Introduction to Quadrotors

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Outline

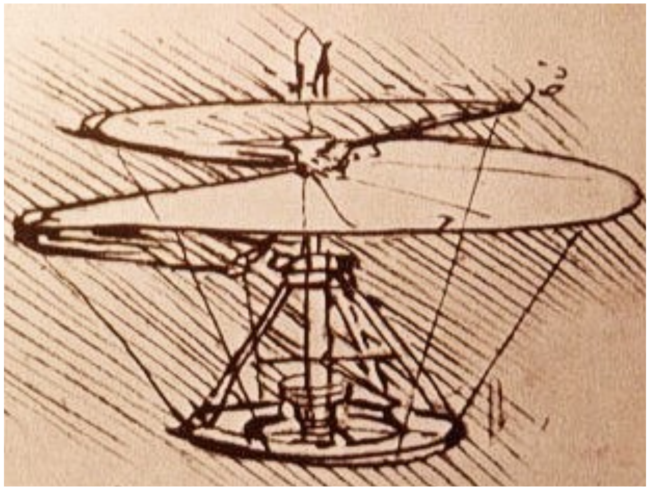
1. **Introduction**
   Why build quadrotors?
2. **Structure and Physics**
   How can a quadrotor stay in the air?
3. **Driving principles**
   How to move a quadrotor in the air?
4. **Automatic controller**
   Why quadrotors need controllers?
5. **Attitude control**
   How to control the attitude of a quadrotor?
6. **Altitude/Position control**
   How to control the altitude/position?

1. Introduction

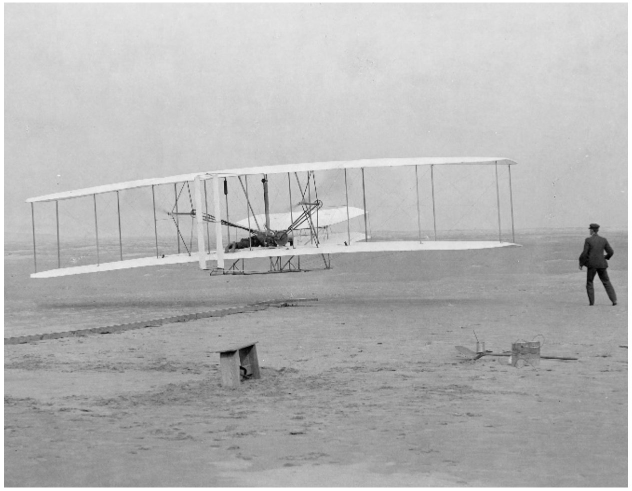
WHAT ARE QUADROTORS?

WHY BUILD QUADROTORS?

History of Flying Machines



Leonardo da Vinci 1493



Wright brothers 1903

- Interest in building flying machines from many centuries ago
- History shows that building flying machines has not been easy

Types of flying machines

(excluding rockets and balloon)

Fixed wing

(airplanes, gliders, …)

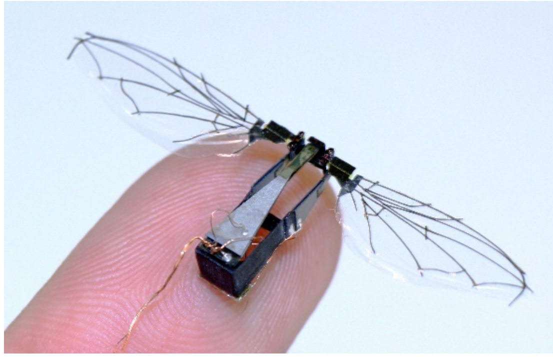
- Fast
- Efficient
- Cannot Hover

rotating wing

(helicopters, multicopters, …)

- Can Hover
- highly maneuverable
- Less efficient

Flapping wing

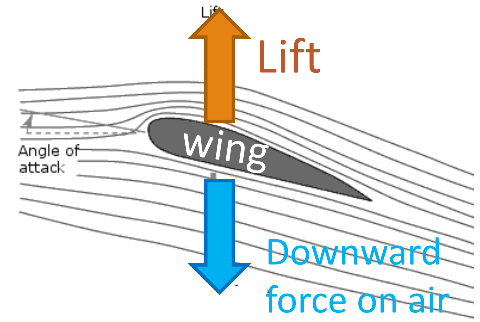


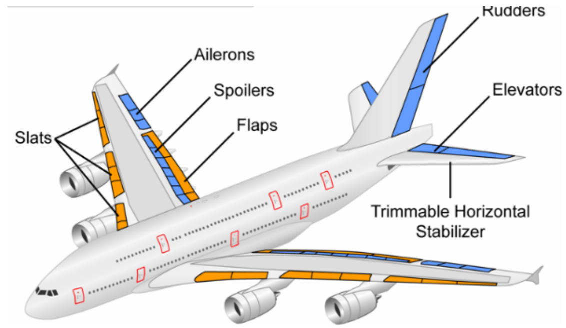
(new robots)

- Scale down in size
- Hard to build and control

Fixed-wing vs Rotating-wing

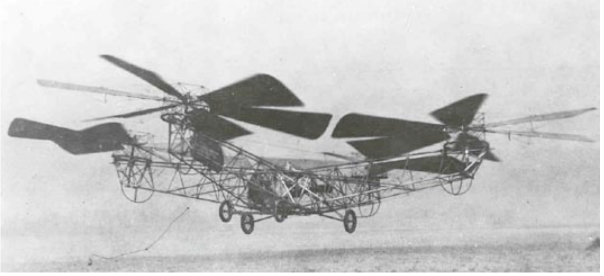
- The force needed to keep an aircraft in the air is called 'Lift.'
- 'Wing' is a component used for for generating lift
- Fixed-wing machines:
  - Forward propulsion
  - Airflow on stationary wings generate lift
  - Control is achieved using movable surfaces
- Rotating-wing machines:
  - Rotating rotor blades generate lift
  - Rotor blades are identical to wings
  - Vetrical take-off and landing (VTOL)
  - Control is achieved by the rotor/rotor blades

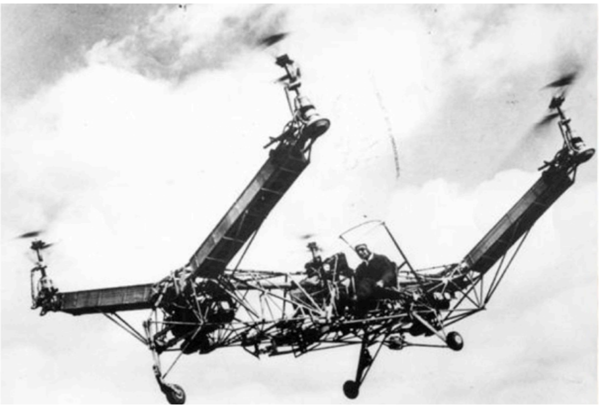




Quadrotor: History

- Quadrotor/ Quadcopter is a rotating-wing machine with four rotors.
- First flight of a quadrotors in 1922 by 'Etienne Oehmichen'
- Experimental aircraft in the 1950's
- Quadrotors overtaken by helicopters due to heavy workload of the pilot.





What is the game changer? Why quadcopters are popular today?

Quadrotor: Today

- More recently quadrotor designs have become popular in unmanned aerial vehicle (UAV) research

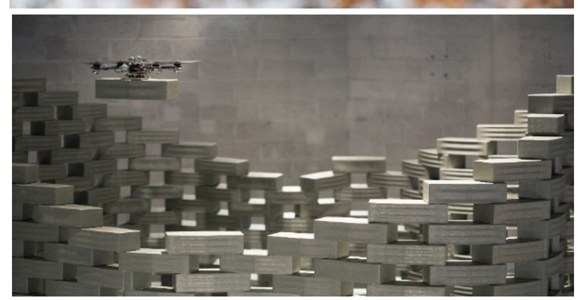
Quadrotor- Advantages

- Mechanically simple: only 4 motors and 4 propellers.
- Does not require any complex mechanical parts.
- Can move around only by changing the motor speeds.
- Can hover, takeoff and land vertically.
- Smaller rotor blades compared to a helicopter's blade.

Easy to build and maintain !

Quadrotor- Applications

- Search and rescue missions
- Aerial photography and mapping
- Aerial inspection
- Transportation
- Construction
- Agriculture
- Environmental studies
- Military
- Entertainment



2. Structure and Physics

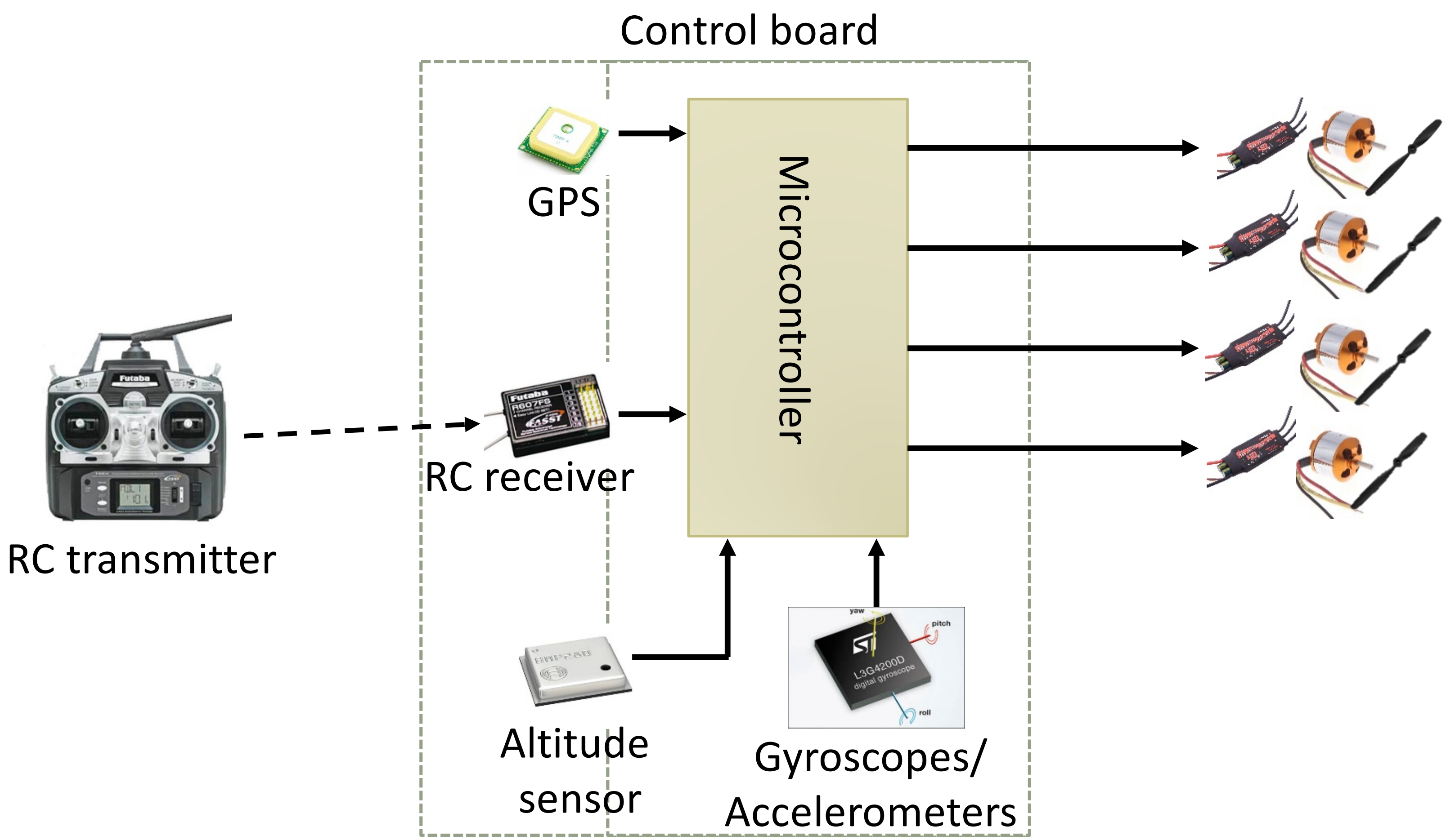
WHAT ARE THE COMPONENTS OF A QUADROTOR?

