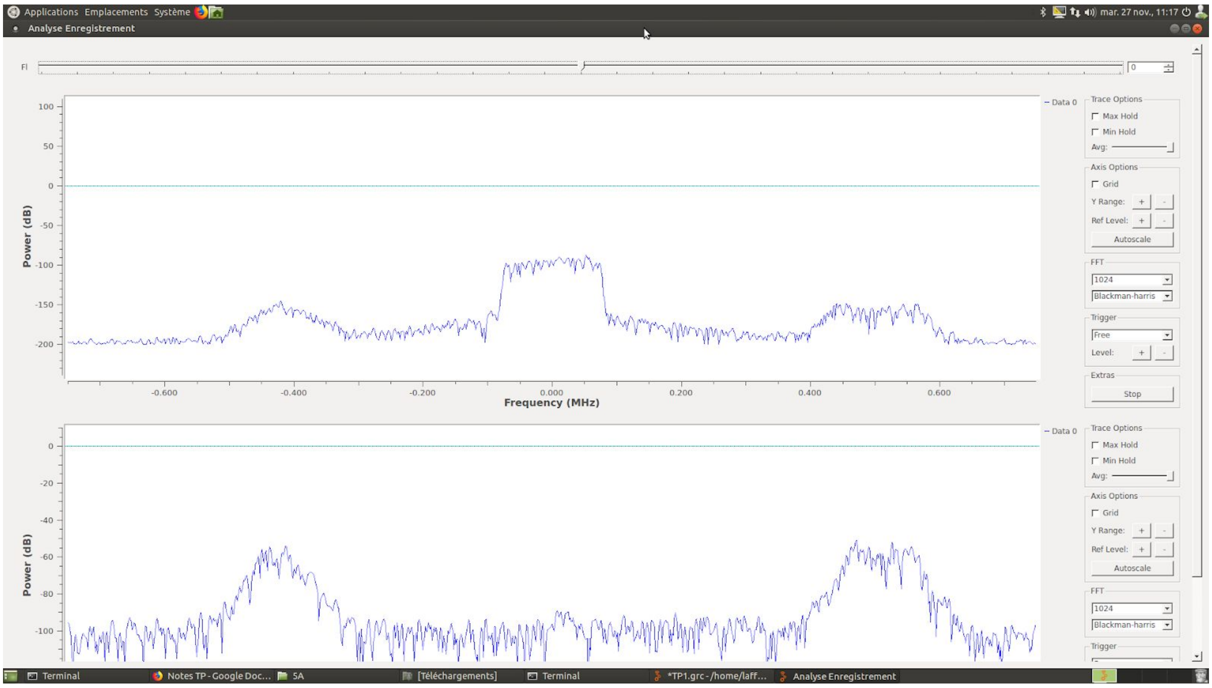


Figure 13: Nostalgie channel after low-pass filtering (top) and before filtering (bottom)

3. Frequency demodulation and restitution

The main part of the process is to demodulate the signal. According to the modulation method used, we have to adapt our demodulation stage.

We know that the transmitted signal $m(t)$ consists of two stereophonic channels, the left one $g(t),\ t \in \Re$ and the right one $d(t),\ t \in \Re$. Both of the stereophonic signals are centered in frequency and have a maximum frequency of $15\ kHz$ or mono-lateral band. But the message can be demodulated by monophonic receivers or stereophonic receivers, so the two signals have to be multiplexed to form the following message, here the expression of the message signal $m(t)$ **before the modulation**:

$$m(t) = g(t) + d(t)\ + A_{sp}.cos(2\pi f_{sp}t)\ +\ [g(t) - d(t)].cos(2\pi f_{sp}t)$$

with the pilot carrier frequency $f_{sp} = 19\ kHz$ and the amplitude $A_{sp} = 2$.

We can represent the spectrum of the message signal $m(t)$:

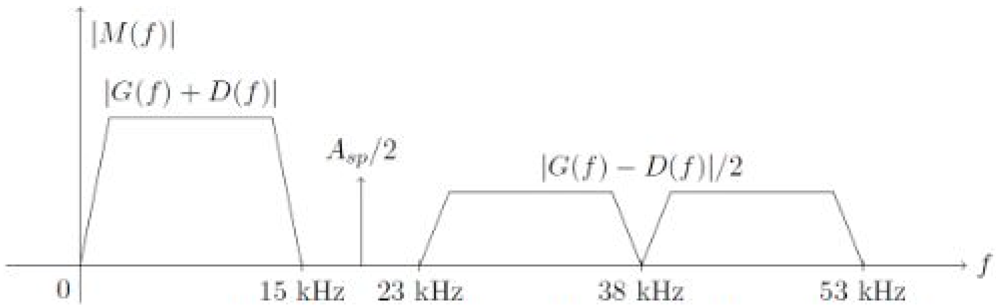


Figure 14: Stereophonic and Monophonic signal before the modulation

We can see on the spectrum the monophonic part on the left ($[g(t) + d(t)]$) and the stereophonic part on the right ($[g(t) + d(t)]/2$).

In the following questions, we want to focus only on the monophonic channel, to keep only this signal we need a low pass filter at $15\ kHz$.

Question 15: Using the Carson rule, check that the bandwidth of the channel measured in the previous part confirms the theory.

The frequency-modulated signal occupies an infinite band, but decreases rapidly, which can be approached via the Carson rule :

$B_{FM} \approx 2 \times (\Delta f + f_m)$ with: $\Delta f = 75\ kHz$ & $f_m = 53\ kHz$

So $B_{FM} \approx 256\ kHz$

This result is close to the second method used in question 11. Thanks to the Carson rule, we know what is the limite frequency the signal has.

Question 16: Define the expression of the signal $s[k]$ after frequency transposition and the filtering

We know that the signal is modulated before the transmission, and its expression is:

$$s_{RF}(t) = A.cos[2\pi f_0 t + \frac{\Delta F}{max(|m(t)|)} \int_{-\infty}^{t} (m(u)du)]$$

with: $$\varphi(t) = \frac{\Delta F}{max(|m(t)|)} \int_{-\infty}^{t} (m(u)du)$$

So, we can rewrite the previous expression:

$$s_{RF}(t) = A.cos[2\pi f_0 t + \varphi(t)]$$

We know the expression of the in-phase and quadrature channels:

$$s_r(t) = A(t).cos(\varphi(t)) \text{ and } s_I(t) = A(t).sin(\varphi(t))$$

And from question 1: $s_{RF}(t) = s_R(t).cos(2\pi f_0 t) - s_I(t).sin(2\pi f_0 t)$

So we have the expression of the transmitted signal:

$$s_{RF}(t) = A(t).cos(\varphi(t)).cos(2\pi f_0 t) - A(t).sin(\varphi(t)).sin(2\pi f_0 t)$$

