

Assignment 3 (ML for TS) - MVA

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1 Introduction

Objective. The goal is to implement (i) a signal processing pipeline with a change-point detection method and (ii) wavelets for graph signals.

Warning and advice.

- Use code from the tutorials as well as from other sources. Do not code yourself well-known procedures (e.g. cross validation or k-means), use an existing implementation.
- The associated notebook contains some hints and several helper functions.
- Be concise. Answers are not expected to be longer than a few sentences (omitting calculations).

Instructions.

- Fill in your names and emails at the top of the document.
- Hand in one report per pair of students.
- Rename your report and notebook as follows:
FirstnameLastname1_FirstnameLastname1.pdf and
FirstnameLastname2_FirstnameLastname2.ipynb.
For instance, LaurentOudre_CharlesTruong.pdf.
- Upload your report (PDF file) and notebook (IPYNB file) using the link given in the email.

2 Dual-tone multi-frequency signaling (DTMF)

Dual-tone multi-frequency signaling is a procedure to encode symbols using an audio signal. The possible symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *, #, A, B, C, and D. A symbol is represented by a sum of cosine waves: for $t = 0, 1, \dots, T - 1$,

$$y_t = \cos(2\pi f_1 t / f_s) + \cos(2\pi f_2 t / f_s)$$

where each combination of (f_1, f_2) represents a symbols. The first frequency has four different levels (low frequencies), and the second frequency has four other levels (high frequencies); there are 16 possible combinations. In the notebook, you can find an example symbol sequence encoded with sound and corrupted by noise (white noise and a distorted sound).

Question 1

Design a procedure that takes a sound signal as input and outputs the sequence of symbols. To that end, you can use the provided training set. The signals have a varying number of symbols with a varying duration. There is a brief silence between each symbol.

Describe in 5 to 10 lines your methodology and the calibration procedure (give the hyperparameter values). Hint: use the time-frequency representation of the signals, apply a change-point detection algorithm to find the starts and ends of the symbols and silences, and then classify each segment.

Answer 1

To decode a sequence of symbols from a sound signal, the following methodology is applied:

- **Preprocessing:** Normalize the signal to ensure it is within an appropriate range. Apply a band-pass filter to retain only the frequencies corresponding to the DTMF ranges (approximately 697-941 Hz for low frequencies and 1209-1633 Hz for high frequencies).
- **Time-Frequency Analysis:** Compute the spectrogram of the signal to obtain a time-frequency representation. Use a suitable window size (e.g., 20 ms) to capture the frequency content of each segment clearly.
- **Change-Point Detection:** Apply a change-point detection algorithm to identify the start and end of each symbol. Silence between symbols can help distinguish individual segments.
- **Frequency Estimation:** For each identified segment, determine the dominant frequencies using peak detection in the frequency spectrum.
- **Symbol Classification:** Map the detected frequencies to the corresponding DTMF symbol using the known frequency combinations for low and high tones.
- **Calibration:** Set hyperparameters such as the band-pass filter cutoff frequencies, spectrogram window size, and threshold for peak detection based on the provided training set.

Question 2

What are the two symbolic sequences encoded in the test set?

Answer 2

- Sequence 1: ['8', '2', '5', 'C', '9', '9', 'B']
- Sequence 2: ['7', '8', '6', '0', '', '']

3 Wavelet transform for graph signals

Let G be a graph defined a set of n nodes V and a set of edges E . A specific node is denoted by v and a specific edge, by e . The eigenvalues and eigenvectors of the graph Laplacian L are $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ and u_1, u_2, \dots, u_n respectively.