HOW CAN QUADROTORS STAY IN THE AIR?

Main components

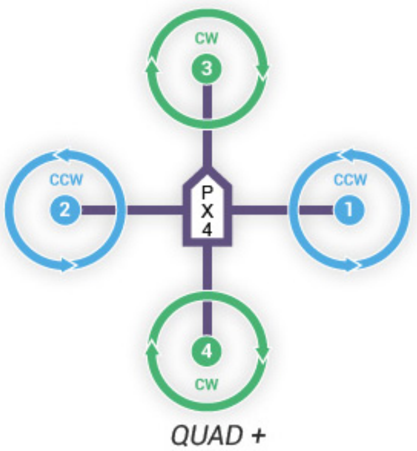
- Frame
- Four motors and motor drivers (ESC)
- Four propellers
- Control board (autopilot)

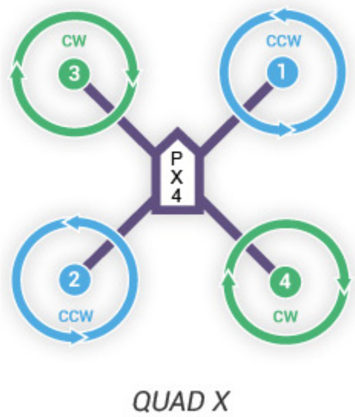
Schematic



Control board
GPS
Microcontroller
RC transmitter
RC receiver
Altitude sensor
Gyroscopes/ Accelerometers

Structure



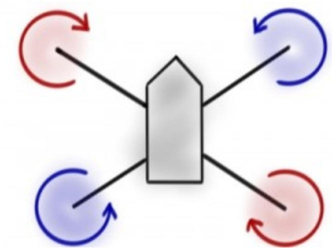


- Two possible configurations: ‘**+**’ & ‘**X**’

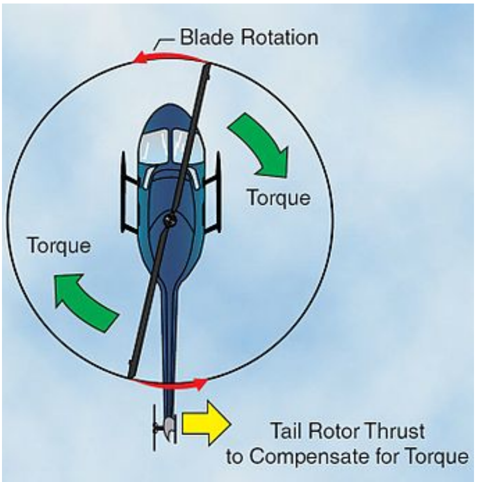
- Four propellers generate four lift forces
- Propellers 3 & 4 rotates CW and propellers 1& 2 rotate CCW
- Propellers 3 & 4 have opposite pitch compared to propellers 1 & 2

Question

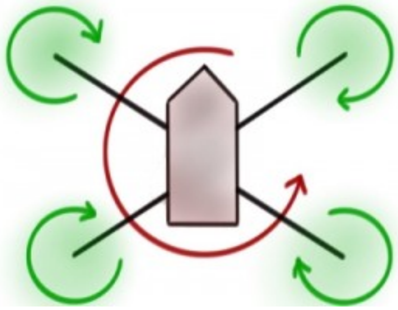
Why do we need to configure motors to have both clockwise and counter-clockwise directions? Cant we have all motors turning in one direction only?



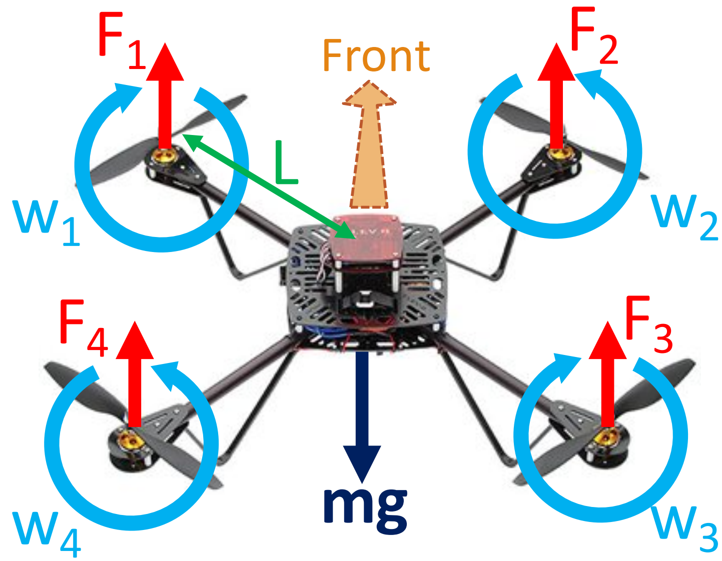
- Newton's third law of motion, "for every action, there is an equal and opposite reaction."
- When a propeller spins in one direction the main body section tends to spin in the opposite direction: "torque effect"



If we were to have all the motors spin clockwise, the quad rotor would start spinning uncontrollably in a counterclockwise direction.



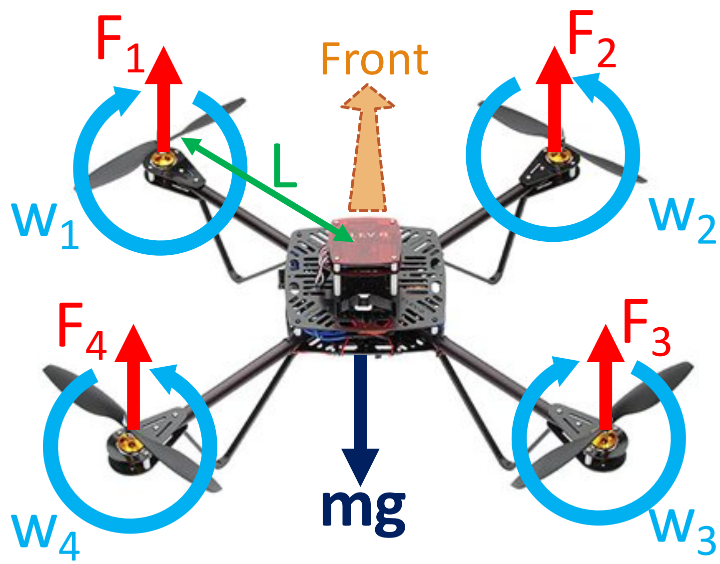
Rotation speeds / Forces / Moments



- w_1 w_2 w_3 w_4 are the rotation speeds of the propellers
- F_1 F_2 F_3 F_4 are the forces generated by the propellers
- Force F is proportional to square of propeller speed: $F_i \propto w_1^2$
- **mg** is the weight of the quadrotor
- Moments generated by the forces are : $M_i = L \times F_i$

The movement of a quadcopter are controlled by changing the rotation speed of the propellers

Hovering Conditions



To stay stationary in mid air (Hover) :

1. All forces must be balanced $F_1 + F_2 + F_3 + F_4 + \mathbf{mg} = 0$
2. Lift forces must be parallel to gravity $F_i \,||\, g$
3. All moments must be balanced $M_1+M_2+M_3+M_4 = 0$
4. Rotor speeds must be balanced (torque balanced) $(w_1+w_3)-(w_2+w_4) =0$

3. Driving principles

WHAT IS THE ORIENTATION OF MY QUADROTOR?

HOW TO MOVE A QUADROTOR AROUND?

3D Orientation of a quadrotor

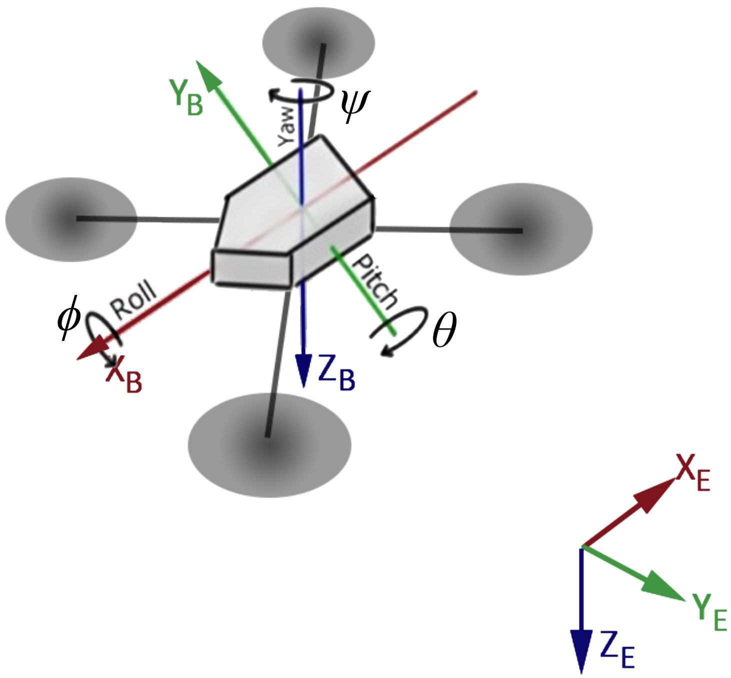
- How to describe the orientation of a quadrotor in space?

