# Machine Learning on Graphs and Sets

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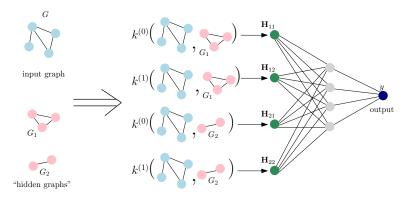
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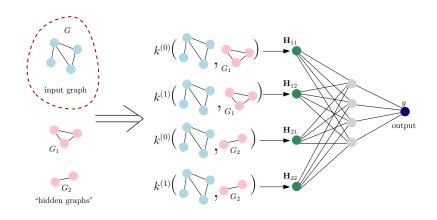
**Idea**: Existing graph neural networks are counter-intuitive  $\hookrightarrow$  features they learn are in the form of vectors

The Random Walk Graph Neural Network is a model

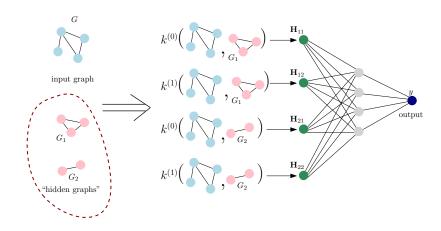
- employs a random walk kernel to learn graph features (in the form of fixed-size graphlets) that contribute to interpretable/intuitive graph representations
- An efficient computation scheme to reduce the time and space complexity of the proposed model

[Nikolentzos and Vazirgiannis, NeurIPS'20]

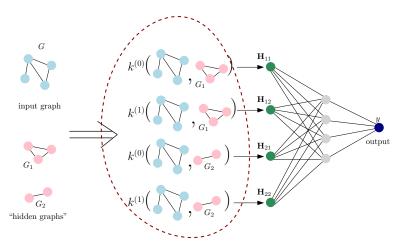




Given an input graph G

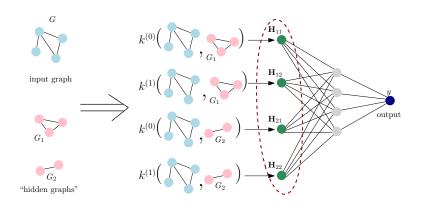


and a set of trainable "hidden graphs"  $G_1, G_2, \ldots$ 

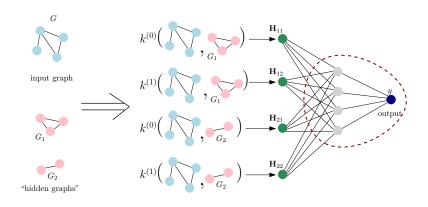


The model computes the following kernel between the input graph G and each "hidden graph"  $G_i$ :  $|V_{\times}| |V_{\times}|$ 

$$k^{(p)}(G, G_i) = \sum_{i=1}^{|V_{\times}|} \sum_{i=1}^{|V_{\times}|} \mathsf{s}_i \mathsf{s}_j \left[\mathsf{A}_{\times}^p\right]_{ij}$$



For each input graph G, we build a matrix  $H \in \mathbb{R}^{N \times P+1}$  where  $H_{ij} = k^{(j-1)}(G, G_i)$ 



 $\label{eq:matrix} \mbox{Matrix H is flattened and fed into a fully-connected neural network to produce the output}$ 

### Experimental Evaluation - Graph Classification

We evaluated RWNN on the following standard graph classification datasets from bio/chemo-informatics and social networks

Dataset	ENZYMES	NCI1	PROTEINS	D&D	IMDB	IMDB	REDDIT	REDDIT	COLLAB
Dataset	LINZTIVILS	IVCII	TROTEINS	DQD	BINARY	MULTI	BINARY	MULTI-5K	COLLAD
Max # vertices	126	111	620	5,748	136	89	3,782	3,648	492
Min # vertices	2	3	4	30	12	7	6	22	32
Average # vertices	32.63	29.87	39.05	284.32	19.77	13.00	429.61	508.50	74.49
Max # edges	149	119	1,049	14,267	1,249	1,467	4,071	4,783	40,119
Min # edges	1	2	5	63	26	12	4	21	60
Average # edges	62.14	32.30	72.81	715.66	96.53	65.93	497.75	594.87	2,457.34
# labels	3	37	3	82	-	-	-	-	-
# graphs	600	4,110	1,113	1,178	1,000	1,500	2,000	4,999	5,000
# classes	6	2	2	2	2	3	2	5	3