If we take the real part and the complex part of the transmitted signal:

$$s(t) = s_r(t) + j.s_it)$$
$$\Rightarrow s(t) = A(t).cos(\varphi(t)) + j.A(t).sin(\varphi(t))$$
$$\Rightarrow s(t) = A(t).e^{j\varphi(t)}$$
$$\Rightarrow s(t) = A(t).e^{j\frac{\Delta F}{max(|m(t)|)}\int_{-\infty}^{t}(m(u)du)}$$

In discrete domain we obtain: $s[k] = A[k].e^{j\frac{\Delta F}{max(|m(t)|)}\sum_{-\infty}^{k} m(i)} + b[k]$

So, by identification we find: $s[k] = A[k].e^{jk_f \sum_{-\infty}^{k} m(i)} + b[k]$, we have: $k_f = \frac{\Delta F}{max(|m(t)|)}$

We know the information of $m(t)$ is contained in the argument, that is why we need to use the **WBFM Receive** bloc which performs this computation to obtain $\overline{m}_l[k]$ with the Carson rule frequency:

$$\overline{m}_l[k] = arg(s[k].s^*[k-1])$$

.

Question 17: Plot the spectrum of the demodulated channel and compare with the Fig. 14.

In order to obtain the demodulated signal, we use the GNU Radio diagram below:

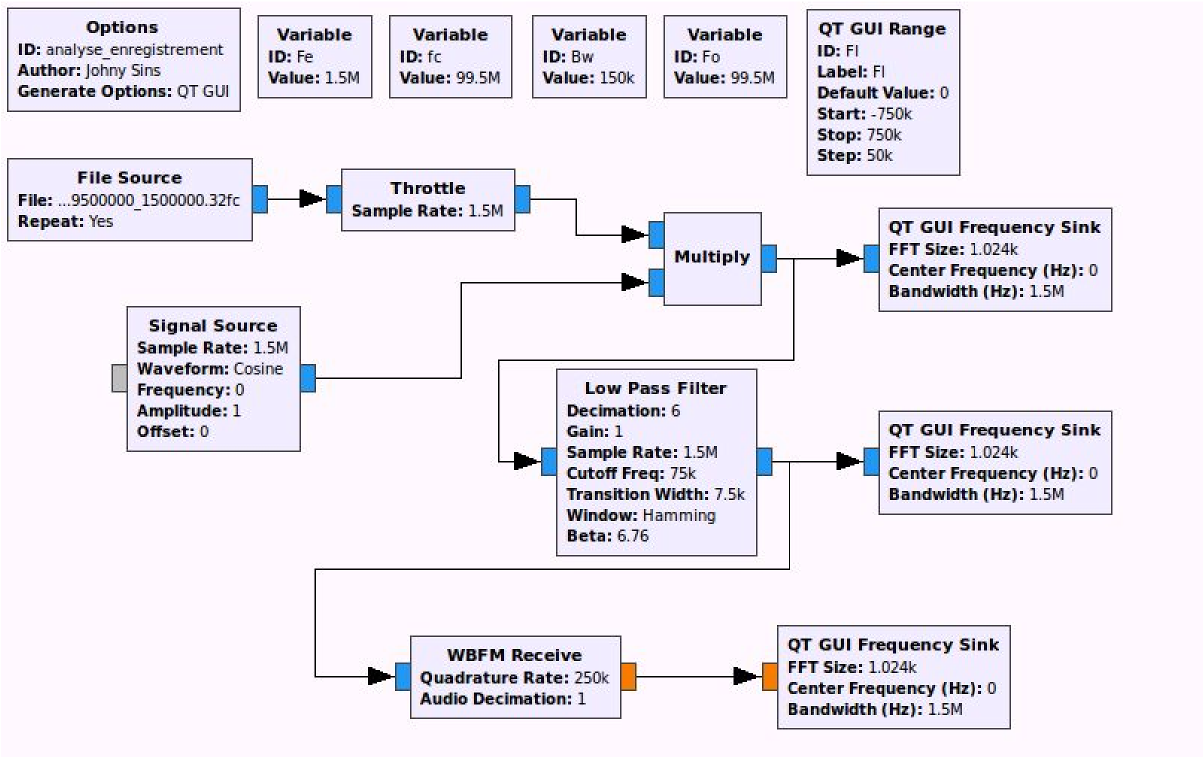


Figure 15: GNU Radio diagram to demodulate the received signal

Here you can find the result of the demodulation:

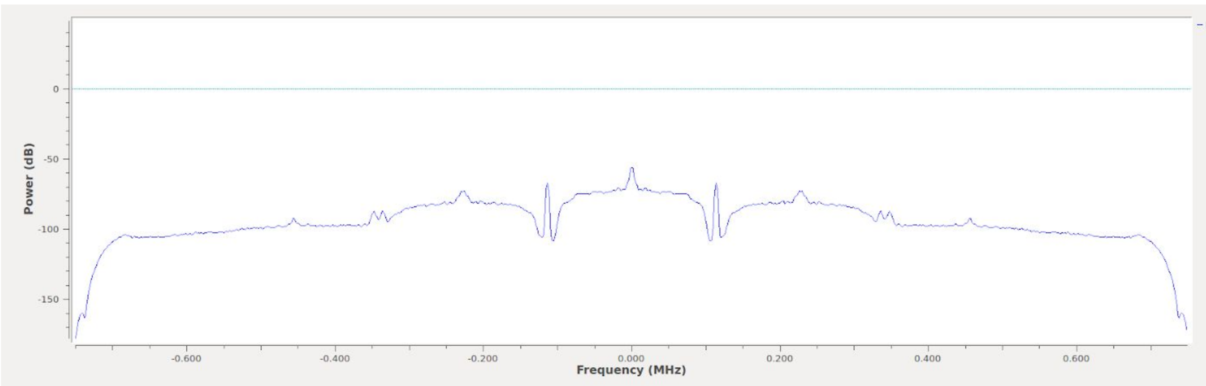


Figure 16: Spectrum of the received signal demodulated

We can see on this figure, the monophonic channel (in the middle) and two stereophonic channels (both side of the monophonic channel). We can also observe the two pics, separating the monophonic and stereophonic channels, which correspond to the frequency $f_{sp} = 19\ kHz$.

Question 18: Who won the Sam Smith album? What do we listen to on other stations?

