

Introduction to *Software-Defined Radio*: Analog demodulation of signals using GNURadio

Organization:

Number of sessions: 3
Duration of session: 2h45
Number of group members: 2

Objectives:

Make the connection between the courses, the theory and the applications.
Understand signal-processing theory in order to implement solutions.
Characterize the tools (USRP and SDR); and know applications.
Utilise the tools to imagine and implement applications.

Evaluation criteria:

A self-sufficient report -to be included in the portfolio- by the group, including everything that has been made. Naturally an introduction and a conclusion are expected..
Absences and delays.

Keywords:

ADC Analog to Digital Converter
DAC Digital to Analog Converter
SDR Software Defined Radio
USRP Universal Software Radio Peripheral

Trigonometric formula:

$$\cos(-x) = \cos(x)$$

$$\sin(-x) = -\sin(x)$$

$$\cos(A) \cdot \cos(B) = \frac{\cos(A - B) + \cos(A + B)}{2}$$

$$\sin(A) \cdot \sin(B) = \frac{\cos(A - B) - \cos(A + B)}{2}$$

$$\sin(A) \cdot \cos(B) = \frac{\sin(A + B) + \sin(A - B)}{2}$$

$$\cos(A) \cdot \sin(B) = \frac{\sin(B + A) + \sin(B - A)}{2}$$

Main Fourier transforms:

$$s(t) \Rightarrow S(f)$$

$$1 \Rightarrow \delta(f)$$

$$\cos(2 \cdot \pi \cdot f_0 \cdot t) \Rightarrow \frac{1}{2} \cdot |\delta(f + f_0) + \delta(f - f_0)|$$

$$\sin(2 \cdot \pi \cdot f_0 \cdot t) \Rightarrow \frac{j}{2} \cdot |\delta(f + f_0) - \delta(f - f_0)|$$

multiply \Rightarrow convolve

Nomenclature:

$s_{RF}(t)$	transmitted signal in time domain
$s_R(t)$	real part of the transmitted signal in time domain (in-phase)
$s_I(t)$	imaginary part of the transmitted signal in time domain (quadrature)
$r_{RF}(t)$	received signal in time domain
$\tilde{r}_R(t)$	real part of the received signal in time domain
$\tilde{r}_I(t)$	imaginary part of the received signal in time domain
$r_R(t)$	real part of the received signal in time domain after filtering
$r_I(t)$	imaginary part of the received signal in time domain after filtering
$r_R(k.T_e)$	real part of the received signal in time domain after filtering and sampling
$r_I(k.T_e)$	imaginary part of the received signal in time domain after filtering and sampling
$s_a(t)$	analytic signal
$s(t)$	complex envelop
$S_R(f)$	Fourier transform of real part of the received signal
$S_I(f)$	Fourier transform of imaginary part of the received signal
$\tilde{R}_R(f)$	Fourier transform of real part of the received signal, after filtering
$\tilde{R}_I(f)$	Fourier transform of imaginary part of the received signal, after filtering

Introduction

During these three practical sessions, we focus on the reception of real communication signals (e.g. broadcasting, aeronautical communications, etc.). A specific part of the wide frequency range of radio spectrum is allocated to each signal, with regards to the targeted applications and constraints. The complete description of the allocations of radiofrequency bands are available at https://en.wikipedia.org/wiki/Electromagnetic_spectrum/. We will see that a single receiver can record almost all these signals and then process them numerically using GNURadio, in order to recover all the information.

Firstly, we will demonstrate that in the case of narrowband signals, the IQ transceivers (In-phase/Quadrature) allow transmissions with frequency transportation without altering data.

Secondly, we will work on FM broadcasting in order to demodulate it in real time signals.

Thirdly, the VOLMET AM broadcasting will be treated, again in real time.

Ultimately, a short project around the OSI physical layer of an Internet-of-Things (IoT) technology will be achieved in autonomy.