### **Experimental Setup**

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# A FAIR COMPARISON OF GRAPH NEURAL NETWORKS FOR GRAPH CLASSIFICATION

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- $\bullet$  10-fold CV for model assessment and an inner holdout technique with a 90%/10% training/validation split for model selection
- ullet After each model selection  $\to$  train 3 times on the whole training fold, holding out a random fraction (10%) of the data to perform early stopping
- Final test fold score obtained as the mean of these 3 runs

### **Baselines**

- Graph Kernels
  - Shortest path kernel (SP) [Borgwardt and Kriegel, ICDM'05]
  - Graphlet kernel (GR) [Shervashidze et al., AISTATS'09]
  - Weisfeiler-Lehman subtree kernel (WL) [Shervashidze et al., JMLR'11]
- Graph Neural Networks
  - DGCNN [Zhang et al., AAAI'18]
  - DiffPool [Ying et al., NIPS'18]
  - ECC [Simonovsky and Komodakis, CVPR'17]
  - GIN [Xu et al., ICLR'19]
  - GraphSAGE [Hamilton et al., NIPS'17]

## Graph Classification - Real World Datasets

	MUTAG	D&D	NCI1	PROTEINS	ENZYMES
SP	80.2 (± 6.5)	<b>78.1</b> (± 4.1)	72.7 (± 1.4)	<b>75.3</b> (± 3.8)	38.3 (± 8.0)
GR	80.8 (± 6.4)	75.4 (± 3.4)	$61.8 (\pm 1.7)$	$71.6 (\pm 3.1)$	$25.1~(\pm~4.4)$
WL	84.6 (± 8.3)	<b>78.1</b> ( $\pm$ 2.4)	<b>84.8</b> ( $\pm$ 2.5)	73.8 ( $\pm$ 4.4)	50.3 (± 5.7)
DGCNN	84.0 (± 6.7)	76.6 (± 4.3)	76.4 (± 1.7)	72.9 (± 3.5)	38.9 (± 5.7)
DiffPool	79.8 ( $\pm$ 7.1)	$75.0~(\pm~3.5)$	76.9 ( $\pm$ 1.9)	$73.7~(\pm~3.5)$	59.5 (± 5.6)
ECC	75.4 (± 6.2)	$72.6 (\pm 4.1)$	$76.2 (\pm 1.4)$	$72.3~(\pm~3.4)$	$29.5~(\pm~8.2)$
GIN	$84.7~(\pm~6.7)$	75.3 $(\pm 2.9)$	$80.0 (\pm 1.4)$	$73.3~(\pm~4.0)$	<u><b>59.6</b></u> (± 4.5)
GraphSAGE	83.6 (± 9.6)	$72.9~(\pm~2.0)$	76.0 ( $\pm$ 1.8)	$73.0~(\pm~4.5)$	$58.2~(\pm~6.0)$
1-step RWNN	<b>89.2</b> (± 4.3)	77.6 (± 4.7)	71.4 (± 1.8)	<u>74.7</u> (± 3.3)	56.7 (± 5.2)
2-step RWNN	$88.1~(\pm~4.8)$	$76.9 (\pm 4.6)$	$73.0 (\pm 2.0)$	74.1 ( $\pm$ 2.8)	57.4 (± 4.9)
3-step RWNN	88.6 ( $\pm$ 4.1)	77.4 (± 4.9)	73.9 ( $\pm$ 1.3)	74.3 ( $\pm$ 3.3)	57.6 (± 6.3)