After the demodulation, we need to apply a low pass filter to keep only the monophonic channel, and then we can listen the audio recorded file with the speaker of the computer. So, we use the following GNU Radio diagram:

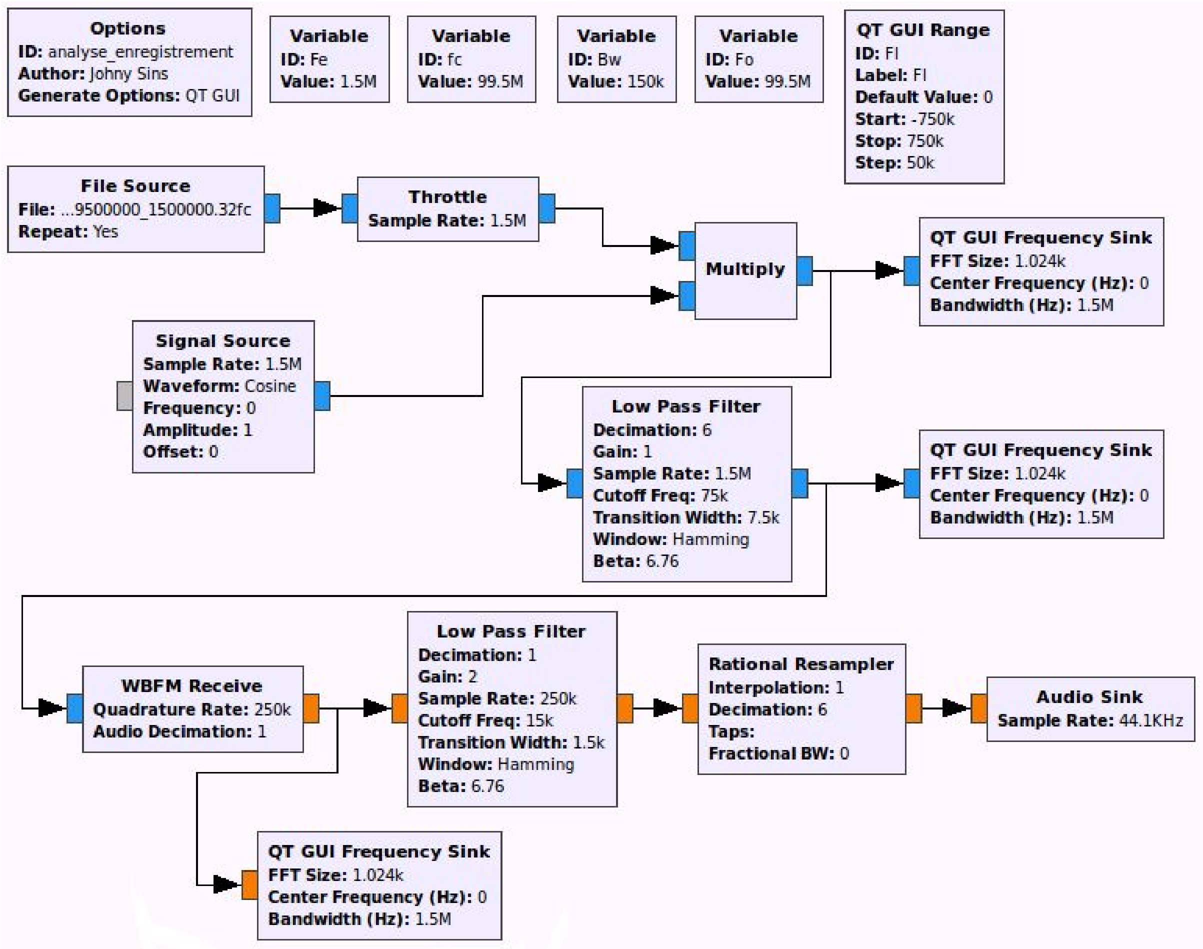


Figure 17: GNU Radio Diagram to listen the recorded file with the speaker of the computer

The sample frequency of the speaker is $44.1\ kHz$ which corresponds to an audio frequency. So, we need to use the **Rational Resampler bloc** to go from a $250\ kHz$ sample frequency to a $44.1\ kHz$ sample frequency.

The operation done by the Rational Resampler is given by this formula:

$$F_{s-out} = F_{s-in} \times \frac{Interpolation}{Decimation}$$

So we choose an $Interpolation = 1$ and a $Decimation = 6$

The decimation allows us to remove samplings, and interpolation to add some.

Remarque: thanks to the decimation, we can remove samplings, this also allows to respect Shannon and also to have a digital to analog converter (DAC) working at a lower frequency, so to have a hardware less expensive.

By listening the Nostalgie radio, we found that Jordy won the Sam Smith album.

On RFM, the music played is *Counting Stars* of *One Republic*.

4. Real time implementation with an USRP receiver

Now we design all the processing chain of the SDR, we can test our system with the USRP receiver in real time to listen to the radio. We have to replace the file source bloc by the UHD USRP source block, like in this diagram:

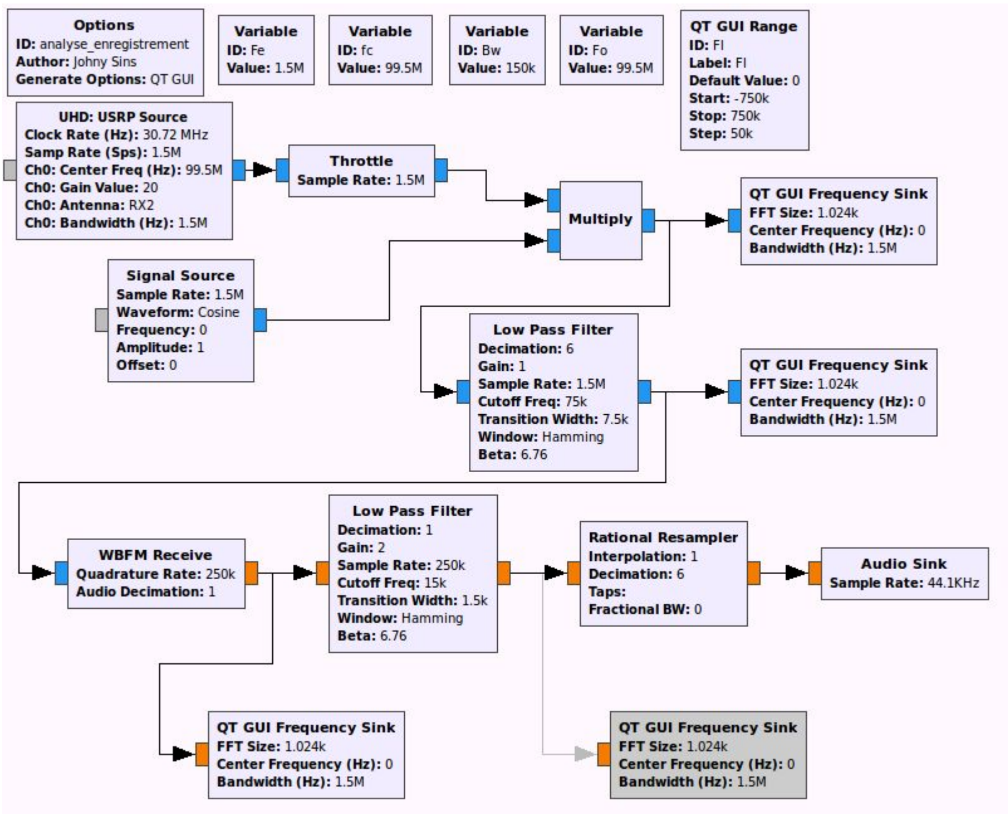


Figure 18: GNU Radio diagram for receiving real time FM radio

Thanks to our program we can listen to RFM, Nostalgie and Skyrock. If we want to receive other stations we need to change the range of the **QT GUI Range block**, like this during the frequency transposition we can focus on other frequencies.