First part: Presentation of the acquisition device: In-phase/Quadrature *Software-Defined Radio* transceiver

The “*Software Defined Radio*” refers to a type of radiofrequency transceiver in which most of the processing is done digitally, whether it is the transmission or the reception of data. For a reception, the received signal is digitized through an ADC, then processed (for instance: filtered, decimated, demodulated, decoded, etc.). For the transmission, the data to transmit are processed (for instance: coded, modulated, etc.), then converted through a DAC and sent to the RF front-end.

Thus, thanks to sufficiently generic hardware (wide frequency range, high sampling frequency, etc.), it is possible to easily develop many applications by only changing the software part of the transceiver (e.g. cellular telephony, digital television, etc.). Moreover, many applications can operate at the same time on the same hardware (e.g. simultaneous received signals -with different technologies, frequencies, modulations and/or encodings- can be processed concurrently, as well as other signals that can be transmitted). In addition, it is possible to update the software without stopping the hardware.

This modular approach is not in consensus with the current trend of creating specific electronic circuits with a view to maximum integration. For instance, in your smartphone there are at least six independent transmission channels: GSM (2G), UMTS (3G), LTE (4G), Wi-Fi (IEEE 802.11), Bluetooth (802.15.1), NFC, etc. Each of these technologies has a specific integrated circuit and antenna. Thus, at each evolution of the technology, the hardware must be changed. In the near future, the *software-defined radio* will mutualise these devices in order to save space and energy and allow reusing and evolving of the system.

During these practical works, we will use a National Instruments USRP-2900 *software defined radio* transceiver which is connected to a computer via a Gigabit Ethernet connection (Fig. 1). The receiver is composed of two stages (Fig. 2): the first allows the transposition of the signals around the zero frequency to facilitate their processing; and the second carries out their analog to digital conversion (CAN) (sampling at a period T_e then quantization scalar uniform over 12 bit). At the end of the reception chain, the computer stores the flow of samples in a file that we use during the following exercises under GNURadio -a free software-.



Fig. 1: National Instruments USRP-2900 transceiver

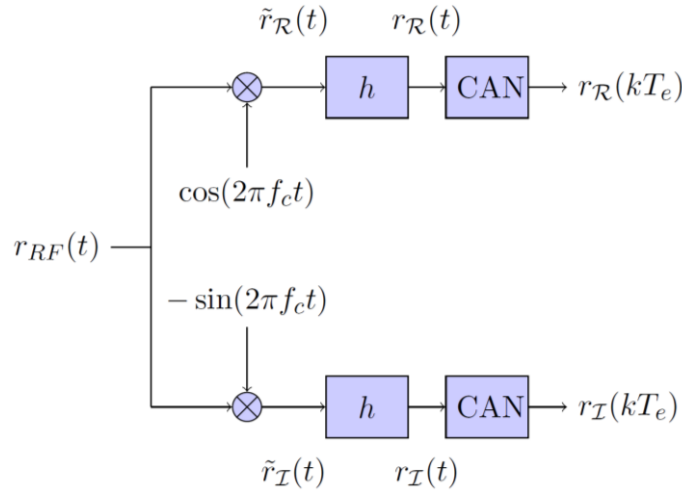


Fig. 2: Block diagram of the receiver

To understand the structure of the receiver, we can note a communication signal transmitted around a carrier frequency f_0 as:

$$s_{RF}(t) = A(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t + \varphi(t)), t \in \mathbb{R} \quad (1)$$

with $A(t) \in \mathbb{R}$ the envelope and $\varphi(t) \in \mathbb{R}$ the phase.

If we consider a message $m(t) \in \mathbb{R}$ to be transmitted, we can distinguish three common modulation schemes described below:

- Amplitude modulation $A(t) \propto m(t)$ and $\varphi(t)$ constante
- Phase modulation $\varphi(t) \propto m(t)$ and $A(t)$ constante
- Frequency modulation $\varphi(t) \propto \int_{-\infty}^t m(u) \cdot du$ and $A(t)$ constante

We can also combine these different modulation schemes in order to have more complex schemes, as amplitude and phase modulation, where $A(t) \propto m(t)$ and $\varphi(t) \propto m(t)$.