- Two reference frames defined:
  1. Earth-fixed frame (X_E, Y_E, Z_E)
  2. Body-fixed frame (X_B, Y_B, Z_B)

- Transformation between the two frames:

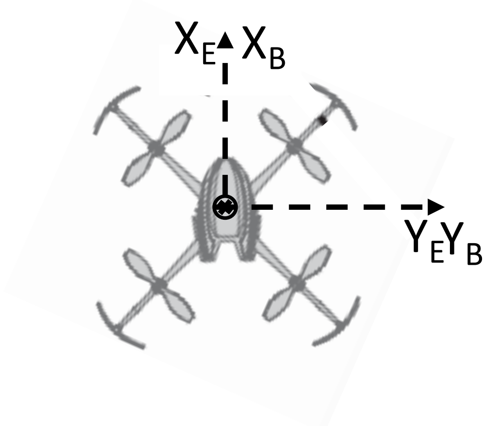
  - Euler Angles (Roll ϕ, Pitch θ, Yaw ψ)

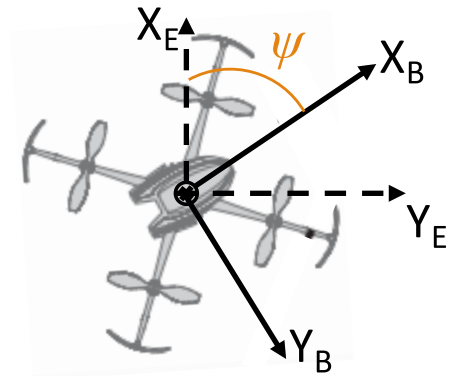
  - Roll ϕ: angle of rotation around X_B
  - Pitch θ: angle of rotation around Y_B
  - Yaw ψ: angle of rotation around Z_B



Orientation: Examples

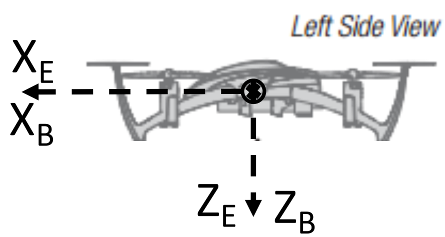
Yaw

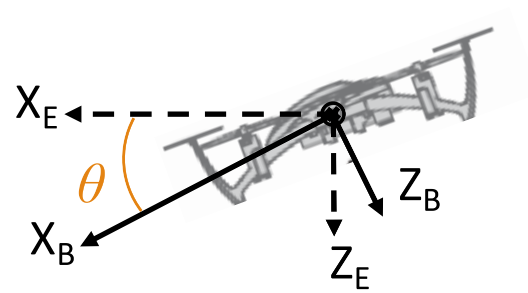




$\psi = 50^o$

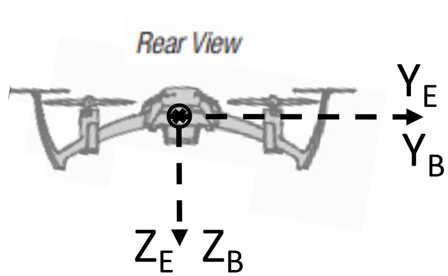
Pitch

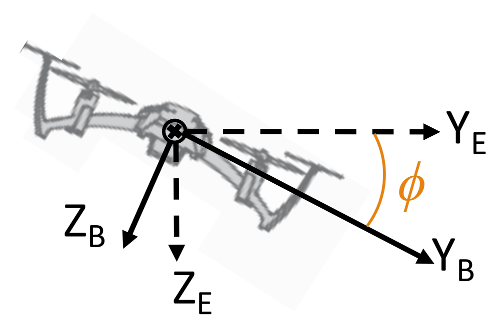




$\theta = -30^o$

Roll



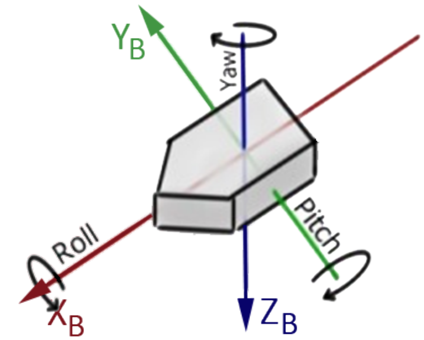


$\phi = 30^o$

Controllable Degrees of Freedom

Angular Speeds

- How fast is the quadrotor turning?
- Angular speeds $(\dot{\phi}, \dot{\theta}, \dot{\psi})$: the rate of change of angles (ϕ, θ, ψ) with respect to time.
- Derivative of angles w.r.t. gives the rate



$$(\dot{\phi}, \dot{\theta}, \dot{\psi}) = \left(\frac{d\phi}{dt}, \frac{d\theta}{dt}, \frac{d\psi}{dt}\right)$$

Example:

$\dot{\psi} = 200$ deg/s

$\dot{\psi} = 330$ deg/s

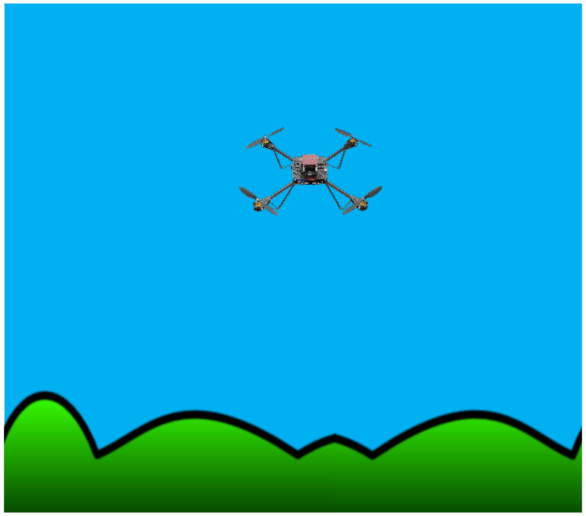
Question

What are the angular speeds of a hovering stationary quadrotor?

Roll rate $\dot{\phi}$ =

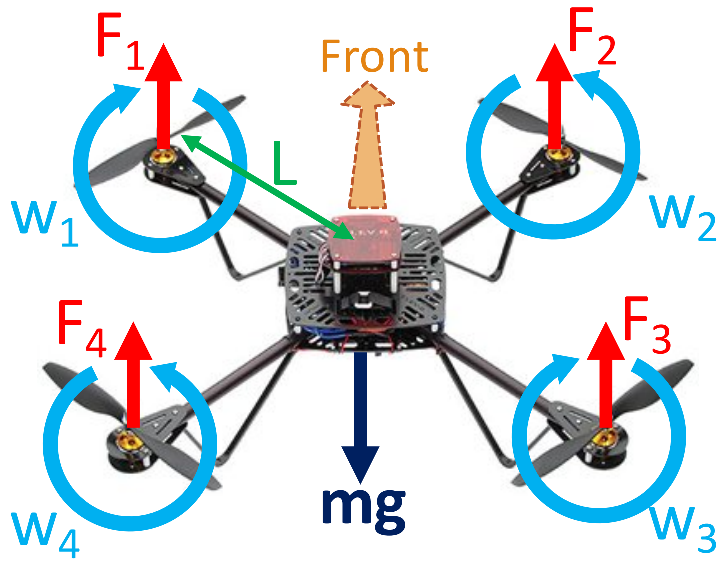
Pitch rate $\dot{\theta}$ =

Yaw rate $\dot{\psi}$ =



NEXT: HOW TO MOVE A QUADROTOR AROUND?

Hovering Conditions



1. Forces must be balanced $F_1 + F_2 + F_3 + F_4 + \mathbf{mg} = 0$
2. Forces must be parallel to gravity $F_i \,||\, g$
3. All moments must be balanced $M_1 + M_2 + M_3 + M_4 = 0$
4. Rotor speeds must be balanced $(w_1 + w_3) - (w_2 + w_4) = 0$

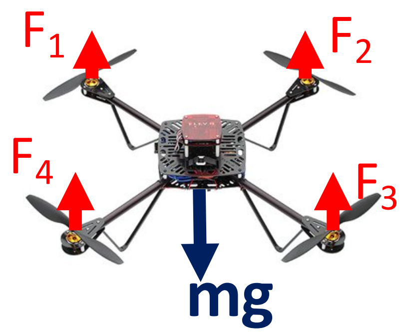
Violating one or more of these conditions implies that the quadcopter starts to move

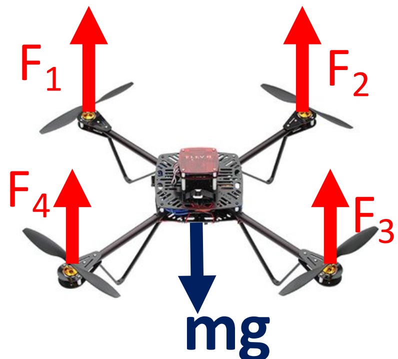
Moving Up and Down

1. Forces Not balanced $F_1 + F_2 + F_3 + F_4 + \mathbf{mg} \neq 0$
2. Forces must be parallel to gravity $F_i \,||\, g$
3. All moments must be balanced $M_1+M_2+M_3+M_4 = 0$
4. Rotor speeds must be balanced $(w_1+w_3)-(w_2+w_4) = 0$

Move Up: $F_1 + F_2 + F_3 + F_4 > \mathbf{mg}$

Move Down: $F_1 + F_2 + F_3 + F_4 < \mathbf{mg}$

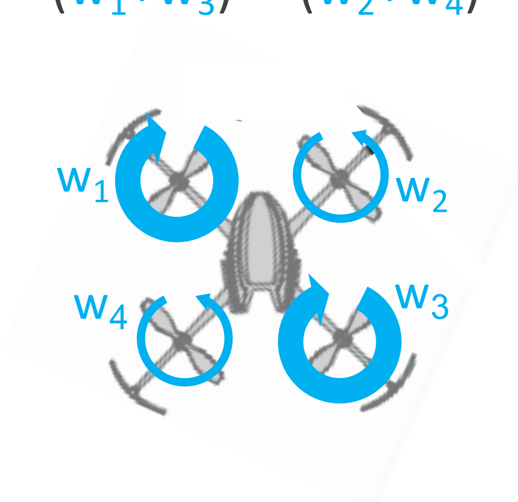




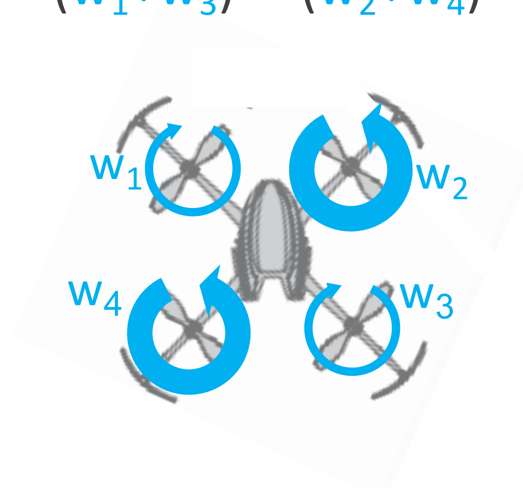
Rotating in Yaw

1. Forces are balanced $F_1 + F_2 + F_3 + F_4 + \mathbf{mg} = 0$
2. Forces must be parallel to gravity $F_i \,||\, g$
3. All moments must be balanced $M_1 + M_2 + M_3 + M_4 = 0$
4. Rotor speeds not balanced $(w_1 + w_3) - (w_2 + w_4) \neq 0$