Third part: Reception of VOLMET messages in AM - SSB

In this part, we will work in the High Frequency (HF) band. The propagation in this band is done by successive reflections on the ionosphere and the ground. The main advantage to this technique is to have an intercontinental links with a reasonable power budget[6]. But you have to adapt the path of the signal according to the evolution of the ionosphere.

We use a file which contains a short record in Toulouse of a subband between $11.175\ MHz$ and $11.4\ MHz$. So the center frequency is $f_o = 11.2965\ MHz$, and the sampling frequency is $F_e = 250\ kHz$.

1. Frequency analysis of the recording

Question 19: Plot the modulus of the Discrete Fourier Transform

As we did on the first part, we used **GUI Frequency Sink** to plot the discrete Fourier Transform in dB, between $F_0 - \frac{F_e}{2}$ and $F_0 + \frac{F_e}{2}$.

Here is the GNU Radio blocks used to do this, followed by the resulting plot:

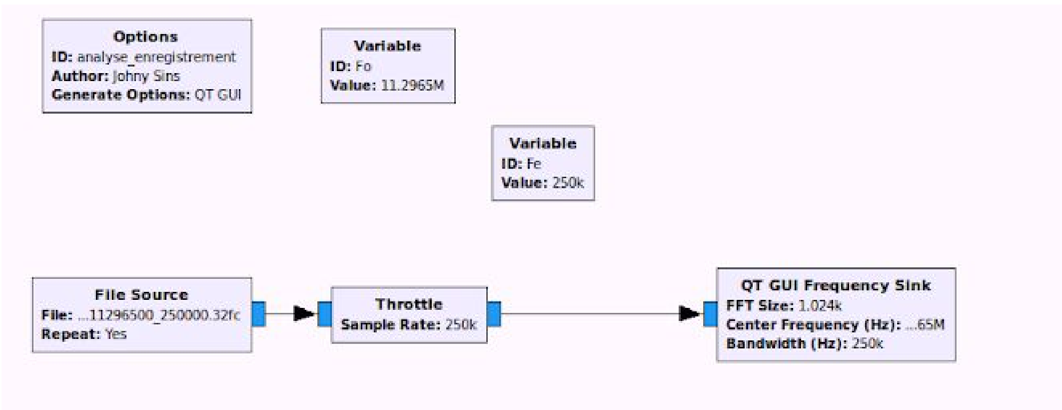


Figure 19: GNU Radio diagram to plot the modulus of the discrete Fourier transform of the File Source

[6] High Frequency (HF): https://en.wikipedia.org/wiki/High_frequency

Thanks to the GNU Radio diagram, we obtain this spectrum:

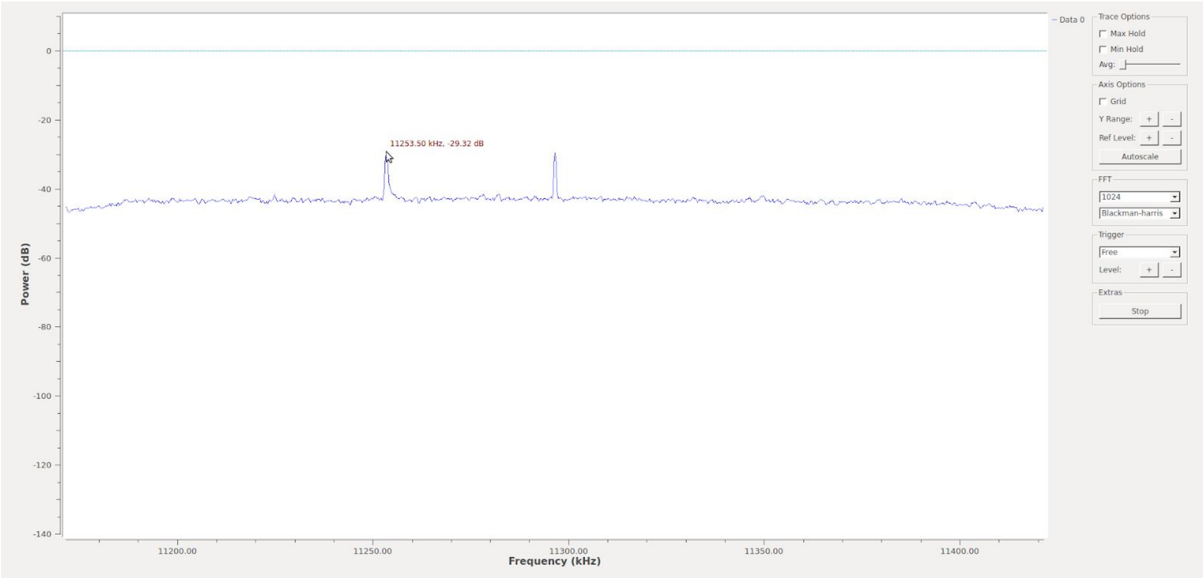


Figure 20: Modulus of the discrete Fourier transform of the recording

On this plot, we can observe a signal at $F_{RAF} = 11,253\ MHz$, which is the one of the VOLMET station located in the Royal Air Force air-base, in United Kingdom[7]:

11.253	*Cont*	MKL	GBR	MILITARY ONE	4000	50 28 58	-5 00 00

2. Frequency transposition

After acquiring the signal from the file source, the first step is to transpose the signal, like we did in the previous part.

Question 20: What are the frequency offsets f1 needed to center the channel with the maximum power at the null frequency?

After computation, we found that the transposition frequency is :

$$f_{offset} = F_0 - F_{RAF} = 43,5\ kHz$$

[7] Worldwide VOLMET broadcasts: http://www.dxinfocentre.com/volmet.htm

If we implement the same blocks, we can apply the transposition frequency and find the same result, it is around $f_{offset} = 45\ kHz$. In fact, we can observe on the graph below that the channel with the max power in centered at the null frequency for this value of f_{offset} .