By noting $s_R(t) = A(t) \cdot \cos(\varphi(t)), t \in \mathbb{R}$ and $s_I(t) = A(t) \cdot \sin(\varphi(t)), t \in \mathbb{R}$ the channels in-phase and quadrature and supposing a bandwidth $\frac{B}{2} < f_0$, so we can rewrite (1) as:

$$s_{RF}(t) = s_R(t) \cdot \cos(2 \cdot \pi \cdot f_0 \cdot t) - s_I(t) \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t), t \in \mathbb{R} \quad (2)$$

The transmitted signal is assumed determinist and of finite energy, so it admits a Fourier transform:

$$S_{RF}(f) = F\{s_{RF}(t)\}(f)$$

To go further:

For $-\frac{1}{2} < t < \frac{1}{2}$, we can demonstrate that the families $\left\{\cos\left(\frac{\pi.n.t}{2}\right)\right\}_{n \in \mathbb{Z}}$ and $\left\{\sin\left(\frac{\pi.n.t}{2}\right)\right\}_{n \in \mathbb{Z}}$ are biorthogonal. In others words,

$$\forall m, n \in \mathbb{Z}, \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos\left(\frac{\pi.n.t}{2}\right) \cdot \sin\left(\frac{\pi.m.t}{2}\right) dt = \delta_{m,n} \quad (3)$$

with $\delta_{m,n}$ the Kronecker symbol defined as:

$$\begin{cases} \delta_{m,n} = 1, & m = n \\ \delta_{m,n} = 0, & \text{otherwise} \end{cases}$$

According to (2), this result suggests there is a receiver capable of perfectly reconstructing the in-phase and quadrature channels.

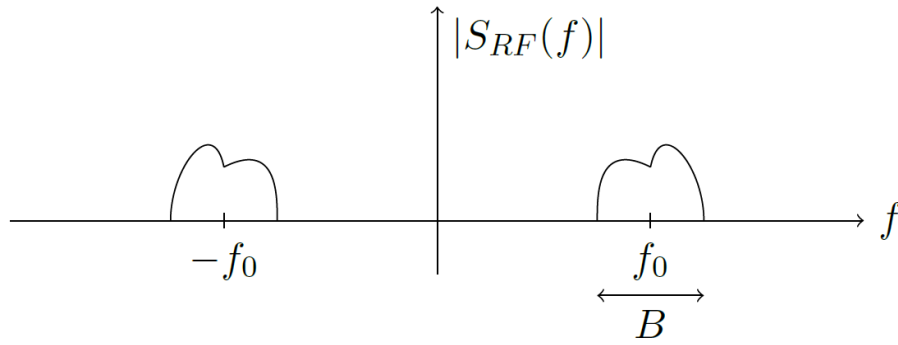


Fig. 3: Spectral representation of the signal $s_{RF}(t)$

Question 1

According to Fig. 2, considering that the received signal is similar to the transmitted one ($r_{RF}(t) = s_{RF}(t)$) and using (2) and trigonometric formulas, express the signal $\tilde{r}_R(t)$ and $\tilde{r}_I(t)$ in function of $s_{RF}(t)$, f_0 and f_c .

Question 2

If we take $f_c = f_0$, what should be the characteristics of the h filters to get $r_R(t) = s_R(t)$ and $r_I(t) = s_I(t)$? You have to make a representation of $\tilde{R}_R(f)$ and $\tilde{R}_I(f)$, after using the Fourier transforms. If you want to work in the time domain, a hypothesis must be clearly stated and checked.

Question 3

Can the receiver presented in Fig. 2 work with wide-band signals? Explain.

Question 4

How must the sampling period T_e be chosen in order to recover $r_R(t), t \in \mathbb{R}$ from $r_R(k.T_e), k \in \mathbb{Z}$?

Question 5

Why do not we interchange the stages of frequency transposition and analog to digital conversion?

We demonstrate that two real signals $s_R(t)$ and $s_I(t)$ can be transmitted on the carrier frequency f_0 and then be perfectly recovered thanks to a well-adapted IQ receiver. Next, we will ignore the operation of transposition on the carrier frequency to study the transmission system.