$(w_1 + w_3) > (w_2 + w_4)$



$(w_1 + w_3) < (w_2 + w_4)$

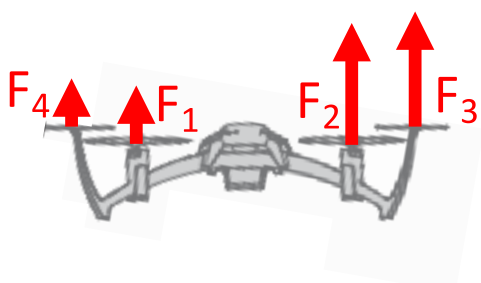


$$\dot{\psi} = k_{\psi}((w_2 + w_4) - (w_1 + w_3))$$

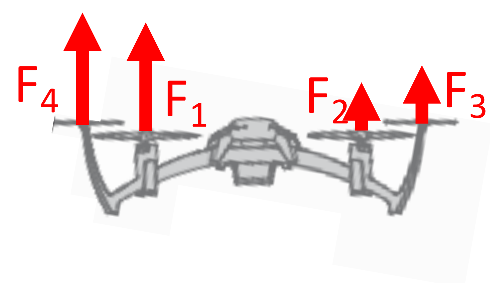
Rotation in Roll

1. Forces are balanced $F_1 + F_2 + F_3 + F_4 + \mathbf{mg} = 0$
2. Forces not parallel to gravity $F_i \not\parallel g$
3. moments not balanced $M_1 + M_2 + M_3 + M_4 \neq 0$
4. Rotor speeds must be balanced $(w_1 + w_3) - (w_2 + w_4) = 0$

$(w_1 + w_4) < (w_2 + w_3)$



$(w_1 + w_4) > (w_2 + w_3)$

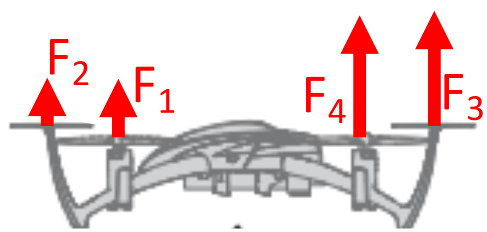


$$\dot{\phi} = k_\phi \left((w_1 + w_4) - (w_2 + w_3) \right)$$

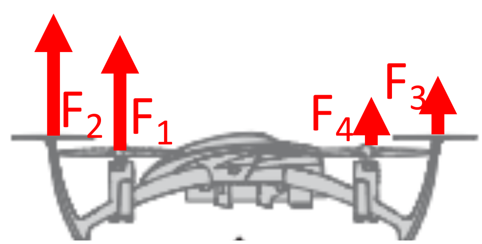
Rotation in Pitch

1. Forces are balanced $F_1 + F_2 + F_3 + F_4 + \mathbf{mg} = 0$
2. Forces not parallel to gravity $F_i \not\parallel g$
3. moments not balanced $M_1+M_2+M_3+M_4 \neq 0$
4. Rotor speeds must be balanced $(w_1+w_3)-(w_2+w_4) = 0$

$(w_1+w_2) < (w_3+w_4)$



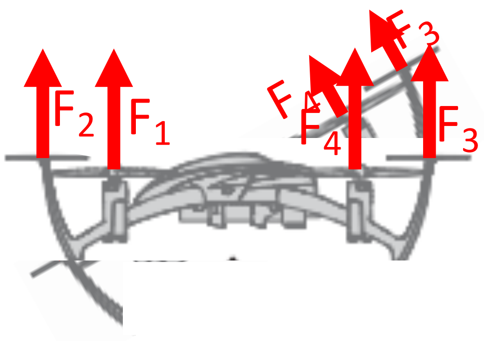
$(w_1+w_2) > (w_3+w_4)$

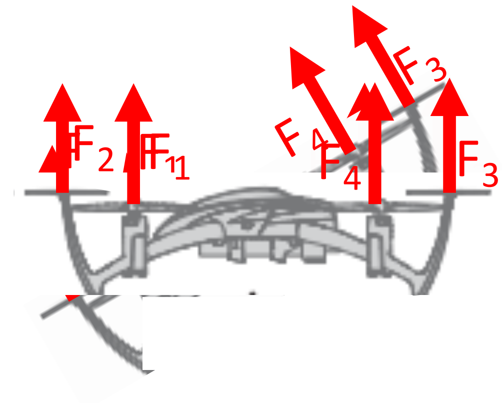


$$\dot{\theta} = k_\theta((w_1+w_2)-(w_3+w_4))$$

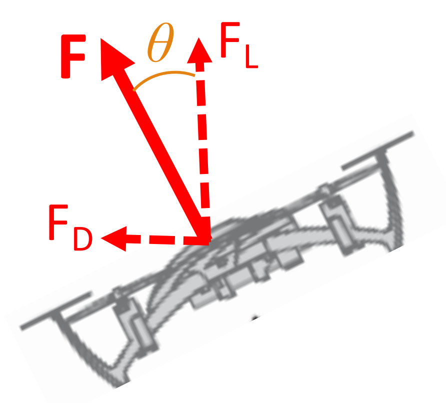
Translated flight

- Example - Moving Forward:





- (Forward, Backward) -> Pitch (Left, right) -> Roll
- At translated flight the total thrust force **F** is decomposed into:
  - Lift force: F_L= ***F*** cos θ
  - Drag force: F_D= ***F*** sin θ
- To avoid diving : ***F*** cos θ = *-mg*
- Translated flight requires more power compared to hovering flight.



Summary of equations

- Assumption: $k = k_\phi = k_\theta = k_\psi$ and $T = \sqrt{F}$

$$\dot{\phi} = k\,((\mathrm{w}_1+\mathrm{w}_4)-(\mathrm{w}_2+\mathrm{w}_3)) \quad = k\,\mathrm{w}_1 - k\mathrm{w}_2 - k\mathrm{w}_3 + k\mathrm{w}_4$$

$$\dot{\theta} = k\,((\mathrm{w}_1+\mathrm{w}_2)-(\mathrm{w}_3+\mathrm{w}_4)) \quad = k\,\mathrm{w}_1 + k\mathrm{w}_2 - k\mathrm{w}_3 - k\mathrm{w}_4$$

$$\dot{\psi} = k\,((\mathrm{w}_2+\mathrm{w}_4)-(\mathrm{w}_1+\mathrm{w}_3)) \quad = -k\,\mathrm{w}_1 + k\mathrm{w}_2 - k\mathrm{w}_3 + k\mathrm{w}_4$$

$$T = k\,((\mathrm{w}_1+\mathrm{w}_3+\mathrm{w}_2+\mathrm{w}_4)) \quad = k\,\mathrm{w}_1 + k\mathrm{w}_2 + k\mathrm{w}_3 + k\mathrm{w}_4$$

- Can also be represented by matrices:

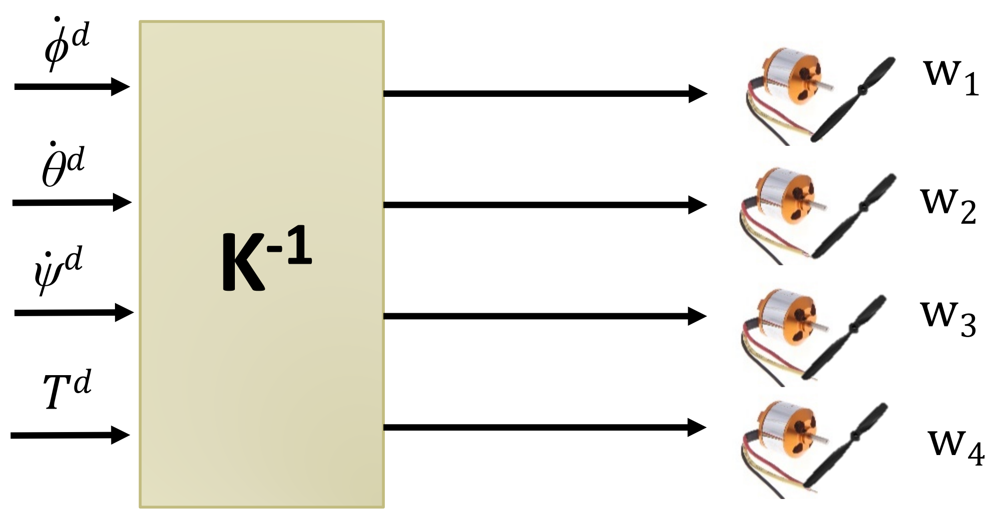
$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ T \end{pmatrix} = \begin{pmatrix} k & -k & -k & k \\ k & k & -k & -k \\ -k & k & -k & k \\ k & k & k & k \end{pmatrix} \begin{pmatrix} \mathrm{w}_1 \\ \mathrm{w}_2 \\ \mathrm{w}_3 \\ \mathrm{w}_4 \end{pmatrix} = \mathbf{K} \begin{pmatrix} \mathrm{w}_1 \\ \mathrm{w}_2 \\ \mathrm{w}_3 \\ \mathrm{w}_4 \end{pmatrix}$$

- These equations give the angular rates/thrust given the motor speeds.

Rotor speeds from angular rates

- But to control the quadrotor we are interested in setting the rotor speeds for obtaining a desired angular rotation.

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix} = \mathbf{K}^{-1} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ T \end{pmatrix} = \begin{pmatrix} k & k & -k & k \\ -k & k & k & k \\ -k & -k & -k & k \\ k & -k & k & k \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ T \end{pmatrix}$$



Demo – Inductrix in action

Name: Blade Inductrix

Width: **83mm**
Height: **28mm**
Weight: **19g**

4. Automatic Controller

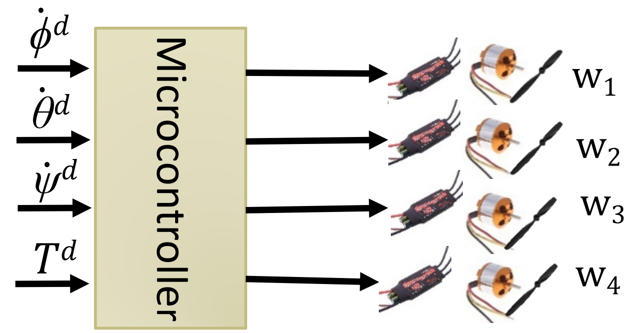
WHY QUADROTORS NEED CONTROLLERS?