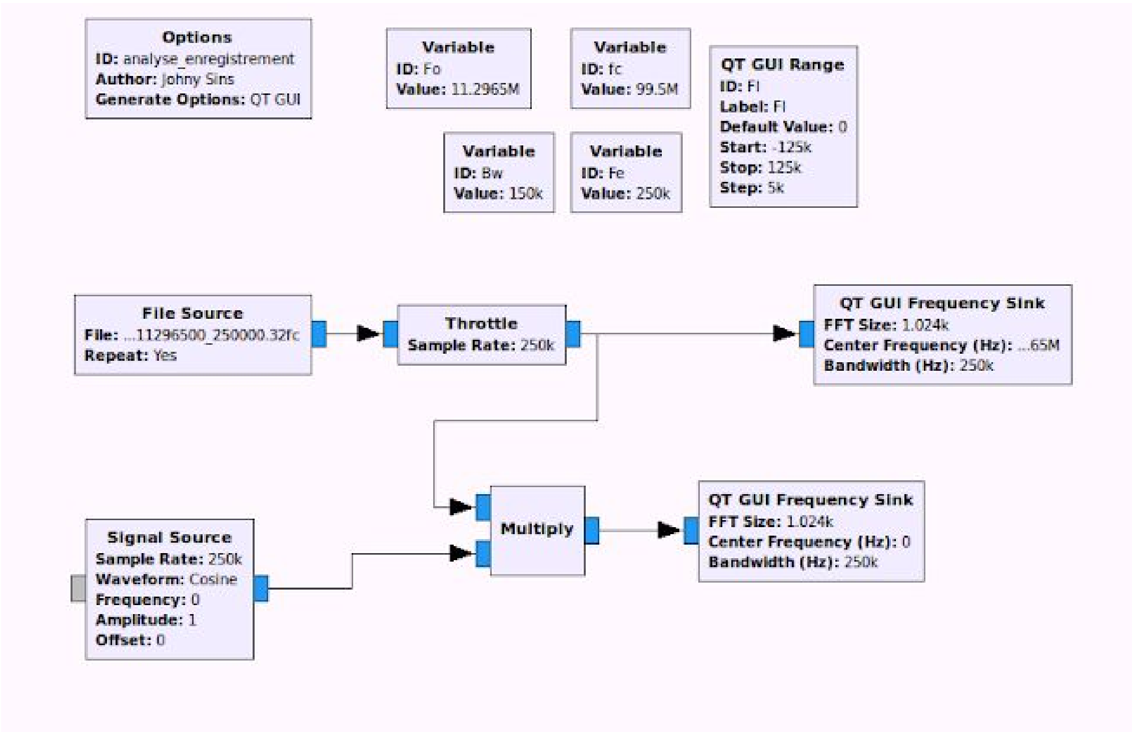


Figure 21: GNU Radio diagram to transpose frequencies

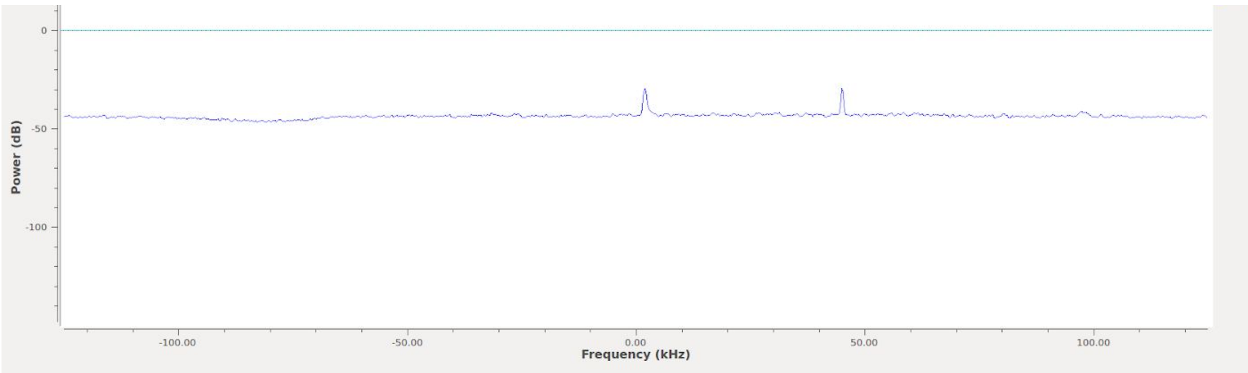


Figure 22: Resulting plot after the frequency transposition

3. Single sideband amplitude demodulation.

The transmission process used by VOLMET service is a single side band amplitude modulation. If we note $m(t),\ t \in \Re$ the vocal message transmitted by the VOLMET service, the message is low-pass filtered in order to occupy a bilateral sideband with a bandwidth of $B = 6\ kHz$ as presented in the figure below:

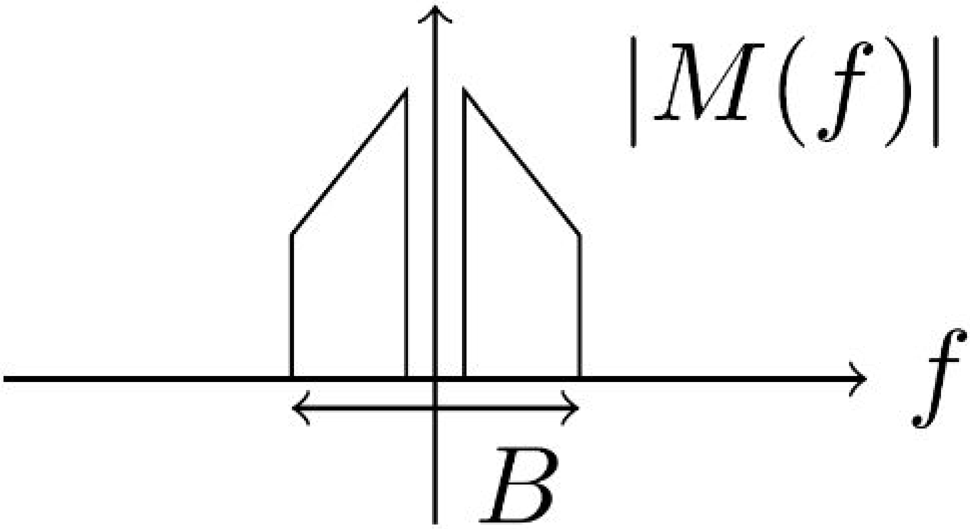


Figure 23: Discrete Fourier Transform of the message $m(t)$

Then signal $m(t)$ is sent through Amplitude Modulation, we can give the expression of the transmitted signal:

$$s_{RF}(t) = R_e\{s(t) \times e^{2\pi f_0 t}\} = R_e\{[m(t) \pm j.H(m(t))] \times e^{2\pi f_0 t}\}$$