	IMDB BINARY	IMDB MULTI	REDDIT BINARY	REDDIT MULTI-5K	COLLAB
SP	57.7 (± 4.1)	39.8 (± 3.7)	89.0 (± 1.0)	51.1 (± 2.2)	<b>79.9</b> (± 2.7)
GR	$63.3 (\pm 2.7)$	$39.6 (\pm 3.0)$	$76.6 (\pm 3.3)$	$38.1 (\pm 2.3)$	$71.1~(\pm~1.4)$
WL	<b>72.8</b> ( $\pm$ 4.5)	51.2 $(\pm 6.5)$	$74.9~(\pm~1.8)$	49.6 ( $\pm$ 2.0)	$78.0~(\pm~2.0)$
DGCNN	69.2 (± 3.0)	45.6 (± 3.4)	87.8 (± 2.5)	49.2 (± 1.2)	71.2 (± 1.9)
DiffPool	$68.4 (\pm 3.3)$	$45.6 (\pm 3.4)$	$89.1~(\pm~1.6)$	$53.8 (\pm 1.4)$	$68.9 (\pm 2.0)$
ECC	$67.7 (\pm 2.8)$	$43.5 (\pm 3.1)$	OOR	OOR	OOR
GIN	$71.2 (\pm 3.9)$	$48.5~(\pm~3.3)$	$89.9 (\pm 1.9)$	$56.1 (\pm 1.7)$	$75.6 (\pm 2.3)$
GraphSAGE	$68.8 (\pm 4.5)$	$47.6 (\pm 3.5)$	84.3 (± 1.9)	$50.0~(\pm~1.3)$	$73.9~(\pm~1.7)$
1-step RWNN	70.8 (± 4.8)	47.8 (± 3.8)	90.4 (± 1.9)	51.7 (± 1.5)	71.7 (± 2.1)
2-step RWNN	70.6 ( $\pm$ 4.4)	$48.8 (\pm 2.9)$	$90.3~(\pm~1.8)$	$51.7 (\pm 1.4)$	71.3 $(\pm 2.1)$
3-step RWNN	70.7 (± 3.9)	47.8 (± 3.5)	89.7 (± 1.2)	53.4 (± 1.6)	71.9 ( $\pm$ 2.5)

### Outline

- Learning on Graphs
  - Node Level
    - Message Passing Models
    - Graph Autoencoders
  - Graph Level
    - Introduction
    - Message Passing Models
    - Other Graph Neural Networks
- 2 Learning on Sets
  - Introduction
  - Neural Networks for Sets
- GNN Applications: Recommendations

#### Sets

### What is a set?

A set is a well-defined collection of distinct objects

Complex data sets decomposed into sets of simpler objects

- → NLP: documents as sets of word embeddings
- $\hookrightarrow$  Graph Mining: graphs as sets of node embeddings

Machine learning on sets has attracted a lot of attention recently

- Set classification
- Set regression

### Set Classification

$$S_1 = \{1, 4, 2\}$$
$$y_1 = -1$$

$$S_2 = \{5, 0, 8, 10\}$$
$$y_2 = -1$$

$$S_6 = \{2, 6, 3, 5\}$$
  
 $y_6 = ???$ 

$$S_3 = \{3, 7\}$$
$$y_3 = 1$$

$$S_4 = \{3, 5, 6\}$$
$$y_4 = 1$$

$$S_7 = \{4, 2, 5\}$$
  
 $y_7 = ????$ 

$$S_5 = \{5\}$$
$$y_5 = -1$$

- Let  $\mathcal{X}$  be a set
- Input data  $S \in 2^{\mathcal{X}}$
- Output  $y \in \{-1, 1\}$
- Training set  $\{(S_1, y_1), \ldots, (S_n, y_n)\}$
- Goal: estimate a function  $f: 2^{\mathcal{X}} \to \{-1, 1\}$  to predict y from f(S)

### Limitations of Standard Machine Learning Models

#### Conventional machine learning models cannot handle sets:

- not invariant to permutations of features
  - a learning algorithm for sets needs to produce identical representations for any permutation of the elements of an input set
  - for instance, a model f needs to satisfy the following for any permutation  $\pi$  of the set's elements:

$$f({x_1,\ldots,x_M}) = f({x_{\pi(1)},\ldots,x_{\pi(M)}})$$

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### Neural Networks for Sets

#### Recent approaches:

- ullet unordered sets o ordered sequences o RNN [Vinyals et al., ICLR'16]
- DeepSets [Zaheer et al., NIPS'17] and PointNet [Qi et al., CVPR'17] transform the vectors of the sets into new representations, then apply permutation-invariant functions
- PointNet++ [Qi et al., NIPS'17] and SO-Net [Li et al., CVPR'18] apply PointNet hierarchically in order to better capture local structures
- Set Transformer [Lee et al., ICML'19], a neural network that uses self-attention to model interactions among the elements of the input set
- RepSet [Skianis et al., AISTATS'20], a neural network that generates representations for sets by comparing them against some trainable sets