We define two equivalent models for $s_{RF}(t)$ based on the principle that all real signals have a symmetrical Hermitian Fourier transform.

Thereby, in the case of narrowband signals -as presented in Fig. 4-, the representation in positive frequencies (or negative) is sufficient and it is possible to hide the value of the carrier frequency f_0

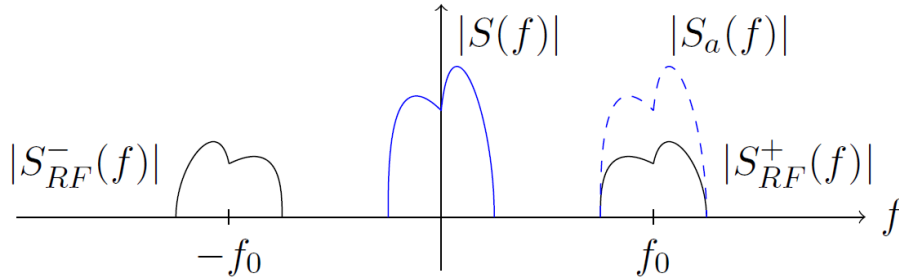


Fig. 4: Spectral representations of a real narrow-band signal, of its analytic signal and of its complex envelope

In its spectral representation, the real narrow-band signal is represented by its negative and positive frequencies, which are symmetric and centred on the ordinate axis. The purely positive analytic signal associated to the real narrow-band signal, to conserve all its power, needs to have an amplitude that is twice as important. The complex envelope is obtained after re-centring the analytic signal on the Y axis. All this is similar to dissociating the carrier frequency f_0 .

The analytic signal can be modelled by:

$$s_a(t) = s_{RF}(t) + j.\mathcal{H}\{s_{RF}(t)\} = s_{RF}(t) + v.p\left\{\int_{-\infty}^{+\infty} \frac{1}{\pi.\tau} . s_{RF}(t - \tau) . d\tau\right\} \quad (4)$$

$$S_a(f) = \mathcal{F}\{s_a\}(f) = S_{RF}(f) + j.\mathcal{H}\{S_{RF}(f)\} = S_{RF}(f) + j.(-j.\text{sgn}(f).S_{RF}(f)) \quad (5)$$

with $\mathcal{H}\{ \}$ the Hilbert transform.

Identically, the complex envelope associated with f_0 is mathematically defined as:

$$s(t) = s_a(t).e^{-j.2.\pi.f_0.t} \quad (6)$$

$$S(f) = \mathcal{F}\{s\}(f) = S_a(f + f_0) \quad (7)$$

Question 6

Supposing a real narrow-band signal $s_{RF}(t) = A(t).\cos(2.\pi.f_0.t + \varphi(t))$, $t \in \mathbb{R}$. Express its analytic signal and its complex envelop in function of f_0 , knowing that $S_{RF}(f) = S_{RF}^*(-f)$.

Now, all the theoretical elements have been introduced. Next, we will implement some receivers thanks to GNURadio SDR software. Generally, the development environments use the complex form *-id est* as a complex envelop relative to the cut-off frequency f_c of the digital filter at the sample period T_e - in order to code the samples sequences of the signal obtained by the use of a SDR receiver.

Second part: Reception of frequency modulation (FM) broadcasting

The Very High Frequency (VHF) band extends from 30MHz to 300MHz. The radiofrequency propagation in this band and for terrestrial communications is nearly geometrical. We focus on the sub-band between 87.5MHz and 108MHz, which is now dedicated to FM broadcasting (https://en.wikipedia.org/wiki/FM_broadcasting/). The different channels are spaced by at least 100kHz so that it is theoretically possible to have 203 simultaneous broadcasting stations. In practise, a particular transmitter broadcasts far fewer stations, allowing the reuse of the frequency channels in the frame of a cellular planning and guaranteeing a good isolation between channels.