For a signal $f \in \mathbb{R}^n$, the Graph Wavelet Transform (GWT) of f is $W_f : \{1, \dots, M\} \times V \longrightarrow \mathbb{R}$:

$$W_f(m, v) := \sum_{l=1}^n \hat{g}_m(\lambda_l) \hat{f}_l u_l(v) \quad (1)$$

where $\hat{f} = [\hat{f}_1, \dots, \hat{f}_n]$ is the Fourier transform of f and \hat{g}_m are M kernel functions. The number M of scales is a user-defined parameter and is set to $M := 9$ in the following. Several designs are available for the \hat{g}_m ; here, we use the Spectrum Adapted Graph Wavelets (SAGW). Formally, each kernel \hat{g}_m is such that

$$\hat{g}_m(\lambda) := \hat{g}^U(\lambda - am) \quad (0 \leq \lambda \leq \lambda_n) \quad (2)$$

where $a := \lambda_n / (M + 1 - R)$,

$$\hat{g}^U(\lambda) := \frac{1}{2} \left[1 + \cos \left(2\pi \left(\frac{\lambda}{aR} + \frac{1}{2} \right) \right) \right] \mathbb{1}(-Ra \leq \lambda < 0) \quad (3)$$

and $R > 0$ is defined by the user.

Question 3

Plot the kernel functions \hat{g}_m for $R = 1$, $R = 3$ and $R = 5$ (take $\lambda_n = 12$) on Figure 1. What is the influence of R ?

Answer 3

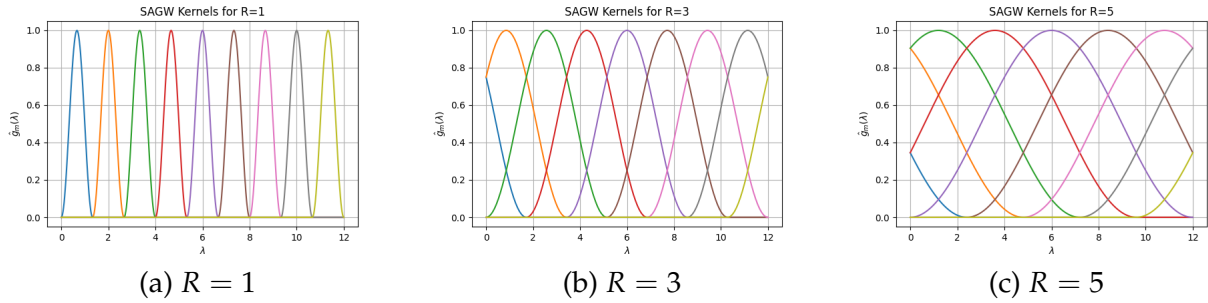


Figure 1: The SAGW kernels functions

We will study the Molene data set (the one we used in the last tutorial). The signal is the temperature.

Question 4

Construct the graph using the distance matrix and exponential smoothing (use the median heuristics for the bandwidth parameter).

- Remove all stations with missing values in the temperature.
- Choose the minimum threshold so that the network is connected and the average degree is at least 3.
- What is the time where the signal is the least smooth?
- What is the time where the signal is the smoothest?

Answer 4

The stations with missing values are BATZ, BEG MEIL, CAMARET, PLOUGONVELIN, RIEC SUR BELON, ST NAZAIRE-MONTOIR, PLOUAY-SA, VANNES-MEUCON, LANNAERO, PLOUDALMEZEAU, LANDIVISIAU, SIZUN, QUIMPER, OUESSANT-STIFF, LANVEOC, ARZAL, BREST-GUIPAVAS, BRIGNOGAN.

The threshold is equal to 0.84.

The signal is the least smooth at 2014-01-21 06:00:00.

The signal is the smoothest at 2014-01-24 19:00:00.

Question 5

(For the remainder, set $R = 3$ for all wavelet transforms.)

For each node v , the vector $[W_f(1, v), W_f(2, v), \dots, W_f(M, v)]$ can be used as a vector of features. We can for instance classify nodes into low / medium / high frequency:

- a node is considered low frequency if the scales $m \in \{1, 2, 3\}$ contain most of the energy,
- a node is considered medium frequency if the scales $m \in \{4, 5, 6\}$ contain most of the energy,
- a node is considered high frequency if the scales $m \in \{6, 7, 9\}$ contain most of the energy.

For both signals from the previous question (smoothest and least smooth) as well as the first available timestamp, apply this procedure and display on the map the result (one colour per class).