### **SOTA** in supervised learning tasks:

- regression: population statistic estimation, sum of digits
- classification: point cloud classification, outlier detection

### DeepSets

## Theorem (Zaheer et al., NIPS'17)

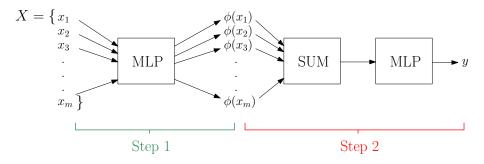
If  $\mathfrak X$  is a countable set and  $\mathcal Y=\mathbb R$ , then a function f(X) operating on a set X having elements from  $\mathfrak X$  is a valid set function, i.e., invariant to the permutation of instances in X, if and only if it can be decomposed in the form  $\rho(\sum_{x\in X}\phi(x))$ , for suitable transformations  $\phi$  and  $\rho$ .

DeepSets achieves permutation invariance by replacing  $\phi$  and  $\rho$  with multi-layer perceptrons (universal approximators)

DeepSets consist of the following two steps:

- **Q** Each element  $x_i$  of each set is transformed (possibly by several layers) into some representation  $\phi(x_i)$
- ② The representations  $\phi(x_i)$  are added up and the output is processed using the  $\rho$  network in the same manner as in any deep network (e.g., fully connected layers, nonlinearities, etc.)

### DeepSets



**Step 1**: The elements  $x_1, \ldots, x_m$  of the input set X are transformed into representations  $\phi(x_1), \ldots, \phi(x_m)$ 

**Step 2**: A representation for the entire set is produced as  $z_X = \phi(x_1) + \ldots + \phi(x_m)$  and is also transformed as follows  $y = \rho(z_X)$  to produce the output

### **Experiments - Point Cloud Classification**

**Objective**: classify point-clouds  $\hookrightarrow$  point-clouds are sets of low-dimensional vectors (typically 3-dimensional vectors representing the x, y, z-coordinates of objects)

**Dataset**: ModelNet40  $\rightarrow$  consists of 3-dimensional representations of 9,843 training and 2,468 test instances belonging to 40 classes of objects

**Setup**: point-clouds directly passed on to DeepSets

Model	Instance Size	Representation	Accuracy
3DShapeNets [25]	$30^{3}$	voxels (using convo- lutional deep belief net)	77%
VoxNet [26]	$32^{3}$	voxels (voxels from point-cloud + 3D CNN)	83.10%
MVCNN [21]	164×164× 12	multi-vew images (2D CNN + view- pooling)	90.1%
VRN Ensemble [27]	$32^{3}$	voxels (3D CNN, variational autoen- coder)	95.54%
3D GAN [28]	$64^{3}$	voxels (3D CNN, generative adversar- ial training)	83.3%
DeepSets	$5000 \times 3$	point-cloud	$90 \pm .3\%$
DeepSets	$100 \times 3$	point-cloud	$82\pm2\%$

### Experiments - Text Concept Retrieval

**Objective**: retrieve words belonging to a "concept" given few words from the concept

**Example**: given the set of words { tiger, lion, cheetah}, retrieve other related words like jaguar and puma, which all belong to the concept of big cats

**Setup**: query word added to set and new set fed to DeepSet which produces a score

	LDA-1 $k$ (Vocab = 17 $k$ )			LDA- $3k$ (Vocab = $38k$ )				LDA- $5k$ (Vocab = $61k$ )							
Method		ecall (9 @100		MRR	Med.		ecall (9 @100		MRR	Med.		ecall (% @100		MRR	Med.
Random	0.06	0.6	5.9	0.001	8520	0.02	0.2	2.6	0.000	28635	0.01	0.2	1.6	0.000	30600
Bayes Set	1.69	11.9	37.2	0.007	2848	2.01	14.5	36.5	0.008	3234	1.75	12.5	34.5	0.007	3590
w2v Near	6.00	28.1	54.7	0.021	641	4.80	21.2	43.2	0.016	2054	4.03	16.7	35.2	0.013	6900
NN-max	4.78	22.5	53.1	0.023	779	5.30	24.9	54.8	0.025	672	4.72	21.4	47.0	0.022	1320
NN-sum-con	4.58	19.8	48.5	0.021	1110	5.81	27.2	60.0	0.027	453	4.87	23.5	53.9	0.022	731
NN-max-con	3.36	16.9	46.6	0.018	1250	5.61	25.7	57.5	0.026	570	4.72	22.0	51.8	0.022	877
DeepSets	5.53	24.2	54.3	0.025	696	6.04	28.5	60.7	0.027	426	5.54	26.1	55.5	0.026	616