Question 21: We want to express $|S_{RF}(f)|$

We know the expression of the transmitted signal:

$$s_{RF}(t) = R_e\{s(t) \times e^{2\pi f_0 t}\} = R_e\{[m(t) \pm j.H(m(t))] \times e^{2\pi f_0 t}\}$$

$$S_{RF}(F) = \Im[s_{RF}(t)](F) = M(F) * \delta(F - F_0)$$
$$S_{RF}(F) = \Im[R_e\{s(t) \times e^{2\pi f_0 t}\}](F)$$

$$S_A(F) = S_{RF}(F) + sgn(F) \times S_{RF}(F)$$
$$S_A(F) = M(F) * \delta(F - F_0) + sgn(F) \times [M(F) * \delta(F - F_0)]$$
$$S_A(F) = \delta(F - F_0) * [M(F) + sgn(F) \times M(F)] = M(F - F_0) + sgn(F - F_0) \times M(F - F_0)$$

$$S_{RF}(F) = \frac{1}{2}[S_A(F) + \widehat{S_A}(F)]$$
$$S_{RF}(F) = \frac{1}{2}[M(F-F_0) + sgn(F-F_0) \times M(F-F_0)$$
$$+ M(-F-F_0) + sgn(-F-F_0) \times M(-F-F_0)]$$

- If $F - F_0 > 0 \ or \ F > F_0$, so we obtain :

$$S_{RF}(F) = \frac{1}{2}[M(F-F_0) + M(F-F_0) + M(-F-F_0) - M(-F-F_0)]$$
$$S_{RF}(F) = M(F-F_0)$$

- If $F - F_0 < 0 \ or \ F < F_0$, so we obtain :

$$S_{RF}(F) = M(-F-F_0)$$

We can choose which side of the signal we want to keep, we can demodulate only the negative part of the signal (lower sideband), or only the positive part (upper sideband) of the signal.

$$H(m(t)) = \ -j.sgn(F).M(F)$$
$$S_{RF}(F) = \Im[R_e\{[m(t) \pm j.H(m(t))] \times e^{2\pi f_0 t}\}](F)$$
$$S_{RF}(F) = \Im[R_e\{m(t) \times e^{2\pi f_0 t} \pm j.H(m(t)) \times e^{2\pi f_0 t}\}](F)$$

Question 22: Using the spectrums plotted in questions 19 and 20, which sideband is conserved for a VOLMET transmission?

By analysing the spectrums plotted in question 19 and 20, and knowing that the bandwidth of the signal is $6\,kHz$ and that the signal is center around $11,253\,MHz$, we can deduce that the sideband conserved by the VOLMET station is the upper one.

In fact, here we keep only one sideband, so if we look at the $11,253\,MHz$ frequency on the plot we can see that the end of the sideband is around $11,255\,MHz$, it means only the upper half of the bandwidth. We can see on the figure below upper sideband of the VOLMET transmission:

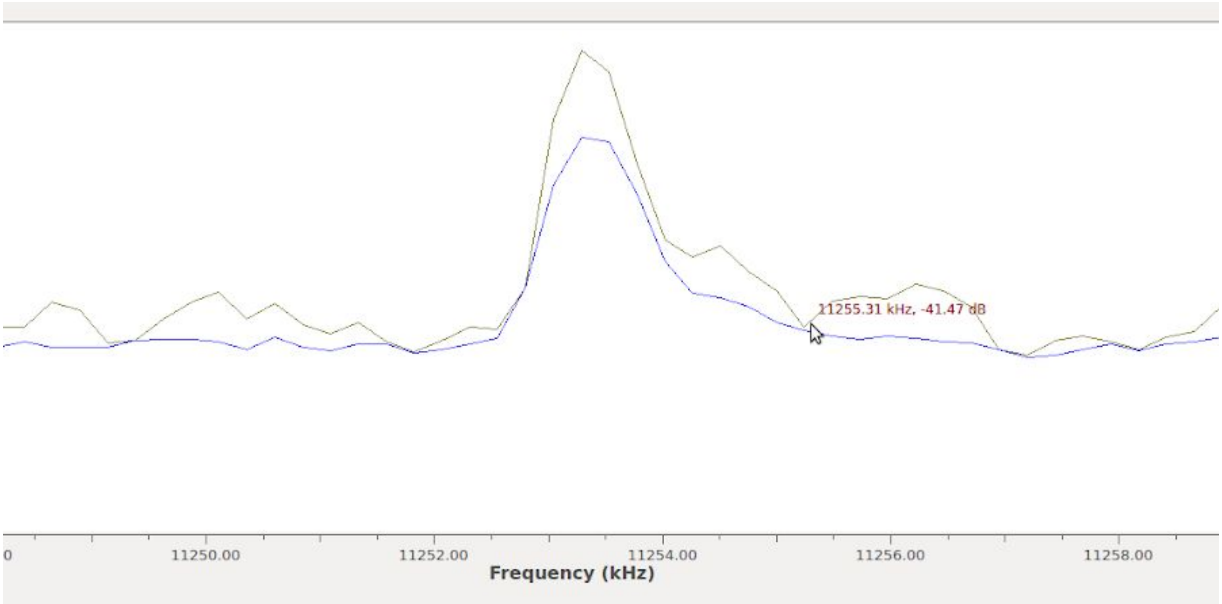


Figure 24: Upper Sideband of the Volmet Transmission

Question 23: Use the Filter Design Tool to create a complex pass-band filter

Thanks to **Filter Design Tool**, we can design a complex pass-band filter with the following characteristics:

- Complex pass-band filter with finite impulse response
- Lower cut-off frequency at $0\ Hz$
- Upper cut-off frequency at $f_c = B/2 = 3\ kHz$
- In-band gain of $0\ dB$
- Out-band gain of $-30\ dB$
- Transition Frequency = 10% of the cut-off frequency:

$$f_T = 10\% * 3\ kHz = 300\ Hz$$

The following graph shows the modulus of the frequency response of this complex pass-band filter:

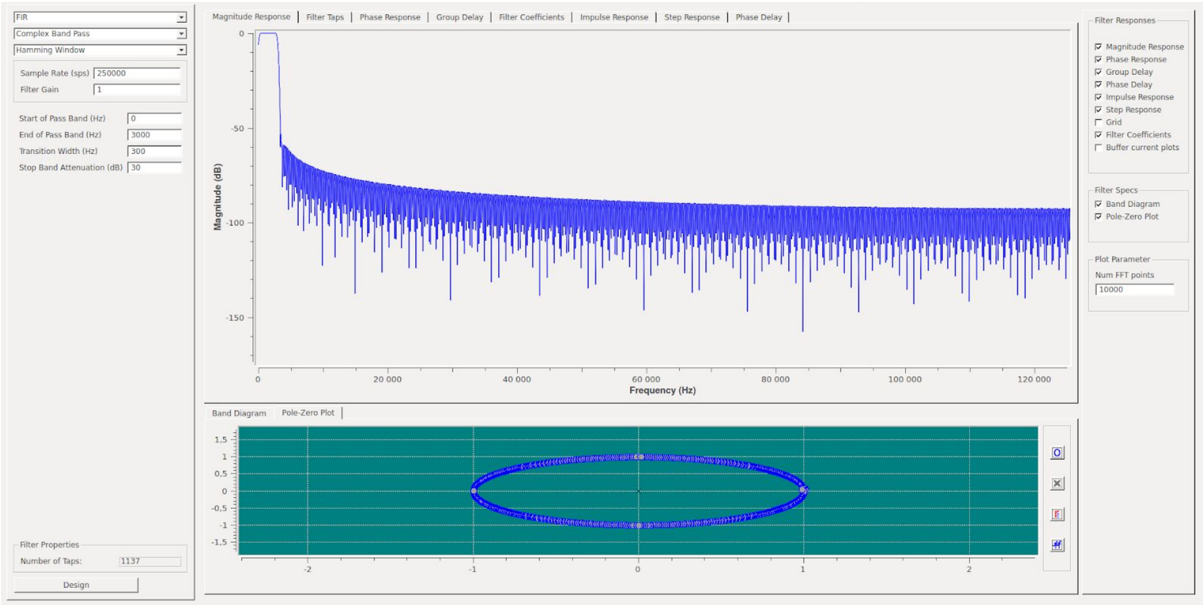


Figure 25: Modulus of the Complex Pass-band Filter

We can see that the characteristics chosen for the filter are quite constraigning, and after the cut-off frequency we have huge oscillations on the modulus. But because the filter is numeric, we can choose a complex filter very powerful, we have to see if the program can run normally.

Question 24: We apply the previous filter to the signal $r_{RF}[k]$

In order to apply the filter we design previously on the signal $r_{RF}[k]$, we use the **FFT Filter Block**:

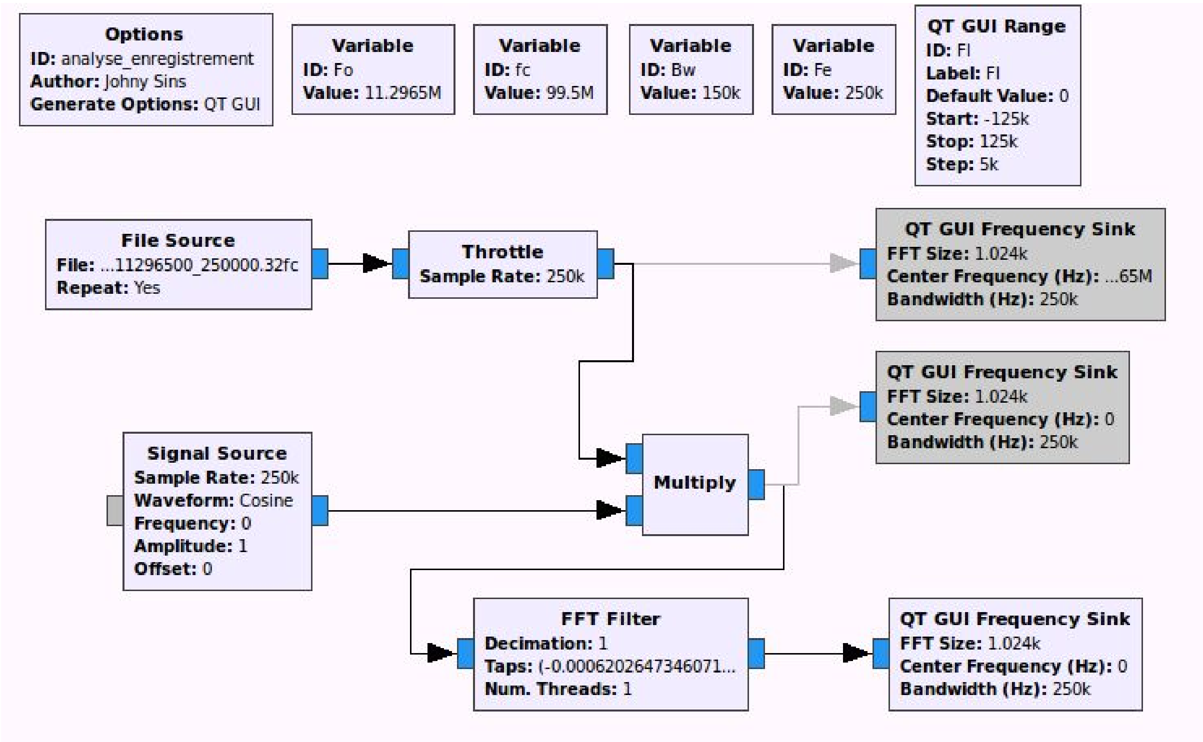


Figure 26: GNU Radio Diagram to transpose and filter the recorded file

And we obtain the following spectrum of the signal $y[k]$, the filter works well.