The FM broadcasting recording file *fm_99500000_1500000.32fc* has been obtained thanks to the acquisition system introduced in the first part. It was recorded at Toulouse in 2015, using a center frequency of $f_0 = 99.5\text{MHz}$ and a sampling frequency of $F_e = 15\text{MHz}$. The transmitter is probably located at the Pech David observatory, near Rangueil Hospital.

The objective of this first exercise is to learn as much as possible about the content of this recording and in particular to restore the audio content. To meet this goal, we will use the GNURadio development environment and precisely the GRC tool with its graphical interface similar to Matlab/Simulink one.

To launch GRC, type “gnuradio-companion” in a terminal.

1. Frequency analysis of the recording

To exploit the recording, we implement the chain presented in Fig. 5. We can save it in our workspace as “recording_analysis.grc”.

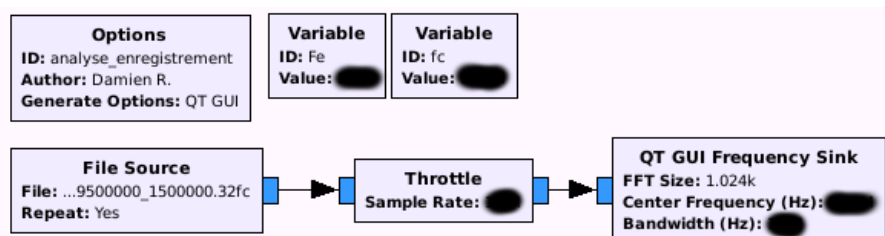


Fig. 5: Frequency analysis processing chain on GRC

Question 7

Present the role of each block used in the processing chain. Help: use the online manual.

Question 8

Specify the values of the missing variables in the characteristics function of the recording file.

Question 9

How many frequency channels -to be noted L- do you observe?

According to the allocation of frequencies in the FM band near Toulouse

(<https://www.annuradio.fr/index.php?mode=searchville&choixville=TOULOUSE/>), which stations are observed?

Question 10

What is the measured signal-to-noise ratio in decibel? Do you think that is enough to be able to demodulate the signal?

Question 11

What is the approximate bandwidth of a channel?

2. Channel extraction by frequency transposition and low-pass filtering

During the previous step, several RF broadcasting stations have been identified. Now, we want to receive each one separately by the use of a new processing chain based on the previous one. We can save it as "channel_extraction.grc". To do this, we propose a two stages reception described as:

- 1- frequency transposition in order to center the useful signal in terms of frequencies
- 2- low-pass filtering in order to attenuate the out-of-band noise.

Noting $r[k]$ the complex envelope (regarding f_c) of the recorded signal, sampled at the frequency F_e , the frequency transposition for a quantity $f_l, l \in \{1; \dots; L\}$ can be wrote:

$$r_l[k] = r[k].e^{-j.2.\pi.\frac{f_l}{F_e}.k} \quad (8)$$

To do this frequency transposition through GRC, we use the next blocks:

- *Signal Source* to generate the complex exponential
- *QT GUI Range* to define dynamically f_l during the software execution
- *Multiply* to make the product (8)

We can use *QT GUI Frequency Sink* to check that the frequency transposition is effective (think about centering the visualization on the null frequency).

Note:

As previously, you can present the role of each block used in the new processing chain.

Question 12

What are the frequency offsets needed to center each channel?

Question 13

What happens if the frequency offset is higher than the sampling frequency F_e ?

Once a channel $r_l[k]$ frequency is centered, we must implement a low-pass filter by considering the channel bandwidth as defined in Part. 2.1. Then we can obtain the signal noted $y_l[k]$.

To do this low-pass filtering through GRC, we use the next block:

- *Low Pass Filter* where the cut-off frequency must ideally be half of the frequency bandwidth of the considering channel. We define the transition width as 10% of the cut-off frequency. After filtering, we can process a decimation with a 6 factor in order to lighten the computational load.

We can use *QT GUI Frequency Sink* to check that the low-pass filtering is effective (think about centering the visualization on the null frequency).