### Experiments - Image Tagging

**Objective**: retrieve all relevant tags corresponding to an image

**Setup**: features of the image are concatenated to the embeddings of the tags, and then the whole set is passed on to DeepSets to assign a single score to the set

Method	ESP game				IAPRTC-12.5			
Method	P	R	F1	N+	P	R	F1	N+
Least Sq.	35	19	25	215	40	19	26	198
MBRM	18	19	18	209	24	23	23	223
JEC	24	19	21	222	29	19	23	211
FastTag	46	22	30	247	47	26	34	280
Least Sq.(D)	44	32	<b>37</b>	232	46	30	36	218
FastTag(D)	44	32	37	229	46	33	38	254
DeepSets	39	34	36	246	42	31	36	247

### Experiments - Set Anomaly Detection

Objective: find the anomalous face in each set

**Architecture**: consists of 9 2d-convolution and max-pooling layers followed by the DeepSets model, and a softmax layer that assigns a probability value to each set member



### RepSet

- A permutation invariant neural network for sets
- Generates a number of "hidden sets" and it compares the input set with these sets using a network flow algorithm (e.g., bipartite matching)
- The outputs of the network flow algorithm form the penultimate layer and are fed to a fully-connected layer which produces the output
- $\bullet$  The model is end-to-end trainable  $\to$  "hidden sets" are updated during training
- For large sets, solving the flow problems can become prohibitive ☺
   → ApproxRepSet is a relaxed formulation (also permutation invariant) that scales to very large datasets

### Permutation Invariant Layer

- A layer whose output is the same regardless of the ordering of the input's elements
- Contains m "hidden sets"  $Y_1, Y_2, \ldots, Y_m$  of d-dimensional vectors  $\hookrightarrow$  may have different cardinalities and their components are trainable
- ullet Measure similarity between input set and each one of the "hidden sets" by comparing their building blocks, i.e., their elements ullet bipartite matching

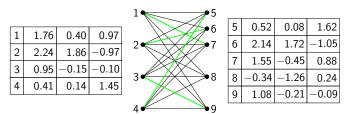


Figure: Example of a bipartite graph generated from 2 sets of 3-dimensional vectors, and of its maximum matching M. Green color indicates that an edge belongs to M

## RepSet - Bipartite Matching Problem

- Input set  $X = \{v_1, v_2, \dots, v_{|X|}\}$  where  $v_1, \dots, v_{|X|}$  vectors
- $\bullet \ \ \text{``Hidden set''} \ \ Y = \{\mathsf{u}_1,\mathsf{u}_2,\ldots,\mathsf{u}_{|Y|}\}$
- Maximum matching between the elements of X and Y by solving the following linear program:

$$\begin{aligned} \max \sum_{i=1}^{|X|} \sum_{j=1}^{|Y|} z_{ij} f(\mathsf{v}_i, \mathsf{u}_j) & \text{subject to:} \\ \sum_{i=1}^{|X|} z_{ij} & \leq 1 \quad \forall j \in \{1, \dots, |Y|\} \\ \sum_{j=1}^{|Y|} z_{ij} & \leq 1 \quad \forall i \in \{1, \dots, |X|\} \\ z_{ii} & > 0 \quad \forall i \in \{1, \dots, |X|\}, \forall j \in \{1, \dots, |Y|\} \end{aligned}$$

where  $f(v_i, u_j)$  is a differentiable function (e.g., inner product), and  $z_{ij} = 1$  if component i of X is assigned to component j of  $Y_i$ , and 0 otherwise