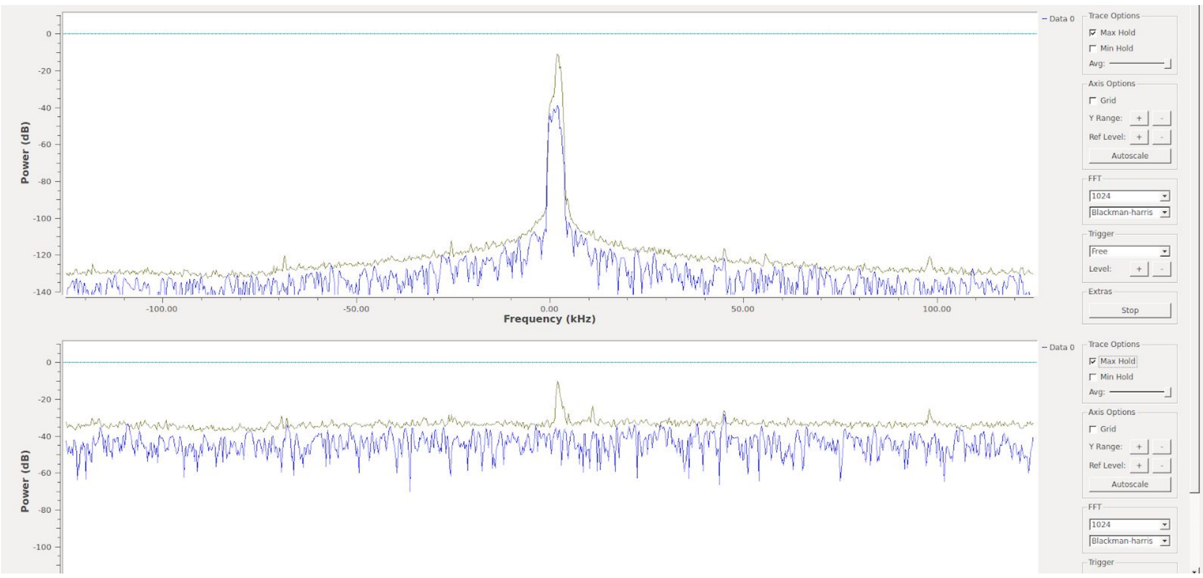


Figure 27: Spectrum of the filtered signal $y[k]$ (top spectrum)
And spectrum of the transposed signal $r_1[k]$

Question 25: How do we recover the real signal ?

To finish, we want to recover the real signal in order to play the audio stream. We want the real signal in the form:

$$\widehat{m}[k] = m[k] + \widehat{b}[k]$$

With: $\widehat{b}[k] = Re\{b[k]\}$ and $b[k]$ the complex noize.
As we want to keep the real part of the signal, we use the block **Complex To Real** that we apply to the previous signal $y[k]$. Finally we use the **Rational Resampler** and **Audio Sink** blocks to play the resulting signal.

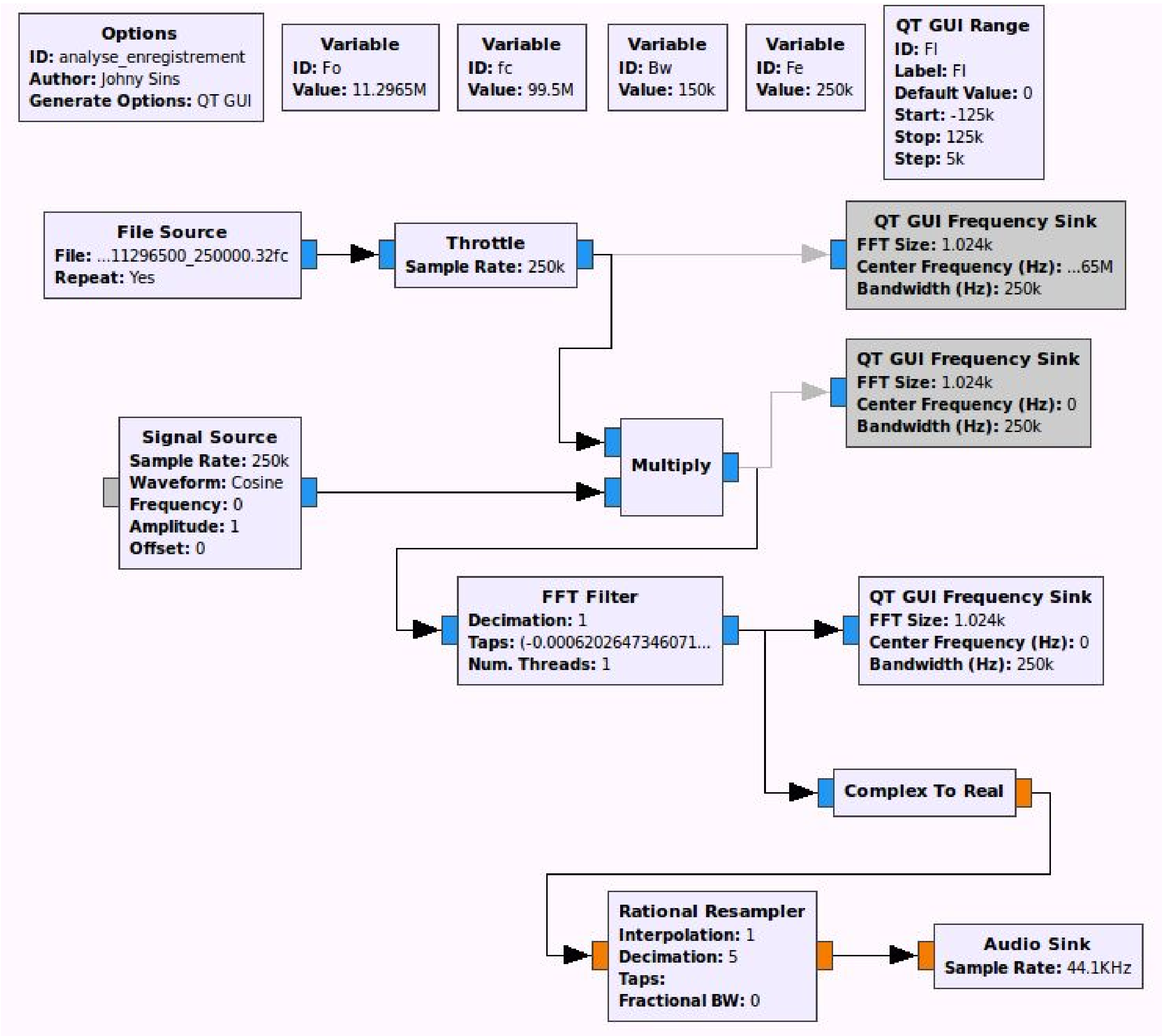


Figure 28: GNU Radio to demodulate the VOLMET signal and to listen to the computer speaker

Unfortunately, we couldn’t make the real time implementation with the USRP because the frequency range of the URSP used for the TP is [70MHz ; 6GHz], but the VOLMET service frequency is between 11.175MHz and 11.4MHz.

Conclusion

These practical works were a good introduction to the Software Defined Radio. We first understood the functioning of an IQ transceiver by analysing the mathematical aspects of the transmitted and received signals.

Then, we focused on the frequency modulation (FM) band and thanks to the GNURadio development environment we were able to restore the audio content of a recording file made by the acquisition system presented in the introduction. By using the *USRP-2900* we could also perform a real time demodulation and listening of the radio channels.

Finally, we study the amplitude demodulation by analysing a recording made in the high frequency band.

Thus, thanks to these practical works, we saw that the main advantage to process digitally the signal is to allow a wide range of applications by only changing the software part. Moreover, the use of an URSP and a SDR is more generic, like this you can received signals from different technologies (Wi-Fi, Bluetooth, GSM, UMTS, LTE) with only one antenna and by only changing the processing software. As today the technology is evolving fast, this allows you update the functionalities of your device without having to change the hardware which is time, cost and energy consuming.