Answer 5

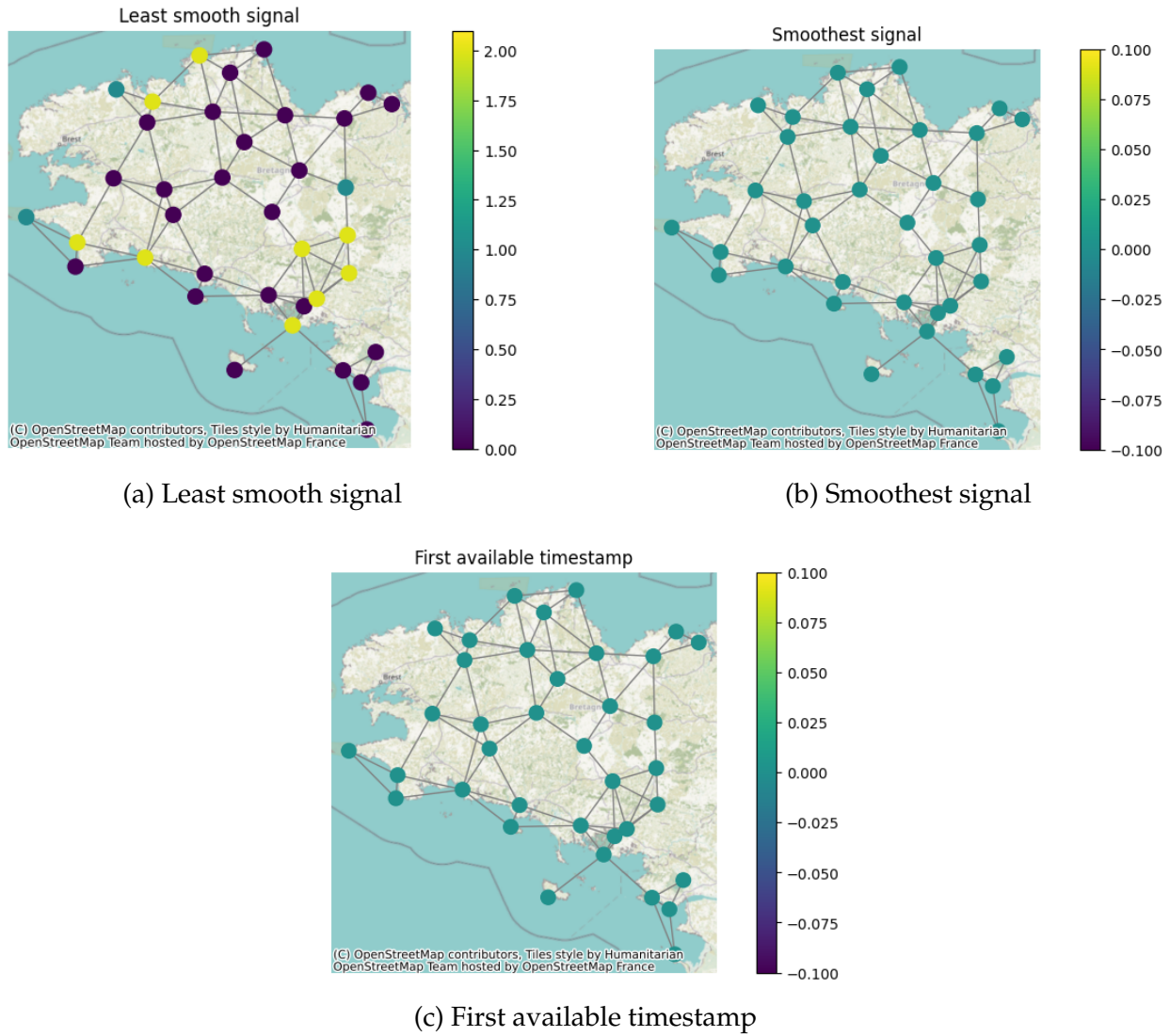


Figure 2: Classification of nodes into low / medium / high frequency

Question 6

Display the average temperature and for each timestamp, adapt the marker colour to the majority class present in the graph (see notebook for more details).