### RepSet - Produced Set Representations

Given an input set X and the m "hidden sets"  $Y_1, Y_2, \ldots, Y_m$ 

- formulate *m* different bipartite matching problems
- ② by solving all m problems, end up with an m-dimensional vector  $\mathbf{x} \to \text{hidden}$  representation of set X
- this *m*-dimensional vector can be used as features for different machine learning tasks (e.g., set regression, set classification)
  - $\hookrightarrow$  For instance, in the case of a set classification problem with  $|\mathcal{C}|$  classes, output is computed as

$$p = softmax(Wx + b)$$

where W is a matrix of trainable parameters and b is the bias term

### RepSet - Architecture

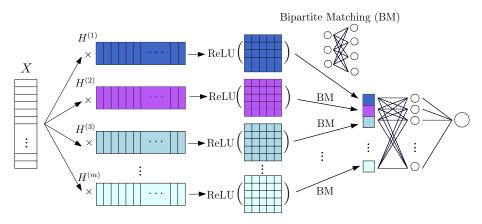


Figure: Each element of the input set is compared with the elements of all "hidden sets", and the emerging matrices serve as the input to bipartite matching. The values of the BM problems correspond to the representation of the input set.

# RepSet - Relaxed Variant (ApproxRepSet)

- Input set  $X = \{v_1, v_2, \dots, v_{|X|}\}$  where  $v_1, \dots, v_{|X|}$  vectors
- "Hidden set"  $Y = \{u_1, u_2, \dots, u_{|Y|}\}$
- Identify which of the two sets has the highest cardinality.
- If  $|X| \ge |Y|$ , we solve the following problem:

$$\begin{split} \max \sum_{i=1}^{|X|} \sum_{j=1}^{|Y|} z_{ij} f(\mathsf{v}_i, \mathsf{u}_j) \text{ subject to:} \\ \sum_{i=1}^{|X|} z_{ij} &\leq 1 \quad \forall j \in \{1, \dots, |Y|\} \\ z_{ij} &\geq 0 \quad \forall i \in \{1, \dots, |X|\}, \forall j \in \{1, \dots, |Y|\} \end{split}$$

- ullet Multiple elements of X (the bigger set) can be matched with the same element of Y
- Optimal solution matches an element  $y_j$  of Y with  $x_i$  of X if  $f(v_i, u_j)$  is positive and  $f(v_i, u_i) = \max_k f(v_k, u_i)$

### Experimental Evaluation

- Text categorization
  - given a document, the input to the model is the set of embeddings of its terms
  - standard text categorization datasets (TWITTER, BBCSPORT etc.)
- Graph classification
  - represent each graph as a set of vectors (i.e., the embeddings of its nodes)
  - node embeddings are extracted by struc2vec [Ribeiro et al., KDD'17]
  - datasets derived from bioinformatics (MUTAG, PROTEINS) and social networks (IMDB-BINARY, -MULTI, REDDIT-BINARY)

### Experiments - Text classification

	BBCSPORT	TWITTER	RECIPE	OHSUMED	CLASSIC	REUTERS	AMAZON	20NG
WMD S-WMD	$\begin{array}{c} 4.60\pm0.70 \\ 2.10\pm0.50 \end{array}$	$\begin{array}{c} 28.70\pm0.60 \\ 27.50\pm0.50 \end{array}$	$\begin{array}{c} 42.60\pm0.30 \\ 39.20\pm0.30 \end{array}$	44.50 34.30	$\begin{array}{c} \textbf{2.88}  \pm  0.10 \\ \textbf{3.20}  \pm  0.20 \end{array}$	3.50 3.20	$\begin{array}{c} 7.40\pm0.30 \\ 5.80\pm0.10 \end{array}$	26.80 26.80
DeepSets NN-mean NN-max NN-attention	$\begin{array}{c} 25.45\pm20.1\\ 10.09\pm2.62\\ 2.18\pm1.75\\ 4.72\pm0.97 \end{array}$	$\begin{array}{c} 29.66 \pm 1.62 \\ 31.56 \pm 1.53 \\ 30.27 \pm 1.26 \\ 29.09 \pm 0.62 \end{array}$	$\begin{array}{c} 70.25\pm0.00 \\ 64.30\pm7.30 \\ 43.47\pm1.05 \\ 43.18\pm1.22 \end{array}$	71.53 45.37 35.88 <b>31.36</b>	$\begin{array}{c} 5.95\pm1.50\\ 5.35\pm0.75\\ 4.21\pm0.11\\ 4.42\pm0.73\\ \end{array}$	10.00 11.37 4.33 3.97	$\begin{array}{c} 8.58 \pm 0.67 \\ 13.66 \pm 3.16 \\ 7.55 \pm 0.63 \\ 6.92 \pm 0.51 \end{array}$	38.88 38.40 32.15 28.73
Set-Transformer	4.18 ± 1.23	27.79 ± 0.47	42.54 ± 1.35	35.68	5.23 ± 0.52	4.52	7.18 ± 0.44	30.01
RepSet ApproxRepSet	$\begin{array}{c} \textbf{2.00}  \pm  0.89 \\ \textbf{4.27}  \pm  1.73 \end{array}$	$\begin{array}{c} \textbf{25.42}  \pm  1.10 \\ \textbf{27.40}  \pm  1.95 \end{array}$	$\begin{array}{c} \textbf{38.57}  \pm  0.83 \\ \textbf{40.94}  \pm  0.40 \end{array}$	33.88 35.94	$\begin{array}{c} 3.38\pm0.50 \\ 3.76\pm0.45 \end{array}$	3.15 2.83	$\begin{array}{c} \textbf{5.29}  \pm  0.28 \\ \textbf{5.69}  \pm  0.40 \end{array}$	<b>22.98</b> 23.82