HOW TO CONTROL THE ANGULAR RATE OF A QUADROTOR?

Real world is not ideal!

- Can we really set the exact motor speeds?

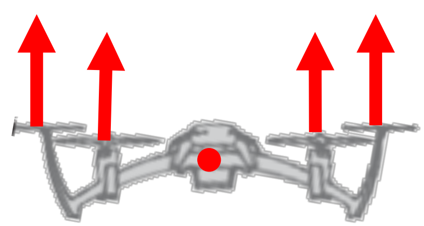
  NO!



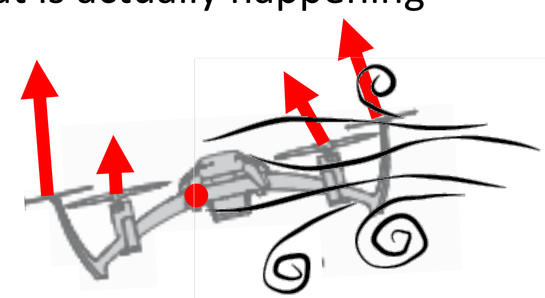
- Problems:
  - Motor drivers are not really the same!
  - Motors are not really identical!
  - Propellers are not really the same!
  - Center of gravity is not exactly in the center!
  - Other disturbances might exist! (i.e. air density, wind)

Example: To hover we give the same inputs to all motors.

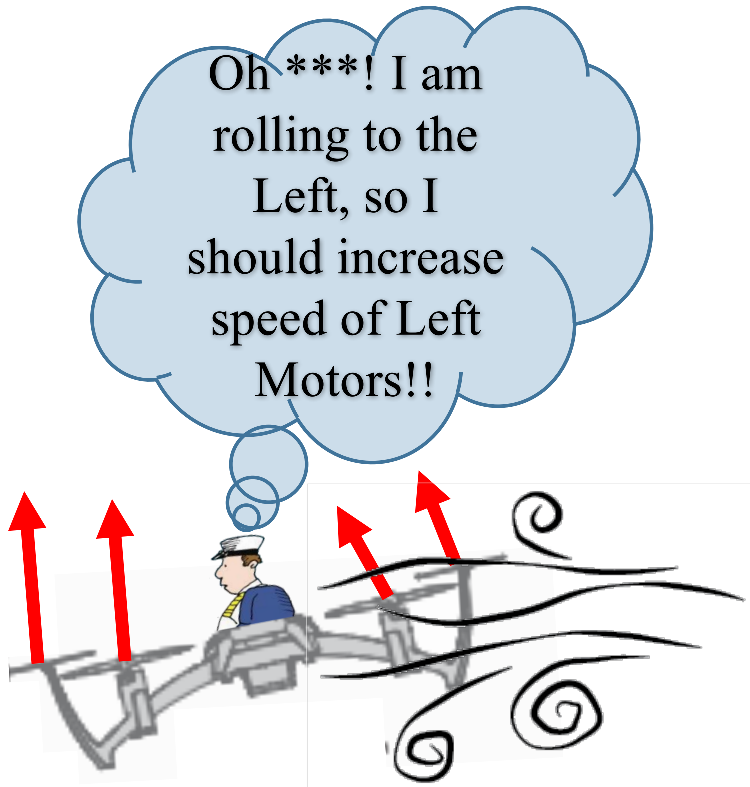
What we think is happening



What is actually happening

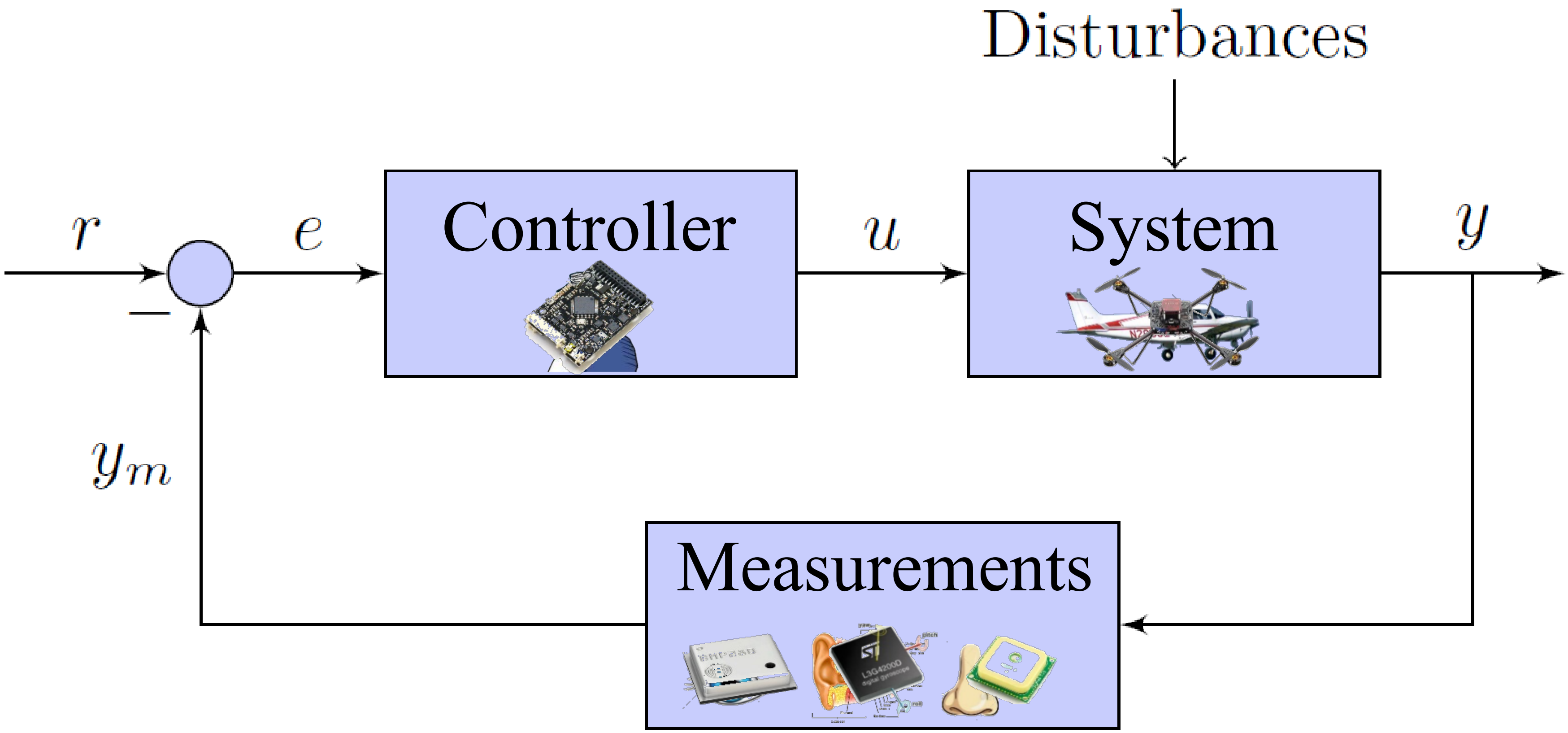


Solution: We need a pilot!



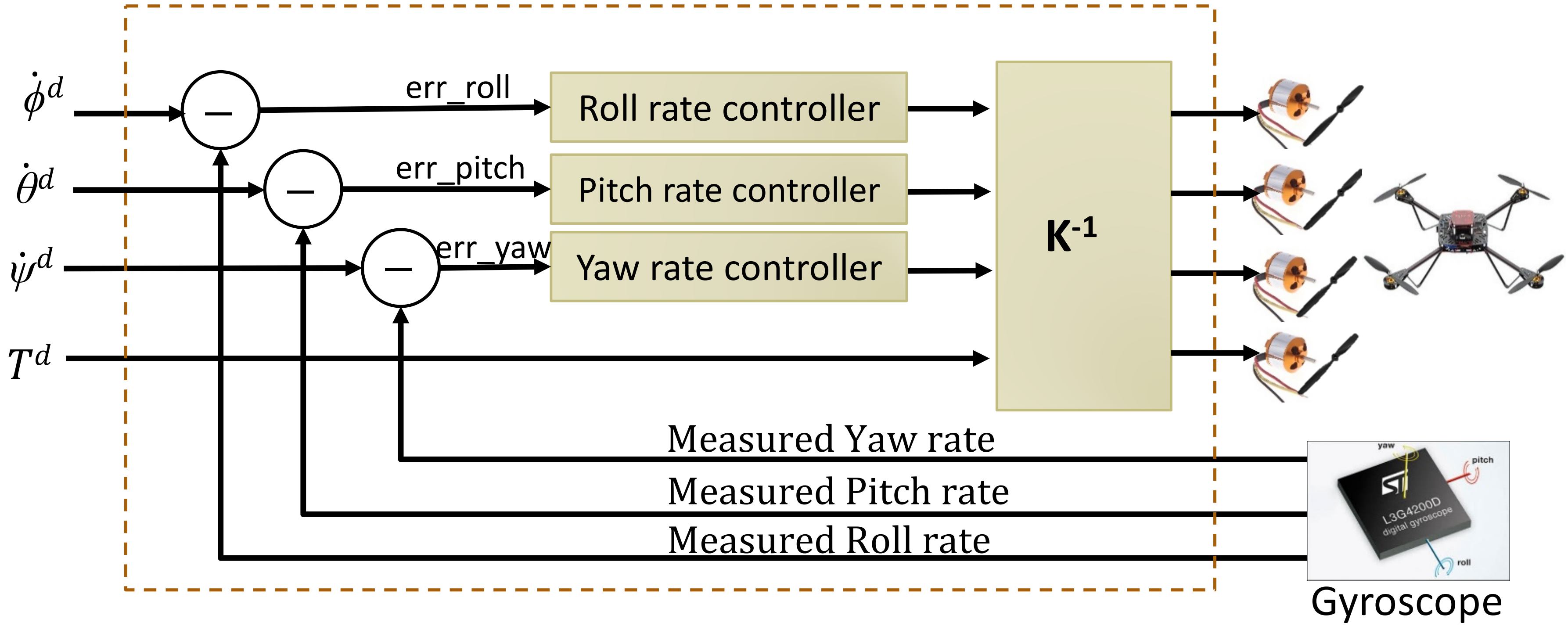
- Feedback is crucial!   (sense and react)
- We cant have a human pilot onboard, but we could design an autopilot.
- Autopilot consists of sensors and automatic controllers.