Answer 6

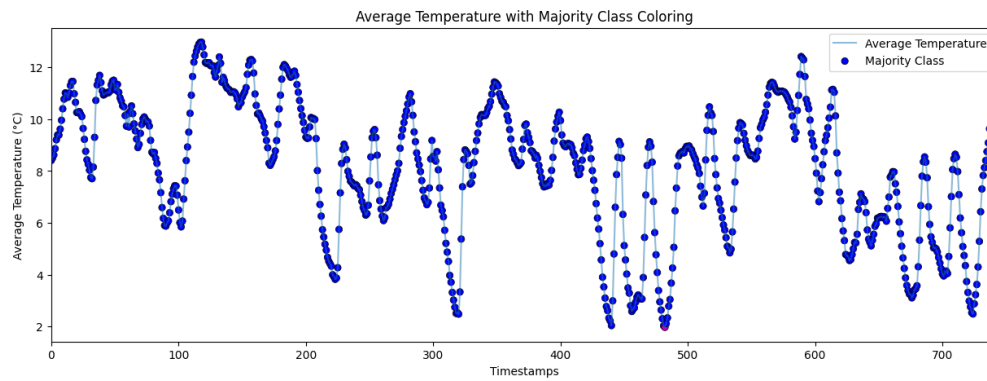


Figure 3: Average temperature. Markers' colours depend on the majority class.

The majority class is the low frequency class.

Question 7

The previous graph G only uses spatial information. To take into account the temporal dynamic, we construct a larger graph H as follows: a node is now *a station at a particular time* and is connected to neighbouring stations (with respect to G) and to itself at the previous timestamp and the following timestamp. Notice that the new spatio-temporal graph H is the Cartesian product of the spatial graph G and the temporal graph G' (which is simply a line graph, without loop).

- Express the Laplacian of H using the Laplacian of G and G' (use Kronecker products).
- Express the eigenvalues and eigenvectors of the Laplacian of H using the eigenvalues and eigenvectors of the Laplacian of G and G' .
- Compute the wavelet transform of the temperature signal.
- Classify nodes into low/medium/high frequency and display the same figure as in the previous question.

Answer 7

The spatio-temporal graph H is the Cartesian product of the spatial graph G and the temporal graph G' :

$$H = G \times G'.$$

The Laplacian of H is expressed as:

$$L_H = L_G \otimes I_m + I_n \otimes L_{G'},$$

where I_n and I_m are identity matrices of size n (nodes in G) and m (nodes in G').

If $\{\lambda_i, v_i\}$ are eigenvalues and eigenvectors of L_G , and $\{\mu_j, u_j\}$ are those of $L_{G'}$, then the eigenvalues and eigenvectors of L_H are:

$$\text{Eigenvalues: } \lambda_i + \mu_j, \quad \text{Eigenvectors: } v_i \otimes u_j.$$

To compute the wavelet transform of the temperature signal:

- Project the signal onto the eigenbasis of L_H .
- Apply wavelet filters $g_m(\lambda)$ to separate frequency components.
- Optionally, invert the transform to analyze node-level features.

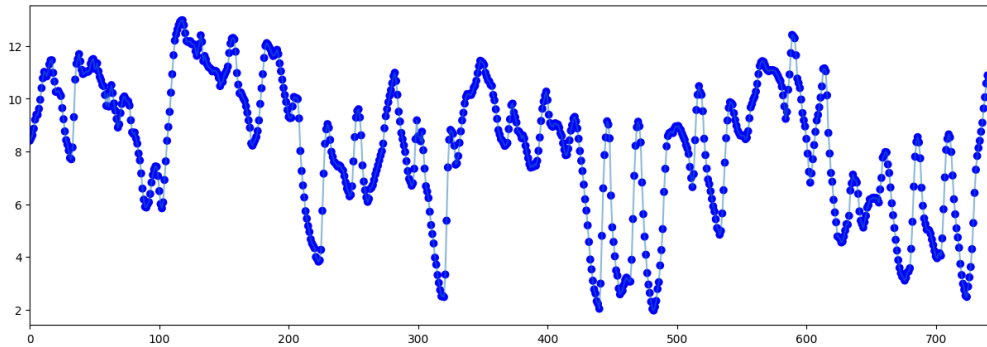


Figure 4: Average temperature. Markers' colours depend on the majority class.

The blue points are the low frequency class.