Note:

As previously, you can present the role of each block used in the new processing chain.

Question 14

What are the low-pass filter parameters, as well as those of the frequency analyser at the output of the filter?

3. Frequency demodulation and restitution

In order to restore the content of each radio broadcasting station using a sound card, we have firstly to understand the modulation method used. The signal to be transmitted consists of two stereophonic channels $g(t) \in \mathbb{R}$ and $d(t) \in \mathbb{R}$, they are centered in frequency and have a maximum frequency of 15kHz (or mono-lateral band). To ensure compatibility between monophonic receivers and stereophonic ones, these two channels are multiplexed to form the message:

$$m(t) = g(t) + d(t) + A_{sp} \cdot \cos(2 \cdot \pi \cdot f_{sp} \cdot t) + [g(t) - d(t)] \cdot \cos(2 \cdot \pi \cdot 2 \cdot f_{sp} \cdot t) \quad (9)$$

with $f_{sp} = 19\text{kHz}$ a pilot carrier frequency with an amplitude of $A_{sp} = 2$. According to the spectrum of the signal presented in Fig. 6, we note that the monophonic receiver consists of implementing a low-

pass filter at 15kHz. The stereophonic receiver (not discussed here) is required to amplitude demodulate the signal around 38kHz and recombine it with the baseband signal to reconstruct the left and right channels.

In practice, the composite signal may comprise of frequencies greater than 53kHz. This is notably the case for the radio data system (RDS) positioned around 57kHz, which allows the transmission of digital information relating to the program (e.g. the name of the radio station, the title and the name of the artist of the played music, etc.). This service is not considered in the rest of the work.

The composite signal $m(t)$ is then frequency modulated and the radiofrequency signal centered on f_0 at the output of the transmitter is noted:

$$s_{RF}(t) = A \cdot \cos \left(2 \cdot \pi \cdot f_0 \cdot t + \frac{\Delta f}{\max(|m(t)|)} \cdot \int_{-\infty}^t m(u) \cdot du \right) \quad (10)$$

with Δf the maximum frequency excursion of the modulation, fixed at 75kHz in the present case. We show that the frequency-modulated signal occupies an infinite band, but decreases rapidly, so that it can be approached via the Carson rule:

$$B_{FM} \approx 2 \cdot (\Delta f + f_m) \quad (11)$$

with f_m the maximum frequency of the composite signal $m(t)$.

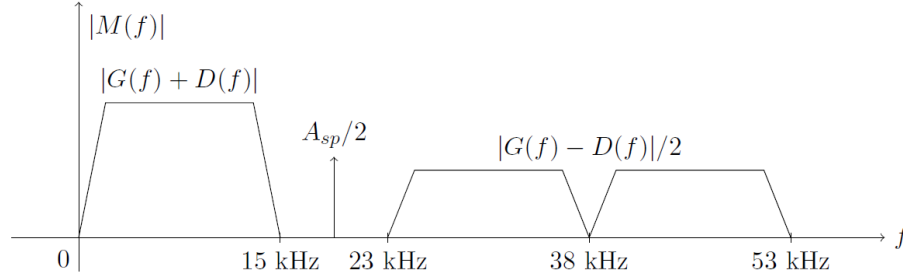


Fig. 6: Stereophonic composite signal before the frequency modulation

Question 15

Using the Carson rule, check that the bandwidth of the channel measured in the previous part confirms the theory.

Question 16

From the expression (10) of the transmitted signal and the affected processes until now (frequency transposition and low-pass filtering), show that the signals $y_l[k]$ can be noted:

$$y_l[k] = A \cdot e^{j \cdot k_f \cdot \sum_{i=0}^k m[i]} + b[k] \quad (12)$$

Define the value of k_f , with $b[k]$ a complex noise term introduced by the propagation channel as well as by the transceiver itself.

From (12), we can show that frequency demodulation can be performed numerically in the following way:

$$\tilde{m}_l[k] = \arg(y_l[k].y_l^*[k-1]) \quad (13)$$

with $*$ the conjugate operator.