Closed-loop control system



Rate controllers

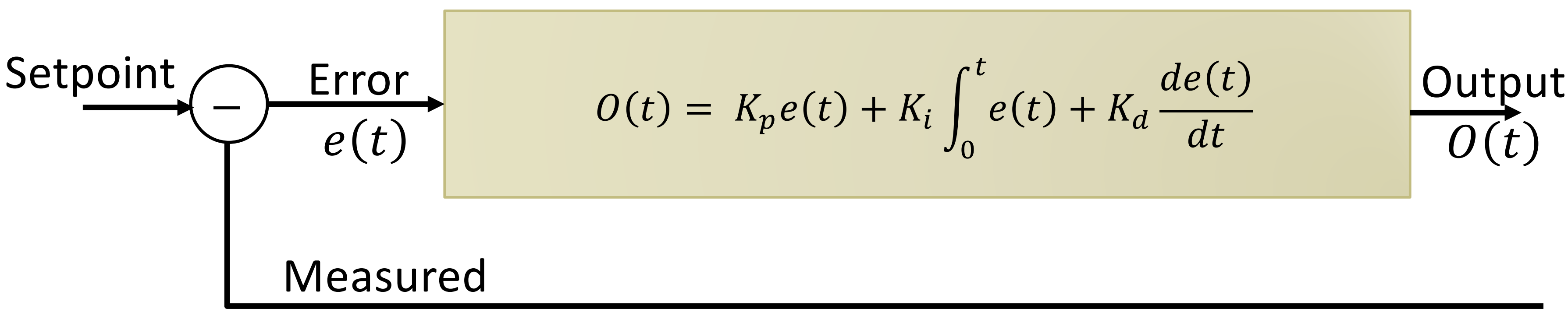
1. Measure the angular rates using a Gyroscope sensor.
2. Compare the measured values with the desired values.
3. Apply corrections to the motor speeds based on the errors.
4. Go to 1



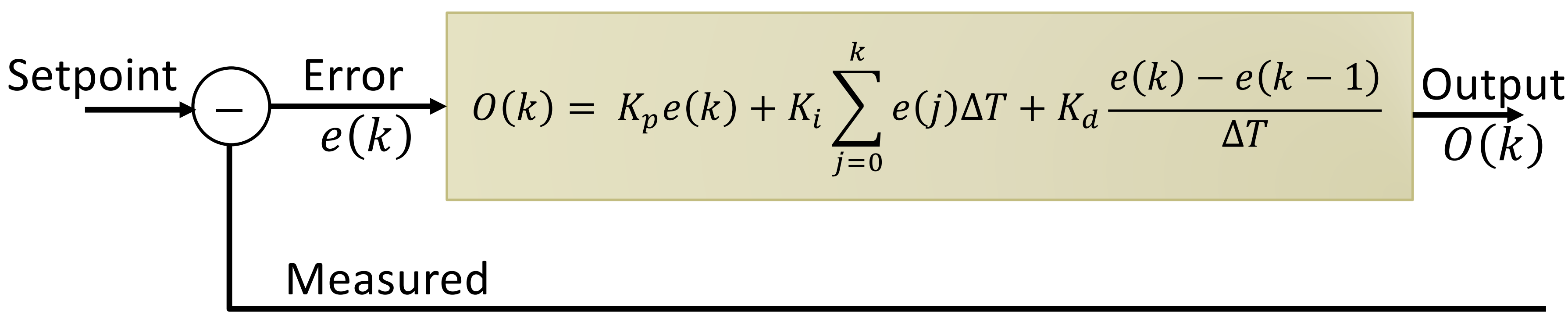
PID Controller

- The most common used controller type is the:
  Proportional Integral Derivative (PID) controller

(PID) controller



(PID) controller (discrete)



PID Controller

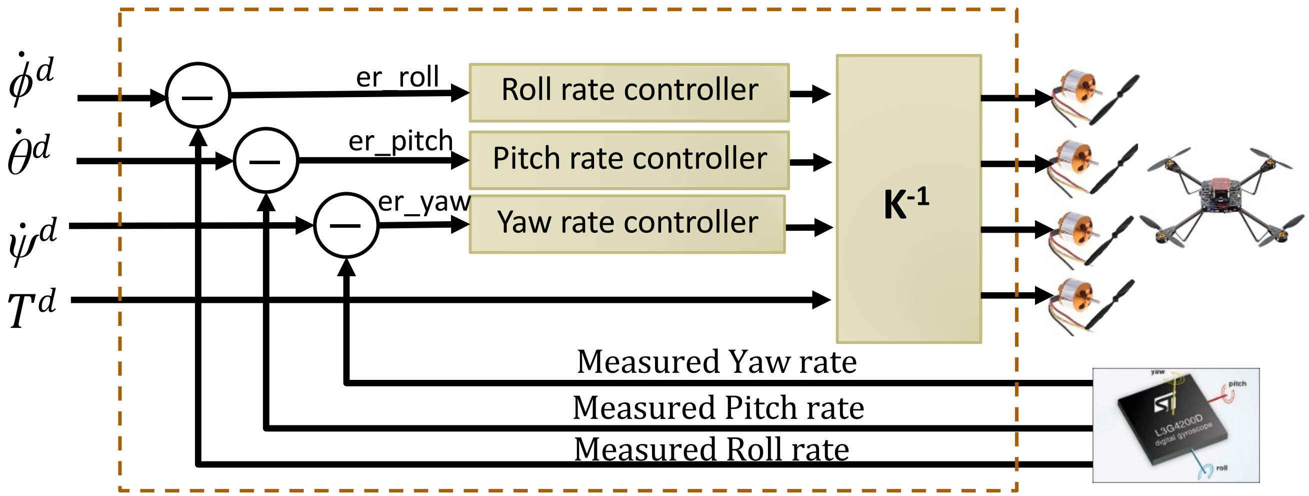
$$O(k) = \underbrace{K_p e(k)}_{\text{Proportional term}} + \underbrace{K_i \sum_{j=0}^{k} e(j)\Delta T}_{\text{Integral term}} + \underbrace{K_d \frac{e(k) - e(k-1)}{\Delta T}}_{\text{Derivative term}}$$

- K_p, K_i, K_d are the gains of the controller and are constants.
- Proportional term produces output proportional to the current error value
- Integral term produces output proportional to both magnitude of the error and the duration of the error
- Derivative term produces output proportional to the rate of change of error over time
- K_p, K_i, K_d are tuned experimentally.

5. Attitude Controller

HOW TO CONTROL THE ATTITUDE OF A QUADROTOR?

Rates are not Angles

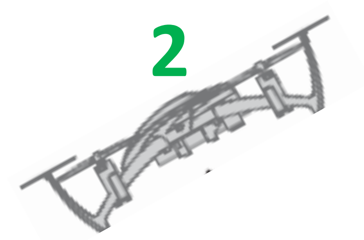


- The above diagram controls the angular rates, but:
- Consider the two cases:
  1. No pitch rotation rate, $\dot{\theta} = 0$, pitch angle of $\theta = 0^o$
  2. No pitch rotation rate, $\dot{\theta} = 0$, pitch angle of $\theta = 20^o$
- Same roll rotation command $\dot{\theta}^d = 0$ results in:
  1. Hovering
  2. Translated flight

We need to control the angles!

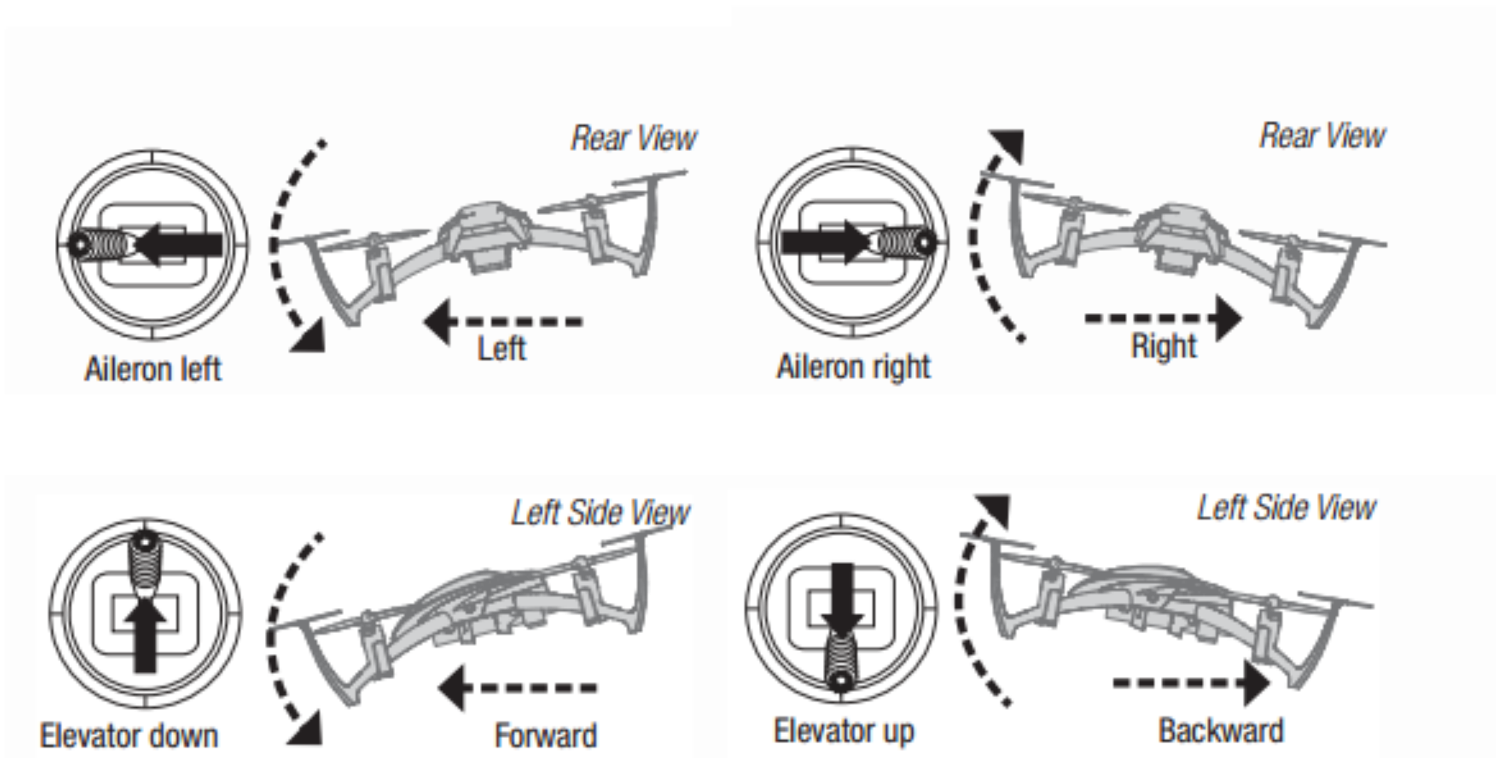
1

2

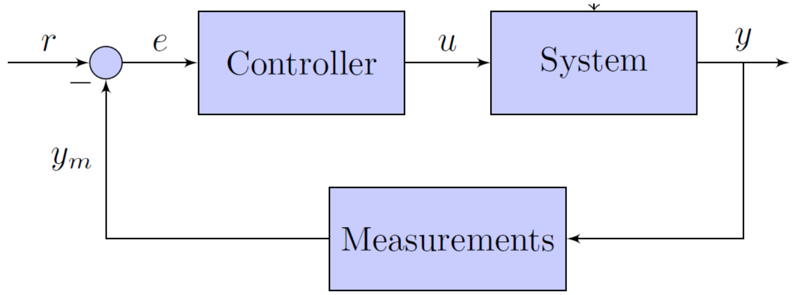


We need to control the quad angles

- No stick movements: $\phi = 0^o$ $\theta = 0^o$
- Change roll angle according to sideway movement of the stick
- Change Pitch angle according to up/down movement of the stick