Table: Classification test error of the proposed architecture and baselines on 8 TC datasets.

Hidden set	Terms similar to elements of hidden sets	Terms similar to centroids of hidden sets
1	chelsea, football, striker, club, champions	footballing
2	qualify, madrid, arsenal, striker, united, france	ARSENAL_Wenger
3	olympic, athlete, olympics, sport, pentathlon	Olympic_Medalist
4	penalty, cup, rugby, coach, goal	rugby
5	match, playing, batsman, batting, striker	batsman

Table: Terms of the employed pre-trained model that are most similar to the elements and centroids of 5 hidden sets.

### Experiments - Graph Classification

	MUTAG	PROTEINS	IMDB BINARY	IMDB MULTI	REDDIT BINARY
PSCN $k = 10$ Deep GR EMD DGCNN SAEN RetGK DiffPool	$88.95 (\pm 4.37)$ $82.66 (\pm 1.45)$ $86.11 (\pm 0.84)$ $85.80 (\pm 1.70)$ $84.99 (\pm 1.82)$ $90.30 (\pm 1.10)$	$75.00 (\pm 2.51)$ $71.68 (\pm 0.50)$ $-$ $75.50 (\pm 0.90)$ $75.31 (\pm 0.70)$ $76.20 (\pm 0.50)$ $76.25$	71.00 ( $\pm$ 2.29) 66.96 ( $\pm$ 0.56) - 70.03 ( $\pm$ 0.86) 71.59 ( $\pm$ 1.20) 72.30 ( $\pm$ 0.60)	45.23 (± 2.84) 44.55 (± 0.52) - 47.83 (± 0.85) 48.53 (± 0.76) 48.70 (± 0.60)	86.30 (± 1.58) 78.04 (± 0.39) - 87.22 (± 0.80) 92.60 (± 0.30)
DeepSets NN-mean NN-max NN-attention	86.26 (± 1.09) 87.55 (± 0.98) 85.84 (± 0.99) 85.92 (± 1.16)	$\begin{array}{c} 60.82\ (\pm\ 0.79)\\ 73.00\ (\pm\ 1.21)\\ 71.05\ (\pm\ 0.54)\\ 74.48\ (\pm\ 0.22) \end{array}$	$69.84~(\pm~0.64)$ $71.48~(\pm~0.48)$ $69.56~(\pm~0.91)$ <b>72.40</b> $(\pm~0.45)$	47.62 (± 1.18) 49.92 (± 0.82) 48.28 (± 0.43) 49.56 (± 0.47)	52.01 (± 1.47) 84.57 (± 0.84) 80.98 (± 0.79) 88.74 (± 0.53)
Set-Transformer	87.71 (± 1.14)	59.62 (± 1.42)	71.21 (± 1.28)	<b>50.25</b> (± 0.74)	83.79 (±) 0.83
RepSet ApproxRepSet	88.63 (± 0.86) 86.33 (± 1.48)	73.04 (± 0.42) 70.74 (± 0.85)	<b>72.40</b> (± 0.73) 71.46 (± 0.91)	49.93 (± 0.60) 48.92 (± 0.28)	87.45 (± 0.86) 80.30 (± 0.56)

Table: Classification accuracy ( $\pm$  standard deviation) of the proposed architecture and the baselines on the 5 graph classification datasets.

# THANK YOU!

Acknowledgements G. Salha

http://www.lix.polytechnique.fr/dascim/