Now, we want to frequency demodulate and play audio stream by the use of a new processing chain based on the previous one. We can save it as “frequency_demodulator.grc”. To do this, we propose the use of the three following blocks:

- *WBFM Receive* to process the frequency demodulation as defined by (13)
- *Low Pass Filter* to conserve only the monophonic signal
- *Audio sink* to play the audio stream.

We can use the block *Rational Resampler* between the *Low Pass Filter* and the *Audio Sink* in order to adapt the sample rates with these available on the sound card.

Note:

As previously, you can present the role of each block used in the new processing chain.

Question 17

Plot the spectrum of the demodulated channel and compare with the Fig. 6.

Question 18

Who won the Sam Smith album? What do we listen to on other stations?

4. Real time implementation with an USRP receiver

In this part, we replace the previously used recording file with a SDR transceiver in order to receive a signal in real time. We can save the new processing chain as “usrp_fm_receiver.grc” and connect an USRP on the computer. To do this, we propose the use of the following block:

- *UHD USRP Source* to interface GRC with the driver of the USRP.
- As the USRP has frequency conversion stage, we do not have to care about the frequency transposition stage treated in Part. 2.2.

Until now, we have just received the monophonic signal. We can use plugins to have a stereophonic reception and/or RDS information ([https://sourceforge.isae.fr/projects/ralf/wiki/Stereo FM receiver and RDS decoder/](https://sourceforge.isae.fr/projects/ralf/wiki/Stereo_FM_receiver_and_RDS_decoder/) or <http://ggrx.dk/>).

Third part: Reception of VOLMET messages in AM-SSB

The frequency band named High Frequency (HF) ranges from 3MHz to 30MHz. The propagation in this band is done by successive reflections on the ionosphere and the Earth's crust. The main advantage of this technique is to have intercontinental links with a reasonable power budget (several tens of Watts). In return, it is necessary to accommodate the multiple paths resulting from this propagation process, as well as the constant evolution of the ionosphere channel (function of time of the day, solar cycles, etc.).

Next, we are interested by the frequency sub-band between 11.175MHz and 11.4MHz, which is now reserved to the international aeronautic communications and in particular to the VOL METEO service (VOLMET). This is a periodic broadcasting of meteorological information, using a single sideband amplitude modulation.

The VOLMET recording file *volmet_raf_11296500_250000.32fc* has been obtained thanks to the acquisition system introduced in the first part. It was recorded at Toulouse in 2015, using a center frequency of $f_0 = 11.2965\text{MHz}$ and a sampling frequency of $F_e = 250\text{kHz}$. There is a single station in this record where the transmitter is located in the Royal Air Force air-base of St-Eval, United Kingdom.

The objective of this last exercise is to learn as much as possible about the content of this recording and in particular to restore the phonic content.

1. Frequency analysis of the recording

Question 19

Plot the modulus of the discrete Fourier transform in decibels, between $f_0 - \frac{F_e}{2}$ and $f_0 + \frac{F_e}{2}$, by using *QT GUI Frequency Sink* block. Check with <http://www.dxinfocentre.com/volmet.htm/> that the VOLMET station located in the Royal Air Force air-base of St-Eval, United Kingdom, is well observed at the expected frequency.

2. Frequency transposition

To do this frequency transposition through GRC, as

$$r_1[k] = r[k].e^{j.2.\pi.\frac{f_1}{F_e}.k} \quad (14)$$

we use the next blocks:

- *Signal Source* to generate the complex exponential
- *QT GUI Range* to define dynamically f_1 during the software execution

- Multiply to make the product (8)

We can use *QT GUI Frequency Sink* to check that the frequency transposition is effective (think about centering the visualization on the null frequency).

Question 20

What are the frequency offsets f_1 needed to center the channel with the maximum power at the null frequency?

Plot the modulus of the discrete Fourier transform in decibels, between $f_0 - \frac{F_c}{2}$ and $f_0 + \frac{F_c}{2}$, by using *QT GUI Frequency Sink* block.