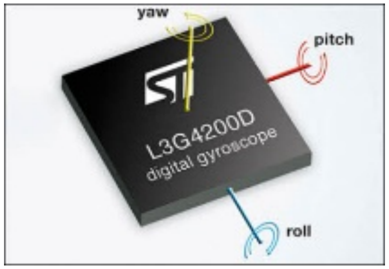
Measuring Angles



- To control the angles we need to first measure the angles (ϕ, θ, ψ) but how?

- Gyroscopes measure angular speeds which can be integrated to obtain the angle:

$$\theta(t) = \int_0^t \dot{\theta}(t)\, dt$$

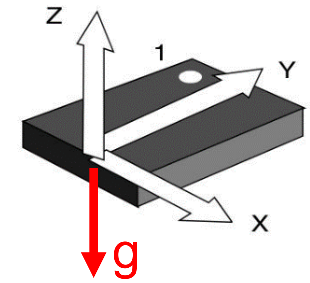


However:

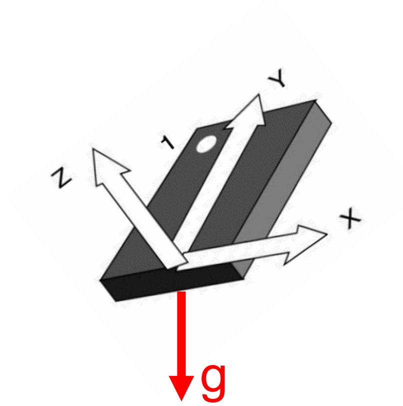
- Gyroscopes are affected by an offset that changes depending on the temperature
- Numerical integration is affected by approximation error
- Measurements are subject to drift

Accelerometer sensor

- Accelerometer sensor measures the acceleration over the three axis (x, y, z)
- If sensor is static the sensed values are the projections of the g vector in the sensor's reference system
- Hence, we can compute roll and pith angles:



$a_z = -9.8 \quad a_y = 0 \quad a_x = 0$



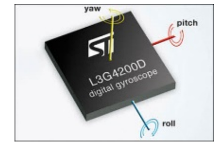
$$\phi = \text{atan}\frac{a_y}{a_z} \qquad \theta = \text{atan}\frac{a_x}{\sqrt{a_y^2 + a_z^2}}$$

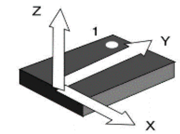
$a_z = -8.2 \quad a_y = -4 \quad a_x = -3.5$

❖ If the sensor is moving or shaking the computed angles are not very reliable!

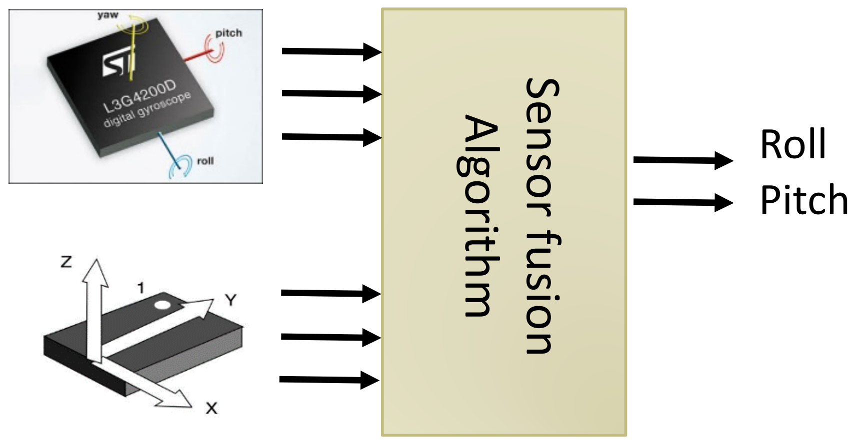
Sensor Fusion

- We have two different source of the same information which are affected by two different error types.





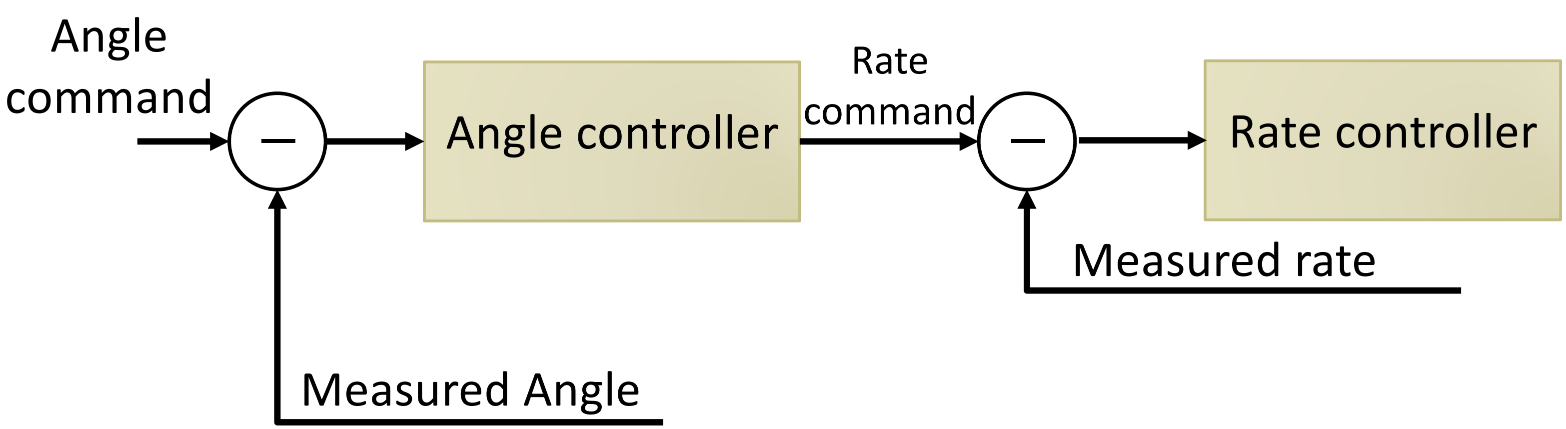
- We can use both measured values and fuse them to obtain a more reliable information:



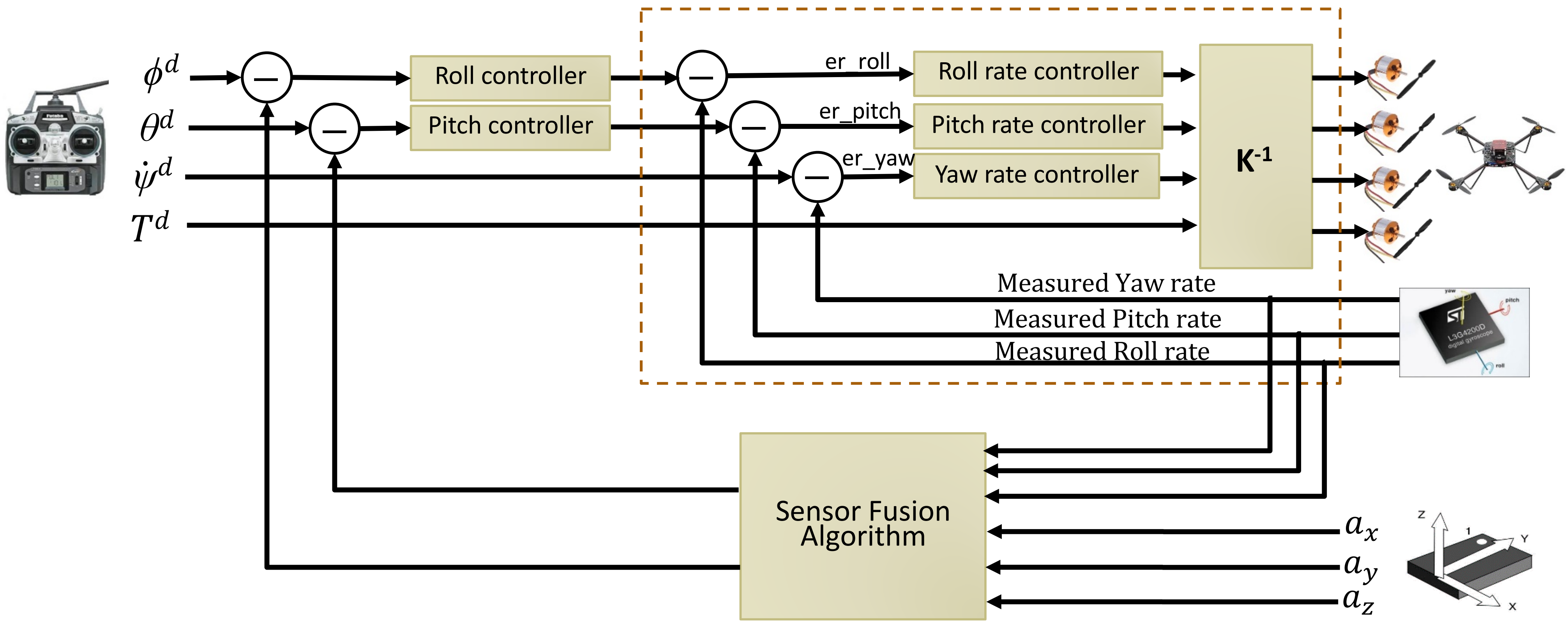
- Fusion techniques: Complementary filters, Kalman Filters, …

Attitude control

- Again, closed-loop control system is used for controlling the attitude
- We set the desired attitude values $(\phi^d, \theta^d, \dot{\psi}^d)$
- Sensor fusion algorithm gives the measured values (ϕ^M, θ^M)
- Error values is sent to PID controllers whose output are the target rates for rate controllers. “ cascading control”



Attitude control schematic



This was how the ladybird quadrotor was able to fly!

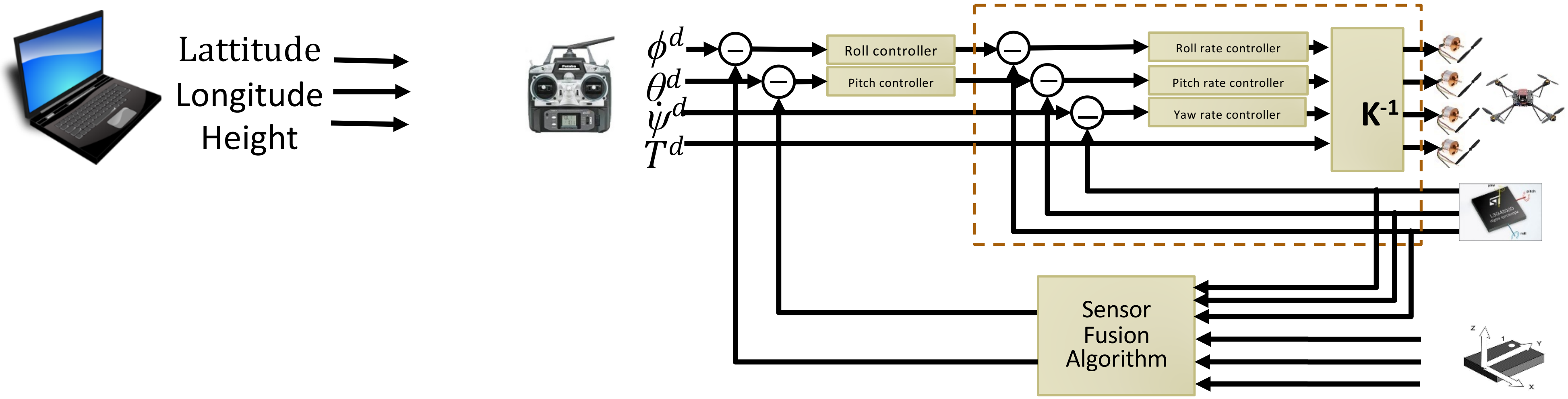
Requirements

- Two control loops:
  - rate control (inner)
  - Attitude control (outer)
- Attitude controller commands the rate controller, so rate controller must have enough time to reach the desired rates.
- Rate control sampling frequency must be higher than the attitude control sampling frequency.
- Example: rate control sampling every 5 ms
  attitude control sampling every 50 ms

6. Altitude/position Control

HOW TO CONTROL ALTITUDE AND POSITION OF A QUADROTOR?

So far:

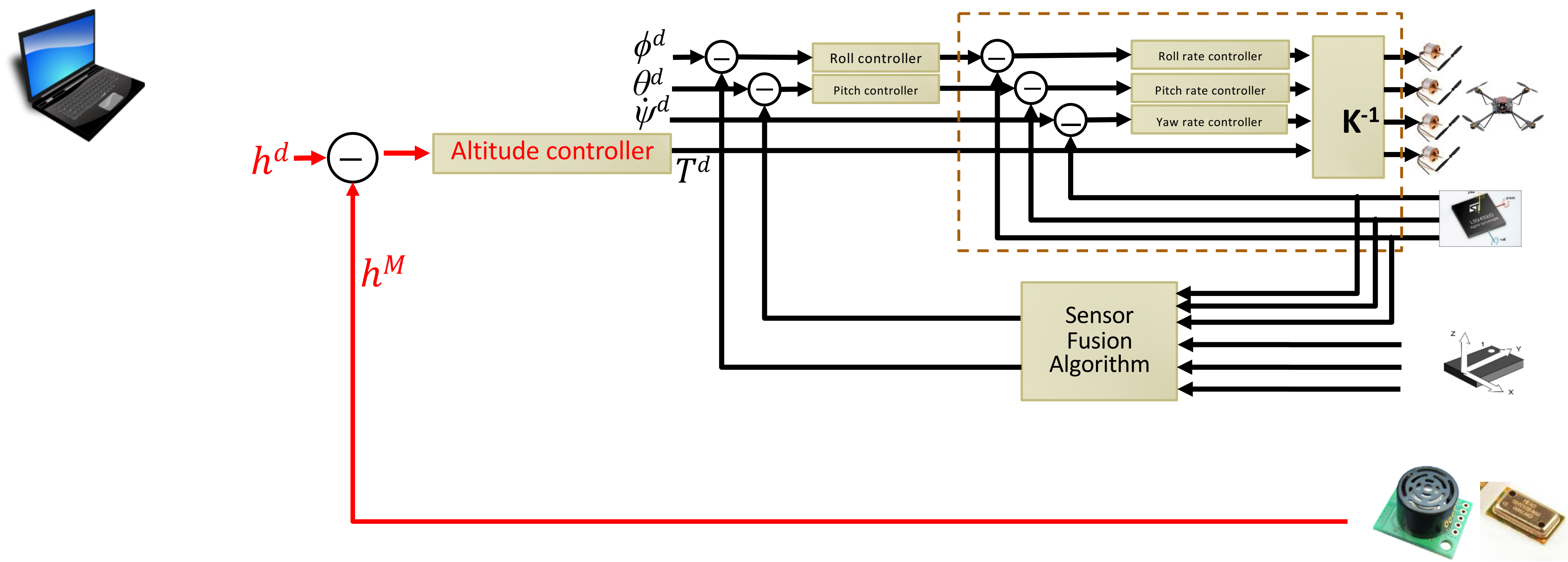


So far we can only set the desired roll, pitch, yaw rate and thrust values. The quadrotor will autonomously follows these desired values.

Can we go one level further and set desired positions for the quadrotor to fly to autonomously?

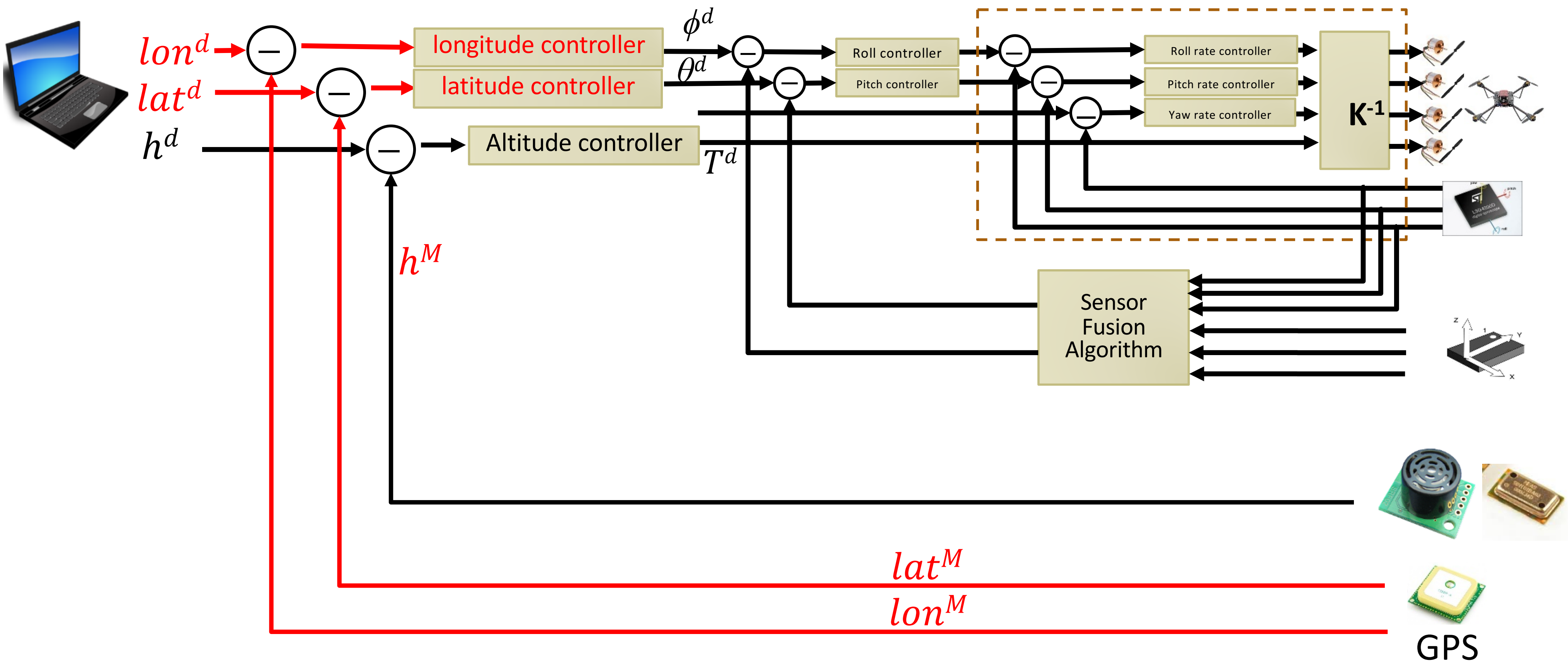
Yes, simply repeat the control technique!

Altitude control



Altitude control: Target: h^d Measured: h^M Output: T^d

Position control



Position control: Target: lat^d, lon^d Measured: lat^M, lon^M Output: ϕ^d , θ^d

Note: A compass and yaw controller could be similarly used to control the heading

Summary of Lecture

1. **Introduction**
   Why build quadrotors?
2. **Structure and Physics**
   How can a quadrotor stay in the air?
3. **Driving principles**
   How to move a quadrotor in the air?
4. **Automatic controller**
   Why quadrotors need controllers?
5. **Attitude control**
   How to control the attitude of a quadrotor?
6. **Altitude/Position control**
   How to control the altitude/position?

Typical Exam Questions

Describe the function and principle of the wing.

Explain the similarities and differences between fixed-wing and rotating-wing flying machines

What are the advantages and applications of Quadrotors?

Why quad-rotors need two different sets of propellers with opposite pitch?

Describe the hovering conditions.

Explain the Euler angles and what they present in quadrotors.

Describe the angular rotations of a quadrotor and explain their relationship with the motor speeds.

Explain the purpose of automatic controllers on quadrotors.

Describe the main components and working principle of a closed loop control system.

What does PID stand for? Explain the task for each of the terms

What sensors are required for attitude control of quadrotors

How can you measure angles from an accelerometer sensor.

What is sensor fusion and why is it required?