3. Single sideband amplitude demodulation.

The transmission process used by the VOLMET service is a single sideband amplitude modulation.

Noting $m(t), t \in \mathbb{R}$, the vocal message to be transmitted. This one is low-pass filtered in order to occupy a bilateral sideband as $B = 6\text{kHz}$, as presented in Fig. 7.

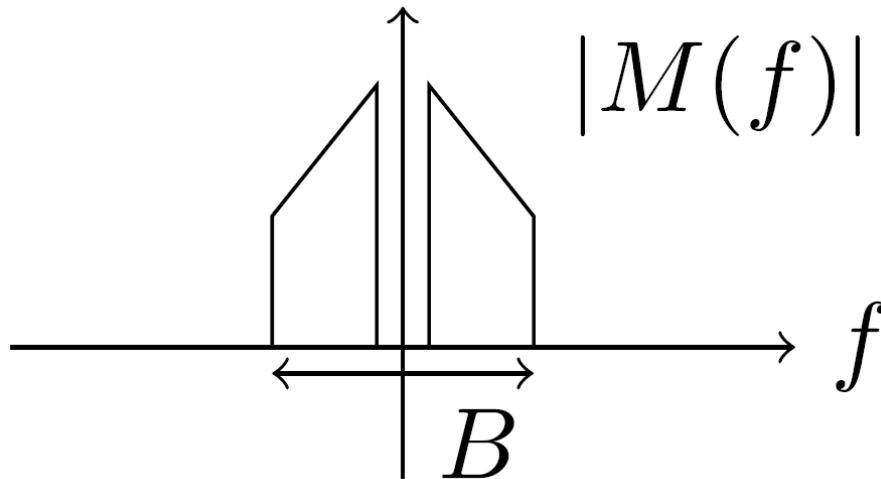


Fig. 7: Spectral representation of the amplitude of the real signal to be transmitted

This signal $m(t)$ can be used to modulate the amplitude of a sinusoid of frequency f_0 generally much higher than B . This is called amplitude modulation and the real signal is noted:

$$s_{RF}(t) = R\{s(t) \cdot e^{j.2.\pi.f_0.t}\} \quad (15)$$

with the complex envelope relative to f_0 defined as:

$$s(t) = m(t) \pm j.\mathcal{H}\{m(t)\} \quad (16)$$

Question 21

Plot $|s_{RF}(f)|$ in function of the polarity of the second term in (16). Interpret the denominations "single lower or upper sideband amplitude modulation".

Question 22

Using the spectrums plotted in questions 19 and 20, which sideband is conserved for a VOLMET transmission?

Considering the expression of a single upper sideband amplitude modulation signal, we can associate the next model to our frequency centred signal (14):

$$r_1[k] = m[k] + j.\mathcal{H}\{m[k]\} + b[k] \quad (17)$$

with $b[k]$ a complex noise term affecting all the frequencies captured during the recording. We note $y[k]$ the result of the filtering of $r_1[k]$ so that to delete the contribution of the noise outside of the interest band, that says $\left[0; \frac{B}{2}\right]$.

Question 23

Using the tool *Filter Design Tool*, create a complex pass-band filter with the characteristics:

- complex pass-band filter with finite impulse response
- lower cut-off frequency at 0Hz
- upper cut-off frequency at $\frac{B}{2}$
- in-band gain of 0dB
- out-band gain of -30dB

Plot the modulus and the phase of frequency response of the filter and comment.

Question 24

Filter $r_1[k]$ thanks to the previous defined pass-band filter in order to get $y[k]$. We can copy the generated filter coefficients to use these in a *FFT Filter* block.

Plot the modulus and the phase of frequency response of the filter and comment regarding the filter parameters.

Question 25

Propose and implement the needed processes in order to recover a real signal in the form:

$$\tilde{m}[k] = m[k] + \tilde{b}[k] \quad (18)$$

Precise the relation between $b[k]$ and $\tilde{b}[k]$.

To finish, play the audio stream by using the blocks *Rational Resampler* and *Audio Sink*. Once achieved, try to implement a real time solution.